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Appendix to the Article "On a Detection Method for Known Finite Sequences"

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Some useful relations for thresholds and detection probabilities are shown for the CFAR quadrature channel detection of finite sequences. Numerical results obtained from the non-central F-distribution are given for some values of false alarm probability, signal/noise ratio, and inversion filter length.

1. INTRODUCTION

In [1], there has been noted that for the quadrature channel detection scheme, the method described represents a generalization of the Siebert CFAR detector [2].

In this Appendix some useful relations will be given and, applying the formulas of [3], some numerical results of computing the detection probabilities will be shown.

2. SOME FORMULAS

For the quadrature channel detection, there may be shown easily that the expression (8) of [1] is to be replaced by

$$0 \leqq \frac{|C_T|^2}{\sum_{i=0}^{N+h} |C_i|^2} \leqq 1\,, \tag{1}$$

where $\{C_n\}$ is the complex output sequence (signal plus noise) of the inversion filter. There is

$$|C_n|^2 = C_{n(1)}^2 + C_{n(2)}^2\,, \tag{2}$$

where $C_{n(1)}$ and $C_{n(2)}$ are the respective quadrature components (real and imaginary parts).

For "good" finite sequences, there has been stated in [1] that (now with complex terms) the correlation coefficients

$$(3) \qquad \varrho(c_i, c_j) = \frac{\bar{a}_0 a_{j-i} + \bar{a}_1 a_{j-i+1} + \ldots + \bar{a}_{N-j+i} a_N}{\sum_{i=0}^{N} |a_i|^2}$$

are $\ll 1$ for $i \neq j$, $\{a_i\}$ $(i = 0, \ldots, N)$ being the weighting sequence of the inversion filter. With this supposition, the numerator in (1) possesses (for the noise alone, supposing that it is Gaussian) approximately the χ^2 distribution with 2 degrees of freedom and the noncentral χ^2 with 2 degrees of freedom for the signal plus noise input. Analogous conclusions are true also for the denominator of (1).

For a prescribed false alarm probability P_{fa}, the threshold Z for (1) will be computed from

$$(4) \qquad (1 - Z)^{N+h} = P_{fa} ,$$

where $h + 1$ is the number of terms of the transmitted sequence.

For $N + h \gg 1$, there is approximately

$$(5) \qquad Z = (2{,}30 \log (1/P_{fa}))/(N + h) ,$$

where log is the common logarithm.

Further, the formulas of [3] may be used substituting therein

$$(6) \qquad a = 1 , \quad b = N + h , \quad \gamma = (S/N)_o , \quad x = Z ,$$

where $(S/N)_o$ is the signal/noise ratio at the output of the inversion filter. One gets (with (4))

$$(7) \qquad P(Z) = P_{fa} \,.\, bx \,.\, \mathrm{e}^{-\gamma} \,.\, \Phi(b, x, \gamma) ,$$

where

$$(8) \quad \Phi(b, x, \gamma) = 1 + \frac{1+b}{2} x \left(1 + \frac{\gamma}{1!}\right) + \frac{1+b}{2} \,.\, \frac{2+b}{3} x^2 \left(1 + \frac{\gamma}{1!} + \frac{\gamma^2}{2!}\right) + \ldots .$$

Finally, the detection probability is

$$(9) \qquad P_{da} = 1 - P(Z) .$$

The signal/noise ratio at the input of the inversion filter is

$$(10) \qquad (S/N)_i = \gamma \,.\, \frac{\sum_{i=0}^{N} |a_i|^2}{c_T^2} .$$

There is seen by inspection of (5), (7), (8) that for $N + h \gg 1$, there holds approximately

$$P_{da} = f(P_{fa}, \gamma)\,, \tag{11}$$

that is P_{da} is function of P_{fa} and $(S/N)_o$ only.

3. SOME NUMERICAL RESULTS

Often, about 10 detection results are used in a second threshold detector, and for definite decision, $P_f \doteq 10^{-6}$, $P_d \doteq 0,9$ are used. The corresponding P_{fa} and P_{da} are about 0,01 and 0,6–0,8.

In what follows, some numerical results computed from (7) will be shown.

Tab. 1.

$P_{fa} = 0{\cdot}005$				
$N + h$	Z	$(S/N)_o$ (dB)		
		6	8	10
10	0·41	0·27	0·50	0·78
20	0·23	0·33	0·59	0·86
30	0·16	0·35	0·62	0·88
40	0·12	0·36	0·63	0·89
50	0·10	0·37	0·64	0·89
100	0·052	0·38	0·66	0·90
200	0·026	0·39	0·66	0·91
300	0·018	0·39	0·67	0·91

Tab. 2.

$P_{fa} = 0{\cdot}02$				
$N + h$	Z	$(S/N)_o$ (dB)		
		6	8	10
10	0·32	0·48	0·72	0·92
20	0·18	0·53	0·77	0·95
30	0·12	0·55	0·79	0·95
40	0·093	0·56	0·80	0·96
50	0·075	0·56	0·80	0·96
100	0·038	0·57	0·81	0·96
200	0·019	0·58	0·82	0·96
300	0·013	0·58	0·82	0·96

In both tables, the validity of (5), (11) may be checked.

4. CONCLUDING REMARKS

For $P_{fa} = 0{\cdot}01$ and $0{\cdot}05$, results analogous to those of preceding tables may be obtained also from charts of the noncentral F-distribution [4], [5]. It is to be noted that "Φ" of these charts is defined as follows

$$\text{“}\Phi\text{”} = \sqrt{\left(\frac{2}{3}\gamma\right)} \tag{12}$$

(γ from (6)).

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