Roman Bek; Milan Růžička
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On Modelling of Large Variable Systems of Higher Degree by Means of Language Systems

ROMAN BEK, MILAN ROZIČKA

There will be presented a few non-traditional definitions of several basic notions from a theory of complicated time variable systems modelling. In particular, a special concern is devoted to a relation between accurate language systems and real (ontic) ones.

1. PRELIMINARIES

The formulations in this article will be based on the set theory concept and principal terms of this theory will be used accordingly. Especially the following ones:

- Cartesian product (symbolically: \( M_1 \times M_2 \) for sets \( M_1, M_2 \)),
- Cartesian power (symbolically: \( M^{\times n} \) the \( n \)-th power of a set \( M \)),
- ordered set (symbolically: \( \langle a_1, \ldots, a_n \rangle \), ordered set consisting of elements \( a_1, \ldots, a_n \)),
- elementhood of a set (symbolically: \( a \in M \)),
- intersection of sets (symbolically: \( M_1 \cap M_2 \) for sets \( M_1, M_2 \)),
- subset (symbolically: \( M_1 \subseteq M_2 \) for sets \( M_1, M_2 \)).

Further we assume the same distribution of ontic reality as accepted in the logical theory of types: individuals of the universe of a language reasoning we consider as entities of the zero degree, their sets and relations as entities of the first degree, sets of these sets and relations and relations among these sets and relations as entities of the second degree, \ldots, sets and relations of entities of the \((s - 1)\)-th degree as entities of the \(s\)-th degree.

The language of language systems we shall discuss is built-up on the predicate-logical basis. Its predicate constants and variables resp. will be denoted by corresponding type symbols again within the requirements of the types theory. For instance: a predicate constant of the type \( 1^R \) denotes an ontic entity of the \(s\)-th degree. This
language will also use time terms, constants and variables. These terms will join individual terms of the language. Statement: \( R^O(a_1, \ldots, a_j, t_i) \) will for example mean: "Objects \( a_1, \ldots, a_j \) are at a moment \( t_i \) in relation \( R^O \).

We shall further suppose existence of a time structure containing a set of moments \( \tau \). Let this set be ordered by a time relation "forecoming"; i.e. the precedence relation. Individual time intervals are ordered subsets of the set \( \tau \). Suppose, to respective moments from the set \( \tau \) have been assigned real numbers by a convenient metric function. To the precedence relation corresponds then a relation "\(<" between these numbers.

2. DEFINITION OF THE NOTION "SYSTEM OF THE \( n \)-TH DEGREE".

\[ \text{D1.} \] An ordered pair \( (U, \mathcal{R}) \) forms at a time interval \( \Delta t \) system of the \( n \)-th degree (symbolically: \( (U, \mathcal{R}) \in \mathcal{P}_{g^S(\mathcal{O})} \)) iff:

- \( U \) is a set of some objects (= individuals = entities of zero degree),
- \( \mathcal{R} \) is a set of all entities of the form \( (s)^{R(k,j)} \), where for \( s = 1 \): \( (s)^{R(k,j)} \in U^{\times s^S} \times \Delta t \) for every \( s > 1 \): there is at least one from \( s_1, \ldots, s - 1 \), where \( s - 1 \) exists, hence

\[
(s)^{R(k,j)} \in U^{\times s^S} \times \{ (s_1)^{R(k_1,j_1)} \times \cdots \times (s_m)^{R(k_m,j_m)} \} \times \Delta t,
\]

\( s_1, s_2, \ldots, s^{-1} \) may or may not be equal in pairs, all \( s_i \) from them which are not equal to \( s - 1 \) are less than \( s - 1 \).

- \( j = j_0 + j_{11} + \ldots + j_{1k} + j_{11} + \ldots + j_{10} + \ldots + j_{11} + \ldots + j_{1k} \),

the greatest \( s \) (from upper left index of these entities \( (s)^{R(k,j)} \in \mathcal{R} \)) is equal to \( n \).

The set \( U \) is said to be "the universe of the system".

Remark. \( (s)^{R(k,j)} \in \mathcal{R} \) is \( (j + 1) \)-th terms relation (on its last place is always time term) the \( s \)-th degree among individuals from the universe of a system and/or other relations from \( \mathcal{R} \) of the lower degree than \( s \). Among them there must be however at least one relation of the \( (s - 1) \)-th degree. Denotation \( k \) in the lower index marks an order of relation \( (s)^{R(k,j)} \) among other \( (j + 1) \)-terms relations of the \( s \)-th degree belonging to \( \mathcal{R} \).

Due to the logical principle of extensionality as it is known, we consider \( j \)-terms relations and sets of ordered \( j \)-tuples objects among those the given relations are defined, as identical ones. For \( j = 1 \) the relation results to a set of objects having corresponding property.

Proposed definition generalizes the notion "system", as it is commonly used. System is usually defined as a pair from the set of objects (the universe of system)
and from a set of relations among these objects. In the suggested definition, system
is specified with respect to a given time interval (within its limits given objects with
the corresponding relations are considered). Further we admit that in the set of relations
there may be not only relations among individuals from the universe of system, but
also relations among relations of individuals, relations of relations among relations
of individual etc. We hope that by doing so we may satisfy needs of the system theory
operating with large variable systems and considering there entities of various degrees.

Examples

P 1. A system of the first degree is on an interval \( \Delta t = \{t_1, t_2, t_3\} \) an ordered pair
\( \langle U, R \rangle \), where:

\[
U = \{a_1, a_2, a_3, a_4\},
\]

\[
R = \{(1)R_1^{(1)}, (1)R_2^{(1)}, (1)R_3^{(1)}\},
\]

and

\[
(1)R_1^{(1)} = \{\langle a_1, t_1 \rangle, \langle a_1, t_2 \rangle, \langle a_1, t_3 \rangle, \langle a_2, t_2 \rangle\},
\]

\[
(1)R_2^{(1)} = \{\langle a_2, t_2 \rangle, \langle a_2, t_3 \rangle, \langle a_3, t_1 \rangle, \langle a_3, t_2 \rangle, \langle a_3, t_3 \rangle, \langle a_1, t_2 \rangle\},
\]

\[
(1)R_3^{(1)} = \{\langle a_1, a_2, t_1 \rangle, \langle a_3, a_2, t_2 \rangle, \langle a_1, a_4, t_2 \rangle, \langle a_1, a_6, t_2 \rangle\}.
\]

Object \( a_1 \) hence belongs to a set \( (1)R_1^{(1)} \) (= has property \( (1)R_1^{(1)} \)) during the whole
period \( \Delta t \), whereas an object \( a_2 \) belongs here (= taking on this property) later
on at a moment \( t_2 \). Obviously: \( (1)R_1^{(1)} \subseteq U^{\times 1} \times \Delta t \), similarly \( (1)R_2^{(1)} \). Objects
\( a_1, a_2 \) are connected by a relation \( (1)R_1^{(1)} \) at a moment \( t_1 \), objects \( a_2, a_3 \) at a moment
\( t_2 \), objects \( a_1, a_4 \) at moments \( t_2, t_3 \). Obviously: \( (1)R_3^{(1)} \subseteq U^{\times 1} \times \Delta t \).

P 2. Let an ordered pair \( \langle U_0, R_0 \rangle = \mathcal{S}_{\phi_0} \) be ontic system of the third degree
on interval \( \Delta t_4 = \{t_1, t_2, t_3, t_4\} \), where the universe

\[
U_0 = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\},
\]

set of relations

\[
R_0 = \{(1)R_1^{(3)}, (1)R_2^{(3)}, (1)R_3^{(3)}, (1)R_4^{(3)}, (1)R_5^{(3)}, (1)R_6^{(3)}, (2)R_1^{(3)}, (3)R_1^{(4)}\},
\]

identification of relations:

\[
(1)R_1^{(3)} = \{\langle a_1, t_1 \rangle, \langle a_1, t_2 \rangle, \langle a_1, t_3 \rangle, \langle a_1, t_4 \rangle, \langle a_2, t_3 \rangle, \langle a_2, t_4 \rangle\},
\]

(continuing)

\[
J_n \subseteq U_n \times \Delta t_n.
\]
(obviously: \((1)R_1^{(2)} \subseteq U_0^{(2)} \times M_1\),
\[\begin{align*}
(1)R_2^{(2)} &= \{(a_1, a_5, t_1), (a_1, a_6, t_1), (a_1, a_9, t_2), (a_2, a_4, t_3)\}, \\
(2)R_1^{(1)} &= \{(1)R_1^{(1)}, (1)R_1^{(1)}, (1)R_1^{(1)}(1), (1)R_1^{(1)}, (1)R_1^{(1)}, (1)R_1^{(1)}, (1)R_1^{(1)}, t_2\}
\end{align*}\]

Relation \((1)R_1^{(1)}\) has for the whole period of time \((2)R_1^{(1)}\); obviously:
\((2)R_1^{(1)} \in \{(1)R_1^{(1)} \times M_1\}\)

Relation \((1)R_2^{(2)}\) is together with relation \((2)R_1^{(1)}\), \((1)R_3^{(1)}\) and object \(a_5\) at moment \(t_3\) in relation \((1)R_4^{(1)}\); obviously:
\((3)R_4^{(1)} \subseteq U_0^{(1)} \times \{(1)R_1^{(1)}, (1)R_1^{(1)}\} \times \{(1)R_1^{(1)} \times M_1\}\)

\[P\ 3.\ \text{Ontic system } S_{O_2}\ \text{of the second degree is on an interval } M_2 = \langle t_1, t_2, t_4, t_5 \rangle\]

set of relations
\[S_{O_2} = \{(1)R_1^{(1)}, (1)R_2^{(1)}, (1)R_2^{(1)}, (1)R_2^{(1)}, (1)R_1^{(1)}, (2)R_1^{(1)}\}\]

identification of relations
\[\begin{align*}
(1)R_1^{(1)} &= \{(a_1, t_1), (a_2, t_4)\}, \\
(1)R_2^{(1)} &= \{(a_2, t_4)\}, \\
(1)R_3^{(1)} &= \{(a_7, a_9, t_4), (a_9, a_7, t_4)\}, \\
(2)R_1^{(1)} &= \{(1)R_1^{(1)}, t_1\}, (1)R_1^{(1)}, t_2)\}
\end{align*}\]

3. DEFINITION OF NOTION "SUBSYSTEM"

D 2. System \(S_{O_2}\) is at a time interval \(M_2\) a subsystem of system \(S_{O_1}\) (symbolically:
\((S_{O_2}, S_{O_1}, M_2) \in \mathcal{D}_{o_o}a_{o_d}\) if there exists \(M_1\) so that \(M_2 \subseteq M_1\),
\((S_{O_1}, M_1) \in \mathcal{D}_{o_o}a_{o_d}\), \((S_{O_2}, M_2) \in \mathcal{D}_{o_o}a_{o_d}\),
there are sets \(U_1, U_2, R_1, R_2\) so that
\[S_{O_1} = \langle U_1, R_1 \rangle, \quad S_{O_2} = \langle U_2, R_2 \rangle,\]
there exists a mapping \(\mathcal{D}_{o_o}\) transforming from set of all moments into the same set so that to every moment \(t \in M_1\) located in the field of given relation \((a)R_{a o} \in S_{O_1} (s_1 \leq s_1)\) uniquely assigning as its image again an element from field of this relation.
There is a mapping \( \mathcal{Z}_0 \) transforming from the universe \( U_1 \) into itself so that to every element \( x \in U_1 \) from the field of a given relation \( (s)^{(t)}R^{(j)} \in \mathcal{R}_1 \) uniquely is assigning as its image again an element from the field of this relation. For every \( s_0 \leq t \leq s_1 \), there further exists a couple of mappings \( \mathcal{Z}_{s_0} \), \( \mathcal{Z}_{s_1} \), \( \mathcal{Z}_{s_0} \) transforms from set of all elements from the field of given relation \( (s)^{(t)}R^{(j)} \in \mathcal{R}_1 \) (apart from elements from set \( \mathcal{M}_1 \) and \( U_1 \)) into the same set using this way: to every object it assigns uniquely as its image again some object of the same degree from the field of this relation. This mapping regards all objects from fields of all relations belonging to \( s_t \). For \( s_t = 1 \), \( \mathcal{Z}_{s_1} \) is identical with mapping \( \mathcal{Z}_0 \), \( \mathcal{Z}_{s_t} \) maps from set of all relations \( s_t \)-th degree belonging to \( \mathcal{R}_1 \), into the set of all its subrelations like this: to every relation of the type \( (s)^{(t)}R^{(j)} \in \mathcal{R}_1 \) uniquely assigns as its image some of its subrelation — including empty subrelation. For each of these mapping, following condition holds: to no object which is being mapped are by two distinct mappings associated two different images. If there is assigned to \( s_a \) as its image \( s_a \) by some mapping, then it must not appear again as image of some object by any mapping and the given object and its image must be identical (i.e. it must be image of itself).

Mentioned mappings \( \mathcal{Z}_{s_t}(\mathcal{Z}_{s_1}, \ldots, (\mathcal{Z}_{s_1}, \ldots, (\mathcal{Z}_1(\mathcal{Z}_0) \ldots)) \ldots)) \) associate with individual relations of the type \( (s)^{(t)}R^{(j)} \in \mathcal{R}_1 \) uniquely relations of the type \( (s)^{(t)}R^{(j)} \in \mathcal{R}_2 \). The universe \( U_2 \) is set of images of mapping \( \mathcal{Z}_0 \), \( \mathcal{M}_2 \) is ordered set of images of \( \mathcal{Z}_0 \).

**Remark.** Mentioned definition of notion “subsystem” is more general from a few viewpoints than usual specification. First there is an introduction and transformation of moments from given time structure (given by mapping \( \mathcal{Z}_1 \)). Result of mapping is a time interval of the existence of given system \( (\mathcal{S}_2) \) as a subsystem of other system \( (\mathcal{S}_1) \). This mapping has been enclosed into the definition for a better correspondence with intuitive imaginations related with construction of subsystem in epistemic process over given system under investigation. We often cannot distinguish two different moments located near to each other because of very limited technical equipment when studying a variable system. In practice we make these moments identical. We consider such a (defined) system as a subsystem of studied system.

**Example.** Let a system \( \mathcal{S}_1 \) be defined on an time interval \( \mathcal{M}_1 = \langle t_1, t_2, t_3, t_4, t_5, t_6 \rangle \). Let mapping \( \mathcal{Z}_1 \) associate:

\[
\begin{align*}
t_1 &\rightarrow t_1, \\
t_2 &\rightarrow t_2, \\
t_3 &\rightarrow t_3, \\
t_4 &\rightarrow t_4, \\
t_5 &\rightarrow t_5, \\
t_6 &\rightarrow t_6.
\end{align*}
\]
Interval $\mathcal{I}_2 = \langle t_1, t_2, t_4, t_6 \rangle$ can be hence a period of existence of some system $\mathcal{S}_2$ as a subsystem $\mathcal{S}_1$ (if the other conditions of definitions are met).

Secondly, there is an analogous transformation of elements from the universe $U_1$ of system $\mathcal{S}_1$ given by mapping $\mathcal{X}_0$. There are similar reasons as in the previous case. Frequently we cannot identify two distinct elements from the universe of studied system and therefore we make them identical. Sometimes we do such a simplification even in a case we can distinguish elements under consideration.

**Example.** Let the universe $U_1$ of a system $\mathcal{S}_1$ be formed by a set of elements $\{a_1, a_2, a_3, a_4, a_5\}$. Let mapping $\mathcal{X}_0$ assign:

$$
\begin{align*}
  a_1 & \rightarrow a_3, \\
  a_2 & \rightarrow a_2, \\
  a_3 & \rightarrow a_3, \\
  a_4 & \rightarrow a_3, \\
  a_5 & \rightarrow a_2.
\end{align*}
$$

The set $U_2 = \{a_2, a_3\}$ forms the universe of system $\mathcal{S}_2$ which is a subsystem $\mathcal{S}_1$, so far the other conditions of definition are satisfied.

It is also obvious that due to specification of $\mathcal{X}_0$ always holds: if $\mathcal{S}_2$ is a subsystem of $\mathcal{S}_1$, then the universe $U_2$ of system $\mathcal{S}_2$ is subset of the universe $U_1$ of system $\mathcal{S}_1$. This statement is a part of traditional specification of notion “subsystem”.

Thirdly, there are analogous transformations of those objects from relations of given system which are relations themselves (of lower degrees). For every degree of these relations there is a mapping (of a form $\mathcal{X}_{n-1}$), having similar function as the transformation $\mathcal{X}_0$.

**Example.** Let a system $\mathcal{S}_1$ have the universe $U_1$ as in the previous example and his set of relations be

$$
\mathcal{R}_1 = \{(1)R_1^{(1)}, (1)R_2^{(1)}, (2)R_1^{(1)}, (2)R_2^{(2)}\}.
$$

(Let relations $(1)R_1^{(1)}, (1)R_2^{(1)}$ belong to fields of relations $(2)R_1^{(1)}, (2)R_1^{(2)}$). Let mapping $\mathcal{X}_{2-1}$ assign:

$$
\begin{align*}
  (1)R_1^{(1)} & \rightarrow (1)R_1^{(1)}, \\
  (1)R_2^{(1)} & \rightarrow (1)R_1^{(1)}.
\end{align*}
$$

Let the unique image $(1)R_1^{(1)}$ belong to set $\mathcal{R}_2$ of system $\mathcal{S}_2$. If $\mathcal{S}_2$ satisfies all other conditions of definition, then $\mathcal{S}_2$ is subsystem of $\mathcal{S}_1$.

Further there is a mapping $\mathcal{X}_n$, defined for every degree (greater than zero) of relation from set $\mathcal{R}_n$ of system $\mathcal{S}_1$. In epistemic process over the system $\mathcal{S}_1$ we can meet (or only specify) merely parts of relations, some relations even we vanish completely.
Example. Let relations of system \( \mathcal{S}_1 \) from recently mentioned three examples be:

\[
(1) R_1^{(1)} = \{ \langle a_1, t_2 \rangle, \langle a_1, t_3 \rangle, \langle a_2, t_3 \rangle, \langle a_3, t_4 \rangle \} ,
\]

\[
(1) R_2^{(1)} = \{ \langle a_1, t_2 \rangle, \langle a_3, t_4 \rangle, \langle a_3, t_2 \rangle, \langle a_4, t_5 \rangle, \langle a_5, t_6 \rangle \} ,
\]

\[
(2) R_1^{(1)} = \{ \langle t_1, t_2 \rangle, \langle t_1, t_3 \rangle, \langle t_2, t_3 \rangle, \langle t_4, t_5 \rangle, \langle t_5, t_6 \rangle \} .
\]

Let mapping \( \mathcal{Z}_1 \) assign to relation \( (1) R_1^{(1)} \) its image \( (1) R_1^{(1)} = \{ \langle a_1, t_2 \rangle, \langle a_3, t_4 \rangle \} \) (relation \( (1) R_1^{(1)} \) has been already eliminated by mapping \( \mathcal{Z}_{2-1} \) to \( (1) R_1^{(1)} \)). Let mapping \( \mathcal{Z}_2 \) assign to relation \( (2) R_2^{(1)} \) its image \( (2) R_2^{(1)} = \{ \langle t_1, t_2 \rangle, \langle t_2, t_3 \rangle, \langle t_4, t_5 \rangle, \langle t_5, t_6 \rangle \} \) to relation \( (2) R_1^{(1)} \) its image \( (2) R_1^{(1)} = \emptyset \) (empty subrelation, i.e. empty set of ordered triples). (Mapping \( \mathcal{Z}_{2-1} \) is obviously not identical with mapping \( \mathcal{Z}_1 \)). It is clear that by simplification of specification for mapping \( \mathcal{Z}_2 \), this in this way: \( \mathcal{Z}_2 \) assigns to every relation of the \( s \)-th degree, belonging to \( \mathcal{R}_1 \) of system \( \mathcal{S}_1 \), the same relation or empty subrelation. Together with simplification \( \mathcal{Z}_0 \) to identical transformation, for set \( \mathcal{R}_3 \) of subsystem \( \mathcal{S}_2 \) will hold: \( \mathcal{R}_2 = \mathcal{R}_1 \). This statement forms the second part of traditional definition of notion "subsystem".

Example. Let system \( \mathcal{S}_2 \) be given on interval \( \mathcal{M}_2 = \langle t_1, t_2, t_3, t_4, t_6 \rangle \), \( \mathcal{U}_2 = \{ a_2, a_3 \} \), \( \mathcal{R}_2 = \{ (1) R_1^{(1)} , (2) R_1^{(1)} \} \), where

\[
(1) R_1^{(1)} = \{ \langle a_1, t_2 \rangle, \langle a_3, t_4 \rangle \} ,
\]

\[
(2) R_1^{(1)} = \{ \langle t_1, t_2 \rangle, \langle t_2, t_3 \rangle, \langle t_4, t_5 \rangle, \langle t_5, t_6 \rangle \} .
\]

Since interval \( \mathcal{M}_2 \) was obtained from interval \( \mathcal{M}_1 \) by the use of mentioned mapping \( \mathcal{Z}_1 \) from previous examples, the universe \( \mathcal{U}_2 \) was obtained from the universe \( \mathcal{U}_1 \) by means of the mentioned mapping \( \mathcal{Z}_0 \) of those examples, set of relations \( \mathcal{R}_2 \) was obtained from set of relations \( \mathcal{R}_1 \) successively by the use of mappings \( \mathcal{Z}_2 \), \( \mathcal{Z}_{2-1} \), \( \mathcal{Z}_1 \), \( \mathcal{Z}_0 \), \( \mathcal{Z}_1 \) from those examples, \( \mathcal{S}_2 \) is subsystem of system \( \mathcal{S}_1 \) on interval \( \mathcal{M}_2 \).

Example. P 4. Ontic system \( \mathcal{S}_0 \) from example P 3 is subsystem of ontic system \( \mathcal{S}_0 \) from example P 2 on interval \( \mathcal{M}_2 \) because of existence of mapping:

\[
\mathcal{Z}_1 : t_1 \to t_1 ,
\]

\[
t_2 \to t_1 ,
\]

\[
t_3 \to t_4 ,
\]

\[
t_4 \to t_4 ,
\]

\[
t_5 \to t_5 .
\]
Let \( \mathcal{F}_0 \) be identical mapping
\[
\mathcal{F}_0 : (1) R^{(1)}_1 \rightarrow (1) R^{(1)}_1,
(1) R^{(2)}_1 \rightarrow (1) R^{(2)}_1,
(1) R^{(3)}_1 \rightarrow 0,
(1) R^{(4)}_1 \rightarrow 0.
\]

Let \( \mathcal{F}_{j-1} \) be identical mapping,
\[
\mathcal{F}_{j-1} : (2) R^{(1)}_1 \rightarrow (2) R^{(1)}_1,
\]
\( \mathcal{F}_j \) be identical mapping,
\[
\mathcal{F}_j : (3) R^{(4)}_1 \rightarrow 0.
\]

4. Definition of Notion "L-System"

D 3. System \( \mathcal{S}_L = \langle U_L, \mathcal{R}_L \rangle \) \( \langle \mathcal{S}_L, \mathcal{M} \rangle \in \mathcal{P} \mathcal{P} \mathcal{L}^{(3)} \) is said to be \( L \)-system on interval \( \mathcal{M} \) if \( U_L \) is set of non-logical constants of language \( L \), \( \mathcal{R}_L \) is set of sets of the type \( R^{(1)}_{L_1} \) correctly formed statements of language \( L \) holding in time interval \( \mathcal{M} \), where each from sets \( R^{(1)}_{L_1} \in \mathcal{P} \mathcal{L} \) consists of statements of the type \( R^{(1)}_{L_1} \) where \( R^{(1)}_{L_1} \in U_L^{<3>} \times \times \mathcal{M}^{<3>} \times \mathcal{S}_L^{<20>}, \)
and \( j \) is number of occurrences of non-logical constants (apart from time constant in this statement),
\( k_\text{t} \) is ordering number of this statement in set \( R^{(1)}_{L_1} \),
\( \mathcal{M} \) is ordered set of names of all moments belonging to \( \mathcal{M} \),
\( \tau \) is number of occurrences of symbols from \( \mathcal{M} \) in this statement,
\( \mathcal{S}_L \) is set of variables, logical connectives, logical operators and brackets in alphabets of language \( L \),
\( l \) is number of occurrences of symbols from set \( \mathcal{S}_L \) in this statement.

Example. Let statement
\[
R^{(3)}_{L_1} = \forall x \exists y (F(x, t_1) \land R(x, y, t_1)) \rightarrow F(y, t_1)
\]
be correctly formed one of language \( L \), whose alphabet contains symbols: \( F, R, x, y, t_1, \land, \to, \forall, \exists, (, ) \) and holding on interval \( \mathcal{M} \). Let \( U_L \) be set of all non-logical constants of \( L \), hence \( F, R \in U_L \), let \( t_1 \) be name of moment \( t_1 \in \mathcal{M} \), let symbols \( x, y, \land, \to, \forall, \exists, (, ) \in \mathcal{S}_L \); then
\[
R^{(3)}_{L_1} \in U_L^{<3>} \times \mathcal{M}^{<3>} \times \mathcal{S}_L^{<20>},
\]
holds because non-logical constants $F$, $R$ occur in this statement on three places, time constant $t$, occurs in this statement on three places, variables $x$, $y$, logical connectives $\land$, $\rightarrow$, operators $\forall$, $\exists$ and brackets $($, $)$ occur in this statement on twenty places.

Remark. "L-system" is specification of "system". It may be a language system, whose correctly formed sentences only describe empirical facts and among them there is not logical relation of deduction. However it may also be an axiomatic system containing some of its sentences as axioms (holding on a given interval $A_t$) and all of them correctly logically derived statements (hence valid also interval $A_t$).

We consider merely $L$-systems of the first degree, since in set $\mathcal{R}_L$, other members of $\mathcal{H}_L$ do not occur in any of its members as elements of corresponding relation. We understand relations involved in set $\mathcal{R}_L$ (as elements of its sets) as relations among symbols of given language $L$, hence as syntactic formations.

Fact that set $\mathcal{R}_L$ does not contain directly these relations, but their sets, is motivated by aim of this reasoning ($L$-systems will be further associated with ontic systems as their language models). More accurate explanation comes out from the following consideration.

Examples

P 5. $L$-system of the first degree $\mathcal{L}_L$ is on interval $A_t = \langle t_1, t_2, t_3, t_4, t_5 \rangle$ an ordered pair $\langle U_{L_1}, \mathcal{R}_{L_1} \rangle$, where

\begin{align*}
U_{L_1} &= \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}^{(1)} R_{11}^{(1)}, a_{11}^{(1)} R_{11}^{(2)}, a_{12}^{(1)} R_{11}^{(3)}, a_{13}^{(1)} R_{11}^{(4)}, a_{14}^{(1)} R_{11}^{(5)} \} , \\
\mathcal{R}_{L_1} &= \{ R_{L_1}^{(1)}, R_{L_1}^{(2)}, R_{L_1}^{(3)}, R_{L_1}^{(4)}, R_{L_1}^{(5)}, R_{L_1}^{(6)}, R_{L_1}^{(7)} \} ,
\end{align*}

and

\begin{align*}
R_{L_1}^{(1)} &= \{ R_{L_1}^{(1)} \} , \\
R_{L_1}^{(2)} &= \forall x \forall t \{ (1) R_{L_1}^{(1)}(x, t) \rightarrow (1) R_{L_1}^{(2)}(x, t, 1) \} ; \\
R_{L_1}^{(3)} &= \{ R_{L_1}^{(3)} \} , \\
R_{L_1}^{(4)} &= \{ (1) R_{L_1}^{(4)}(a_1, t_1), (1) R_{L_1}^{(4)}(a_2, t_1) \} , \\
R_{L_1}^{(5)} &= \{ (1) R_{L_1}^{(5)}(a_3, t_1), (1) R_{L_1}^{(5)}(a_4, t_1) \} , \\
R_{L_1}^{(6)} &= \{ (1) R_{L_1}^{(6)}(a_5, t_1), (1) R_{L_1}^{(6)}(a_6, t_1) \} , \\
R_{L_1}^{(7)} &= \{ (1) R_{L_1}^{(7)}(a_7, t_1), (1) R_{L_1}^{(7)}(a_8, t_1) \} , \\
R_{L_1}^{(8)} &= \forall x \forall y \forall t (1) R_{L_1}^{(8)}(x, y, t) \rightarrow (1) R_{L_1}^{(8)}(y, x, t)) \}
\end{align*}

("(1) $R_{L_1}^{(8)}$ is symmetric relation at every moment")

\begin{align*}
R_{L_1}^{(9)} &= \{ R_{L_1}^{(9)} \} ,
\end{align*}
\[ R^{(2)}_{L_2} = \{ R^{(2)}_{L_1}, R^{(2)}_{L_3} \}, \]
\[ R^{(2)}_{L_3} = \{ R^{(2)}_{L_3} \}, \]
\[ R^{(2)}_{L_3} = (1)R^{(1)}_{L_3}(a_2, t_5); \]
\[ R^{(3)}_{L_3} = \{ R^{(3)}_{L_3}, R^{(3)}_{L_3} \}, \]
\[ R^{(3)}_{L_3} = (1)R^{(1)}_{L_3}(a_2, a_7, t_5); \]
\[ R^{(3)}_{L_3} = \{ R^{(3)}_{L_3}, R^{(3)}_{L_3} \}, \]
\[ R^{(3)}_{L_3} = (1)R^{(1)}_{L_3}(a_2, a_7, t_5); \]
\[ R^{(3)}_{L_3} = \{ R^{(3)}_{L_3}, R^{(3)}_{L_3} \}, \]
\[ R^{(3)}_{L_3} = (1)R^{(1)}_{L_3}(a_2, a_7, t_5); \]

\[ L \text{-systems can also contain their subsystems, which are clearly again } L \text{-systems.} \]

Notion "subsystem of } L \text{-system" is again specified by definition D 2.

P 7. } L \text{-system } \mathcal{S}_{L_2} \text{ (from example P 5.) is subsystem of } L \text{-system } \mathcal{S}_{L_3} \text{ (from example P 4.) on interval } \mathcal{A}_{L_2} = \{ t_2, t_5, t_6, t_5 \} \text{ as a consequence of existence of the following mappings:}

\[ \mathcal{S}_1: \text{identical mapping} \]
\[ \mathcal{S}_0: \text{identical mapping} \]
\[ \mathcal{S}_{L_3}: R^{(1)}_{L_3} \rightarrow 0, \]
\[ R^{(1)}_{L_3} \rightarrow R^{(1)}_{L_3}, \]
\[ R^{(1)}_{L_3} \rightarrow R^{(1)}_{L_3}; \]
\[ R^{(1)}_{L_3} \rightarrow 0, \]
\[ R^{(1)}_{L_3} \rightarrow 0, \]
\[ R^{(1)}_{L_3} \rightarrow R^{(1)}_{L_3}; \]
\[ R^{(1)}_{L_3} \rightarrow R^{(1)}_{L_3}; \]
\[ R^{(1)}_{L_3} \rightarrow 0, \]
\[ R^{(1)}_{L_3} \rightarrow R^{(1)}_{L_3}. \]

\( (L \text{-system } \mathcal{S}_{L_2} \text{ was created obviously from } L \text{-system } \mathcal{S}_{L_3} \text{ by eliminating general sentences, existential sentence } R^{(1)}_{L_3}. \text{ There remain statements describing individual concrete events).} \)
5. DEFINITION OF NOTIONS "ISOMORPHY OF SYSTEMS", "GENERALIZED HOMOMORPHY OF SYSTEMS"

D 4. System \( S_f \) is at a time interval \( At \) isomorphic with system \( S_i \) (symbolically: \( \langle S_f, S_i, At \rangle \in \mathfrak{I}_{iso} \)),
iff there exist intervals \( At_1, At_2 \) so that \( At = At_1 \cap At_2 \),

\[ \langle S_f, At_1 \rangle \in \mathfrak{I}_{iso} (^{(s)}), \quad \langle S_f, At_2 \rangle \in \mathfrak{I}_{iso} (^{(s)}) \]

there exist sets \( U_1, U_2, \mathcal{R}_1, \mathcal{R}_2 \) so that \( S_f = \langle U_1, \mathcal{R}_1 \rangle, S_i = \langle U_2, \mathcal{R}_2 \rangle \).

to every relation of the type \( (^{(s)}) R_{ij}^{(s)} \in \mathcal{R}_1 \) there exists at every moment \( t_i \in At \) just one relation of the type \( (^{(s)}) R_{ij}^{(s)} \in \mathcal{R}_2 \) so that \( (^{(s)}) R_{ij}^{(s)} \) is isomorphic with respect to \( (^{(s)}) R_{ij}^{(s)} \) and at the same time corresponding one-to-one mapping (setting up isomorphy) from set \( \mathcal{R}_1 \) preserves in set \( \mathcal{R}_2 \) relation of type superiority, moreover set \( \mathcal{R}_2 \) does not contain any more relation, which would not be an image of some relation from \( \mathcal{R}_1 \) at this mapping.

Definition of notion "relation of type superiority":

D 5. Relation \( (^{(s)}) R_{ij}^{(s)} \) is in relation of type superiority with respect to relation \( (^{(s)}) R_{ij}^{(s)} \) iff \( s_1 > s_2 \) (i.e. degree of relation \( (^{(s)}) R_{ij}^{(s)} \) is greater than that of relation \( (^{(s)}) R_{ij}^{(s)} \)).

D 6. System \( S_2 \) is on time interval \( At \) generally homomorphic with respect to system \( S_1 \), symbolically \( \langle S_2, S_1, At \rangle \in \mathfrak{I}_{gshom} \),
iff there exist intervals \( At_1, At_2 \) so that \( At = At_1 \cap At_2 \),

\[ \langle S_2, At_1 \rangle \in \mathfrak{I}_{gshom} (^{(s)}), \quad \langle S_2, At_2 \rangle \in \mathfrak{I}_{gshom} (^{(s)}) \]

there exist sets \( U_1, U_2, \mathcal{R}_1, \mathcal{R}_2 \) so that \( S_1 = \langle U_1, \mathcal{R}_1 \rangle, S_2 = \langle U_2, \mathcal{R}_2 \rangle \),

there exists single-valued mapping \( 3 \) so that to every relation of the type \( (^{(s)}) R_{ij}^{(s)} \in \mathcal{R}_1 \), it assigns at every moment \( t_i \in At \) not more than one relation of the type \( (^{(s)}) R_{ij}^{(s)} \in \mathcal{R}_2 \) where \( 3 \) is homomorphism of \( (^{(s)}) \mathcal{R}_1^{(s)} \) into \( (^{(s)}) \mathcal{R}_2^{(s)} \), \( 3 \) preserved in set \( \mathcal{R}_2 \) among its relations the same correspondence of type superiority which was in the set \( \mathcal{R}_1 \) among patterns of these relations, moreover set \( \mathcal{R}_2 \) does not contain any other relation which would not associate with some relation from \( \mathcal{R}_1 \) due to this mapping.

Remark. Discussed relation among systems are called "generally homomorphic", since its definition is a certain generalization of well-known definition of two-relation homomorphy.

By comparing definitions D 2, D 4, D 6 we can obtain following fact: if system \( S_2 \) is a subsystem of system \( S_1 \) on interval \( At \) and system \( S_3 \) is isomorphic with system \( S_2 \) at \( At \), then system \( S_3 \) is generally homomorphic with system \( S_4 \) at \( At \).
6. DEFINITION OF NOTIONS "MODEL_1", "MODEL_2", "MODEL_3"

D 7. System \( S_f \) is model \( x \) (or model \( 2 \) or model \( 3 \) resp.) of system \( S_f \) on time interval \( At \), symbolically \(< S_f, S_f, At \> \in \text{mod}_x \) (or \( \text{mod}_2 \) or \( \text{mod}_3 \) resp.) iff there exist intervals \( At_1, At_2 \) so that \( At = At_1 \cap At_2 \), 
\[
\langle S_f, At_1 \rangle \in \mathcal{P} \gamma \text{ol}(x), \quad \langle S_f, At_2 \rangle \in \mathcal{P} \gamma \text{ol}(x),
\]
moresove holds \(< S_f, S_f, At \> \in \text{Isyol}, \langle S_f, S_f, At \rangle \in \text{Isystem} \) resp., there are such systems \( S_f', S_f' \) resp. that \(< S_f', S_f', At \> \in \text{Isyol}, \langle S_f', S_f', At \rangle \in \text{Isystem} \) and simultaneously: \(< S_f', S_f', At \rangle \in \text{Isystem} \).

7. DISCUSSION OVER THE DEFINED NOTIONS

We shall use the defined notions for discussion of notions "language model of ontic system" and "semantic model of language system".

D 8. L-system \( S_f \) is on time interval \( At \) language model of ontic system \( S_f \) iff 
\[
\langle S_f, S_f, At \rangle \in \text{mod}_x \) (or \( \text{mod}_2 \) or \( \text{mod}_3 \) resp.) 
moresove there exists mapping setting-up isomorphy \( S_f \) to \( S_f \) (or isomorphy \( S_f \) to some subsystem \( S_f \) resp., or isomorphy of some subsystem \( S_f \) to a subsystem \( S_f \) resp.), on interval \( At \) and this mapping satisfies following requirements: to individual non-logical predicate constants of the type \( (s)R^j \) from the universe of system \( S_f \) (or its subsystem resp.) individual relations of the type \( (s)R^j \) of system \( S_f \) (or its subsystem resp.) are mutually assigned, to individual non-logical constants of the type \( a_m \) from the universe of system \( S_f \) (or to its subsystem resp.) individual objects \( a_m \) from the universe \( S_f \) (or its subsystem resp.) are uniquelly and mutually assigned, to individual symbols of the type \( t_i \) (to names of time moments) from alphabet of language of system \( S_f \) individual time moments \( t_i \) from interval \( At \) of existence of system \( S_f \) are mutually uniquely associated, hence: to individual statements \( S_f \) (or their subsystem resp.) of the type \( (s)R^j[a_1, \ldots, a_p, t_i] \) elements of relations of \( S_f \) of the type \( (s)R^j \) are uniquely associated for \( s = 1 \) and to particular statements \( S_f \) (or their subsystem resp.) of the type \( (s)R^j \) elements of relations \( S_f \) of the type \( (s)R^j \) are uniquely associated by this mapping(to individual logical connectives and operators from statements are assigned meanings in accordance with rules of interpretation known from logical semantic, similarly are associated individual and predicate variables from these statements their domains of variability).

D 9. System \( S_f \) is on time interval \( At \) semantic model of language system \( S_f \) iff 
\[
\langle S_f, S_f, At \rangle \in \text{mod}_x \) where mapping setting-up isomorphy \( S_f \) to \( S_f \) satisfies the same requirements which were formulated in D 8.
Remark. In the previous definition D 9 we did not admit in correspondence $\mathcal{S}$ and $\mathcal{L}$ alternatives “mod 2”, “mod 3” according to traditional concept of notion “semantic model” in logical semantic. Semantic model of language system can be ontic system, but also some other language system can be applicable (due to common semantic practice).

Examples.

P 8. Systems $\mathcal{S}_0$, and $\mathcal{L}_l$, from examples P 3 and P 6 are mutually isomorphic on interval $\mathcal{M}_2$, because relation $(1)R_l^{(1)}$ is isomorphic with relation $R_l^{(2)}$,

\begin{align*}
\langle a_1, t_1 \rangle &\in (1)R_l^{(1)} \Leftrightarrow R_{l,2}^{(2)}(a_1, t_1), \\
\langle a_2, t_2 \rangle &\in (1)R_l^{(1)} \Leftrightarrow R_{l,2}^{(2)}(a_2, t_2),
\end{align*}

relation $(1)R_l^{(2)}$ is isomorphic with relation $R_{l,2}^{(2)}$,

\begin{align*}
\langle a_2, t_3 \rangle &\in (1)R_l^{(1)} \Leftrightarrow R_{l,2}^{(2)}(a_2, t_3),
\end{align*}

relation $(1)R_l^{(2)}$ is isomorphic with relation $R_{l,2}^{(2)}$,

\begin{align*}
\langle a_7, a_9, t_4 \rangle &\in (1)R_l^{(1)} \Leftrightarrow R_{l,2}^{(2)}(a_7, a_9, t_4), \\
\langle a_9, a_7, t_5 \rangle &\in (1)R_l^{(1)} \Leftrightarrow R_{l,2}^{(2)}(a_9, a_7, t_5),
\end{align*}

relation $(1)R_l^{(2)}$ is isomorphic with relation $R_{l,2}^{(2)}$,

\begin{align*}
\langle a_8, a_10, t_1 \rangle &\in (1)R_l^{(1)} \Leftrightarrow R_{l,2}^{(2)}(a_8, a_10, t_1), \\
\langle a_1, a_10, t_1 \rangle &\in (1)R_l^{(1)} \Leftrightarrow R_{l,2}^{(2)}(a_1, a_10, t_1),
\end{align*}

relation $(2)R_l^{(1)}$ is isomorphic with relation $R_{l,2}^{(2)}$,

\begin{align*}
\langle (1)R_l^{(1)}, t_1 \rangle &\in (2)R_l^{(1)} \Leftrightarrow R_{l,2}^{(2)}((1)R_l^{(1)}, t_1), \\
\langle (1)R_l^{(1)}, t_2 \rangle &\in (2)R_l^{(1)} \Leftrightarrow R_{l,2}^{(2)}((1)R_l^{(1)}, t_2).
\end{align*}

Therefore: $\langle \mathcal{S}_0, \mathcal{L}_l, \mathcal{M}_2 \rangle \in \text{mod}_1$, $\langle \mathcal{L}_l, \mathcal{S}_0, \mathcal{M}_2 \rangle \in \text{mod}_3$ (correspondence of isomorphy is symmetric).

Simultaneously we can consider $\mathcal{L}_l$ as language model of ontic system $\mathcal{S}_0$, and conversely system $\mathcal{S}_0$ as semantic model of language model $\mathcal{L}_l$, if conditions of definitions D 8, D 9 are satisfied:
to constant (1)R'(1) of system $\mathcal{L}_1$ relation (1)R(1) of system $\mathcal{L}_1$ is uniquely associated to constant (1)R'(2) of system $\mathcal{L}_1$ relation (1)R(2) of system $\mathcal{L}_1$ is uniquely associated to constant (1)R(3) of system $\mathcal{L}_1$ relation (1)R(1) of system $\mathcal{L}_1$ is uniquely associated to constant (1)R(2) of system $\mathcal{L}_1$ relation (1)R(2) of system $\mathcal{L}_1$ is uniquely associated to constant (1)R(3) of system $\mathcal{L}_1$ relation (1)R(3) of system $\mathcal{L}_1$ is uniquely associated

to constant $a_1$ of system $\mathcal{L}_1$, object $a_1$ from the universe of system $\mathcal{L}_1$ is uniquely associated
to constant $a_2$ of system $\mathcal{L}_1$, object $a_2$ from the universe of system $\mathcal{L}_1$ is uniquely associated
to constant $a_7$ of system $\mathcal{L}_1$, object $a_7$ from the universe of system $\mathcal{L}_1$ is uniquely associated
to constant $a_9$ of system $\mathcal{L}_1$, object $a_9$ from the universe of system $\mathcal{L}_1$ is uniquely associated
to constant $a_{10}$ of system $\mathcal{L}_1$, object $a_{10}$ from the universe of system $\mathcal{L}_1$ is uniquely associated
to symbol $t_1$ of system $\mathcal{L}_1$ moment $t_1$ from interval $\Delta t_1$ is uniquely associated
to symbol $t_4$ of system $\mathcal{L}_1$ moment $t_4$ from interval $\Delta t_1$ is uniquely associated
to symbol $t_5$ of system $\mathcal{L}_1$ moment $t_5$ from interval $\Delta t_1$ is uniquely associated

P 9. System $\mathcal{L}_1$ is on time interval $\Delta t_1$ generally homomorphic to a system $\mathcal{L}_2$, from example P 2, since there is system $\mathcal{L}_0$, which is on $\Delta t_1$ subsystem $\mathcal{L}_0$ and $\mathcal{L}_2$ is on $\Delta t_1$ isomorphic to $\mathcal{L}_0$. Easily we can find out that mapping founding generalized homomorphy $\mathcal{L}_1$ to $\mathcal{L}_0$, we obtain by composition of two mappings from P 4 and P 8:

\[
\begin{align*}
(1)R'(1) & \Rightarrow R_{a_1}^{(2)}, \\
(1)R'(2) & \Rightarrow R_{a_2}^{(2)}, \\
(1)R'(3) & \Rightarrow 0, \\
(1)R'(1) & \Rightarrow R_{t_1}^{(3)}, \\
(1)R'(2) & \Rightarrow R_{t_4}^{(3)}, \\
(1)R'(3) & \Rightarrow 0, \\
(1)R'(1) & \Rightarrow R_{t_5}^{(2)}, \\
(3)R'(1) & \Rightarrow 0.
\end{align*}
\]

Hence $\mathcal{L}_1$ is also model of ontic system $\mathcal{L}_0$ on interval $\Delta t_1$ and, if conditions from D 8 are met, is also its language model on $\Delta t_1$. 


By examples P4 and P7 $\mathcal{S}_0$ is subsystem of $\mathcal{S}_4$, on interval $\Delta t_2$, $\mathcal{S}_4$ is subsystem of $\mathcal{S}_0$, on interval $\Delta t_2$. Therefore we can assume $\mathcal{S}_4$ as model$_1$ of ontic system $\mathcal{S}_0$, on $\Delta t_2$ and conversely also $\mathcal{S}_0$ as model$_1$ of system $\mathcal{S}_4$, on $\Delta t_2$. If conditions from D 8 are satisfied, we can consider $\mathcal{S}_4$ as language model$_3$ of system $\mathcal{S}_0$.

Proposed definition of notion "model" and its specification for notions "model$_1$", "model$_2$", "model$_3$" may look unusual and perhaps even artificial and strange. However consider that in practice of language (especially mathematical) modelling of real ontic systems we often use language and axiomatic system with general axioms. To these general axioms, we cannot associate mutually and uniquely any elements of relations (or collections of these elements) of real systems. Thus there is no possibility to take those language systems for models$_1$ of given real systems. Despite that we do talk about "language (for instance mathematical) model of real system "also in these cases. For this reason we have introduced notion "model$_2$".

Similar reasoning leads to acceptance of specification "model$_3$". In practice we hardly detect existing large real and variable systems in their all completeness. We cannot hence form language system modelling those real systems in complete complexity due to very strict demand from definition of notion "model$_1$". This fact is strengthened by known desires of finiteness imposed on modelling language systems when modelling real systems by modern computers (finite automata). Conversely, when we shall use axiomatic language systems with general axioms for modelling, then we cannot apply even notion "model$_2$". Therefore we have introduced notion "model$_3$".

Applicability of proposed concept will be particularly stressed when we consider possibility of prediction of future events due to logical operations in modelling language system.

Without formulating the whole concept on general level, we shall present very simple example.

Let us return to examples P2—P10. Suppose that system $\mathcal{S}_4$, is given to our disposal already at moment $t_4$. From assumptions $\mathcal{R}^{(1)}_{t_4}$ and $\mathcal{R}^{(1)}_{t_4}$ it is possible by means of logical deduction (by substitution and detachment) to obtain statement $\mathcal{R}^{(2)}_{t_4}$:

$$
^{(1)}\mathcal{R}^{(1)}(a_2, t_3),
$$

describing (at given interpretation), that at future moment $t_4$ object $a_2$ will have property $^{(1)}\mathcal{R}^{(1)}$. The first proposition $\mathcal{R}^{(2)}_{t_4}$ is general sentence which we have accepted into system $\mathcal{S}_4$, for instance from a theory. The second assumption $\mathcal{R}^{(2)}_t$ is sentence which we can use to describe an event which passed by in system $\mathcal{S}_0$, at the same moment $t_4$ ($\langle a_2, t_4 \in ^{(1)}\mathcal{R}^{(1)}\rangle$) in terms of language of system $\mathcal{S}_t$. Since language system $\mathcal{S}_t$ contains this sentence, we cannot classify that at moment $t_4$ as model$_1$ of system $\mathcal{S}_0$, (which presumably at $t_4$ we cannot empirically detect completely).
System $\mathcal{F}_0$, anyway encloses events with no corresponding sentences in $\mathcal{F}_1$ — for example event: $\langle a_6, t_1 \rangle \in (1)[1]$. For this reason we cannot characterise correspondence $\mathcal{F}_1$ and $\mathcal{F}_0$ for interval $\Delta t$ as “model”1. However due to existence of model of this kind we can “predict future”.

At the end of this reasoning we introduce example dealing with the correspondence “model” among studied system, known only partially, method “black box” and deterministic automata.

Let us have system $\mathcal{F}_0$, whose structure we do not know and we can determine on it two limit points $a_1, a_2$. We shall make large series of experiments associating thus one of the elements $a_1$ impulsion of various kinds and we shall investigate reaction of system on the other limit point $a_2$. Hence we study correspondence between assigned value of input element $a_1$ and values of output element $a_2$ taken on after duration of given time period. Let experiments be created during given time interval $\Delta t_B$. We describe their results in the Tab. 1.

**Tab. 1.**

<table>
<thead>
<tr>
<th>to element $a_1$ was assigned</th>
<th>at moment</th>
<th>element $a_2$ took on</th>
<th>at moment</th>
</tr>
</thead>
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<tr>
<td>$\lambda$</td>
<td>$t_1$</td>
<td>$\lambda$</td>
<td>$t_2$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$t_3$</td>
<td>$\lambda$</td>
<td>$t_4$</td>
</tr>
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<td>$t_5$</td>
<td>$\lambda$</td>
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<td>..</td>
<td>..</td>
<td>..</td>
</tr>
<tr>
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<td>$t_{10}$</td>
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</tr>
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</tr>
<tr>
<td>$A$</td>
<td>$t_{14}$</td>
<td>$A$</td>
<td>$t_{15}$</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
<td>..</td>
<td>..</td>
</tr>
</tbody>
</table>

Let results of experiments be resolved and generalized for the whole interval $\Delta t_B$ as shown in Tab. 2.

We shall now define system $\mathcal{F}_B$ based on mentioned experiments and existing on interval $\Delta t_B$: $\mathcal{F}_B = \langle U_B, A_B \rangle$.

$U_B = \{a_1, a_2\}$, $A_B = \{\lambda, A, B, C\}$,

$\lambda = \{\langle a_1, t_1 \rangle, \langle a_1, t_3 \rangle, \langle a_1, t_5 \rangle, \langle a_1, t_{13} \rangle, \ldots, \langle a_2, t_2 \rangle, \langle a_2, t_4 \rangle, \langle a_2, t_6 \rangle, \langle a_2, t_{14} \rangle, \ldots\}$,

$A = \{\langle a_1, t_{11} \rangle, \langle a_1, t_{14} \rangle, \ldots, \langle a_2, t_{12} \rangle, \langle a_2, t_{15} \rangle \ldots\}$.
Tab. 2.

<table>
<thead>
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<th>input word</th>
<th>at moments</th>
<th>output word</th>
<th>at moments</th>
</tr>
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<td>t_{i+1}</td>
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<td>λA</td>
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</tr>
<tr>
<td>λB</td>
<td>λB</td>
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</tr>
<tr>
<td>λC</td>
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<td>t_{i+1}, t_{i+2}</td>
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<td>AA</td>
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<td>AB</td>
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<tr>
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<tr>
<td>BB</td>
<td>BA</td>
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</tr>
<tr>
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<td>t_{i}, t_{i+1}</td>
<td>BC</td>
<td>t_{i+1}, t_{i+2}</td>
</tr>
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<tr>
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</tr>
</tbody>
</table>

(Due to Table 1). $S_B$ does not contain other relations except those which were found from experiments summarized in Table 1.

We define further axiomatic language system $S_{LB}$ holding on interval $At_B$: $S_{LB} = \langle U_{LB}, S_{LB} \rangle$.

$U_{LB} = \{a_1, a_2, \lambda, A, B, C\}$.

$S_{LB}$ contains:

1. general statements

   $P_1 = \forall t (\lambda(a_1, t) \rightarrow \lambda(a_2, t_{i+1}))$,

   $P_2 = \forall t ((\lambda(a_1, t) \land A(a_1, t_{i+1})) \rightarrow (\lambda(a_2, t_{i+1}) \land A(a_2, t_{i+2})))$,

   $P_{16} = \forall t ((C(a_1, t_i) \land C(a_1, t_{i+1})) \rightarrow (C(a_2, t_{i+1}) \land C(a_2, t_{i+2})))$;

(they are description of result of generalization from Tab. 2).

2. particular statements:

   $V_{11} = \lambda(a_1, t_i)$,

   $V_{12} = \lambda(a_2, t_i)$,

(statements regarding entry $a_1$ are description of recalled situations in experiments described in Table 1, statements concerning output $a_2$ are logical consequence of general statements and particular statements about entry $a_1$ and are in accordance with description in Table 1).
Further there are here statements obtained from general ones $P_1, \ldots, P_{16}$ by substitution of names of time moments from interval $\Delta t_2$ and time variables:

$$V_{n_1} = (\lambda(a_1, t_1) \rightarrow \lambda(a_2, t_2)),$$
$$V_{n_2} = ((\lambda(a_1, t_1) \land A(a_1, t_2)) \rightarrow (\lambda(a_2, t_2) \land A(a_2, t_3))),$$

Let us form the automaton shown on Fig. 1 from technical elements.

**Fig. 1.**

Automaton has two binary inputs $X_1, X_2$ and two binary outputs $Y_1, Y_2$. $Q_1, Q_2$ are binary parts of state. We can describe relations of automaton by canonic equations (by means of Boolean algebra):

$$Y_1 = Q_1X_1 + Q_2X_2,$$
$$Y_2 = Q_1[Q_2X_2 + Q_2(X_1X_2 + X_1\overline{X}_2)] + Q_1X_1,$$
$$Q_1' = \overline{Q}_1Q_2X_1,$$
$$Q_2' = \overline{Q}_1Q_2X_2.$$
Let us translate (mutually uniquelly) binary symbols to letters of alphabet according to code:

<table>
<thead>
<tr>
<th>Input Symbols</th>
<th>Letter of Input Alphabet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1X_2$</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>$X_1X_2$</td>
<td>$A$</td>
</tr>
<tr>
<td>$X_1X_2$</td>
<td>$B$</td>
</tr>
<tr>
<td>$X_1X_2$</td>
<td>$C$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Output Symbols</th>
<th>Letter of Output Alphabet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_1Y_2$</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>$Y_1Y_2$</td>
<td>$A$</td>
</tr>
<tr>
<td>$Y_1Y_2$</td>
<td>$B$</td>
</tr>
<tr>
<td>$Y_1Y_2$</td>
<td>$C$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Binary Parts of State</th>
<th>Denotation of State</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_1Q_2$</td>
<td>$S_1$</td>
</tr>
<tr>
<td>$Q_1Q_2$</td>
<td>$S_2$</td>
</tr>
<tr>
<td>$Q_1Q_2$</td>
<td>$S_3$</td>
</tr>
<tr>
<td>$Q_1Q_2$</td>
<td>$S_4$</td>
</tr>
</tbody>
</table>

Because of this translation it is possible to consider automata as a system with one input element $x_i$ and one output element $y_i$. Both input and output element can take on values $\lambda, A, B, C$.

Not taking in to mind internal (technical) structure of automaton let us define on that subsystem $\mathcal{S}_A$:

$$\mathcal{S}_A = \langle U_A, \mathcal{R}_A \rangle,$$

$$U_A = \{x_i, y_i\}, \quad \mathcal{R}_A = \{\lambda, A, B, C, R_1, R_2, \ldots\}.$$

Let $\mathcal{S}_A$ exist again on interval $A B$. Relations which are elements of set of relations $\mathcal{R}_A$ are more terms relations between input, its values, corresponding moments, output, its values and corresponding moments. To element of a relation $R_i \in \mathcal{R}_A$:

$$\langle x_i, y_i, A, t_1, B, t_2, C, t_3 \rangle$$

there will be uniquely assigned a statement of language system $\mathcal{S}_{LB}$:

$$V_i = ((A(a_1, t_1) \land B(a_1, t_2)) \land (A(a_2, t_2) \land C(a_2, t_3)))$$

which is element of a set of statements $V_i \in \mathcal{S}_{LB}$.

The automaton has been formed to assign to every relation $R_i \in \mathcal{R}_A$ uniquely a set of statements from set $\mathcal{S}_{LB}$ of language system $\mathcal{S}_{LB}$. If this association is mutually unique, there exists a correspondence "model", on interval $A B$ between both systems.

On the contrary, between system $\mathcal{S}_B$ and $\mathcal{S}_{LB}$ there is only correspondence
“model,” on interval $\mathcal{A}_B$, since to general statements of system $\mathcal{S}_{B,n}$ cannot be assigned any relations of system $\mathcal{S}_B$ — system $\mathcal{S}_B$ was defined merely for summarizing particular empirically obtained knowledge on issuing data of input and taking on values of output in individual moments.

Obviously it is possible to define subsystem $\mathcal{S}_{B,n}$ on system $\mathcal{S}_{B,n}$ isomorphic on interval $\mathcal{A}_B$ with system $\mathcal{S}_B$. Between $\mathcal{S}_A$ and $\mathcal{S}_{B,n}$ is therefore also correspondence “model,” on interval $\mathcal{A}_B$.

System $\mathcal{S}_B$ was however defined as subsystem on system $\mathcal{S}_B$. It is therefore also correspondence “model,” on interval $\mathcal{A}_B$ between $\mathcal{S}_{B,n}$ and $\mathcal{S}_A$. There surely are relations in $\mathcal{S}_{B,n}$ connecting input and output words for those moments in which no corresponding experiments were done on $\mathcal{S}_B$. Because of that it is possible by experimenting with $\mathcal{S}_A$ (made as experiments with automaton, on which $\mathcal{S}_A$ was defined), to predict or postdict or explanate resp. processes which can run on in model system $\mathcal{S}_B$.

Let us further consider a possibility (which will not be discussed here in details) to assess structure of incompletely known automata $\mathcal{S}_{B,n}$ on the base of knowledge of internal (technical) structure given automata. We can easily construct considered deterministic automaton if we know its canonic equations.

Discussed example corresponds with real procedures in technical practice. On incompletely known system “black box” we define, on the base of really created experiments, subsystem. To that we search adequate language model. If this model is axiomatic system, then statements describing results of made experiments belong to its theorems. To such a language system we look for corresponding automata modelling (at least some) statements of system. Experiments with automata can be replaced by further experiments with system “black box”. We assess structure of system “black box” due to knowledge regarding structure of modelling automata.

Let us notice at the end that system $\mathcal{S}_A$ enclosing relations among input, output, their values and moments is a system of the second degree (values of input or output elements resp. are considered as its properties).

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REFERENCES