Kybernetika

Akihiro Nozaki

On the notion of universality of Turing machine

Kybernetika, Vol. 5 (1969), No. 1, (29)--43

Persistent URL: http://dml.cz/dmlcz/125257

KYBERNETIKA ČÍSLO 1, ROČNÍK 5/1969

On the Notion of Universality of Turing Machine

A. NOZAKI

Several definitions of universal Turing machines are formulated and compared. Following to a definition recommended by the author, some simple Turing machines are shown to be non-universal.

0. INTRODUCTION

As it is well known, A. M. Turing proved in 1936 the existence of a universal computing machine which is called a *universal Turing machine* today. To this pioneering work of Turing, C. Shannon added the following results:

(S1) There exist universal Turing machines (UTM's) having only two states.

(S2) There is no UTM having only one state.

(S3) There exist UTM's having only two symbols.

(S4) There is no UTM having only one symbol.

Several years later, A. Minsky showed a simple example of UTM:

(M5) There is a UTM having 6 states and 7 symbols.

Recently, S. Watanabe (University of Tokyo) demonstrated an extremely simple machine which is also universal in a sense.

(W6) There is a UTM having 3 states and 7 symbols.*

It seems strange, however, that there is no general definition of the notion of universality: these scholars used implicitly their particular definitions which were quite different from each other at several points. In fact, there arise some delicate

* To my knowledge, this machine gives the minimum value of state-symbol product among well-known UTM's.

 conflicts among their results (S2), (M5) and (W6) as it shall be explained later. Here a rigorous consideration is desirable.

Roughly speaking, a UTM is defined to be a machine which can "simulate" the behavior of any Turing machine (TM), starting with any given initial state and given initial tape. Therefore a definition of "simultability" leads immediately to a definition of universality.

In the next section, we shall describe a rather broad definition of simultability in order to give an accurate comparison among several definitions of universality.

1. GENERAL DEFINITION

Here we assume that we have a so-called Goedel Numbering System G as follows:

a) By G, a positive integer $(>0)\, f^*$ is associated to each partially computable function f. We shall call the number f^* the *goedel number of the function* f (gn. of f).

b) By G, a positive integer w^* (>0) is associated to each 'total state' w of each TM M.*

We call the number w^* an *instantaneous description of the machine* M (an Id of M).

By this numbering system G, we can associate to every TM M a pair

$$M^* = (N, f^*)$$

of the whole set N of Id's of the machine M and the gn. of its "next-state function" f which maps N into $N \cup \{0\}$.**

This pair M^* is hereafter called the goedel number of the machine M (gn. of M).

Definition 1. An *abstract machine* $\mathfrak{A}$ is a pair of a set N of positive integers and a gn. of a partially computable function f whose domain contains N and whose image $f(N)$ is contained in $N \cup \{0\}$.

Definition 2. Let $\mathfrak{A}_1 = (N_1, f_1^*)$, $\mathfrak{A}_2 = (N_2, f_2^*)$ be abstract machines.

$\mathfrak{A}_2$ is called a factor machine of $\mathfrak{A}_1$ over N_2, $\mathfrak{A}_2 = \mathfrak{A}_1/N_2$ in symbol, if the following conditions are satisfied.

a) $N_2 \subseteq N_1$,

b) If $x \in N_2$ and $y = f_2(x)$, then there exist a finite number of elements $x_0, x_1, \ldots, x_n$ of N_1 such that $x_0 = x$, $x_n = y$ and $x_i = f(x_{i-1})$ for $i = 1, \ldots, n$.

* Here the word "total state" means a combination (s, α, n) of an internal state s of a machine M, a sequence α of symbols on its tape and a position n of the read-write head of the machine.

** Here, the number 0 is added to the range of f to indicate machine-stops; if the machine stops at a total state w, then the value of $f(w^*)$ of f at w^* is defined to be 0.

(If M is a TM and $\mathfrak{A}_1 = M^*$, then every node of the first sequence in the above diagram (Fig. 1) represents a total state of the machine M. However, even in this case, $\mathfrak{A}_2$ may not be represented as $\mathfrak{A}_2 = (M')^*$ by any TM M'.)

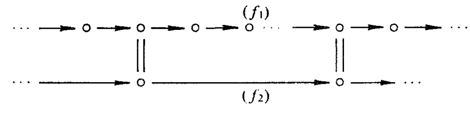


Fig. 1.

Definition 3. Let $\mathfrak{A}_1 = (N_1, f_1^*)$, $\mathfrak{A}_2 = (N_2, f_1^*)$ be abstract machines.
Let λ be a partially computable function which maps N_2 into N_1.
Let

$$N_1' = \lambda(N_2) \cup f(\lambda(N_2)) \cup f_2^2(\lambda(N_2)) \cup \ldots = \bigcup_{n=0}^{\infty} f^n \,.\, \lambda(N_2)\,.$$

Let μ be a partially computable function which maps $N_1' \cup \{0\}$ into $N_2 \cup \{0\}$.

We say that *the machine $\mathfrak{A}_1$ simulates synchronously a machine $\mathfrak{A}_2$ under initial setting by λ and interpretation by μ, and denote*

$$\mathfrak{A}_1 \xrightarrow[\text{syn}]{\lambda,\,\mu} \mathfrak{A}_2\,,$$

if and only if the following diagrams (Fig. 2) commute.

a)
$$\begin{tikzcd} N_1 \arrow[r, "f_1"] & N_1 \cup \{0\} \arrow[d, "\mu"] \\ N_2 \arrow[u, "\lambda"] \arrow[r, "f_2"'] & N_2 \cup \{0\} \end{tikzcd}$$

b)
$$\begin{tikzcd} N_1' \arrow[r, "f_1"] \arrow[d, "\mu"'] & N_1 \cup \{0\} \arrow[d, "\mu"] \\ N_2 \arrow[r, "f_2"'] & N_2 \cup \{0\} \end{tikzcd}$$

Fig. 2.

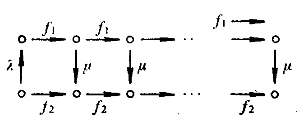


Fig. 3.

Consequently, we have the following commutative diagram (Fig. 3).
In other words, if $y = f_2^n(x)$ $(n > 1)$, then $y = \mu \,.\, f_1^n \,.\, \lambda(x)$. Thus y can be obtained by computing λ, f and μ.

Note that in this definition, nothing is mentioned about machine-stops. We should at least assume that the set $\mu^{-1}(0)$ is recursive. In the next section, we shall assume rather strong condition at this point.

Definition 4. Let M_1, M_2 be TM's.
We say that M_1 *simulates* M_2 and denote

$$M_1 \rightarrow M_2$$

if the following conditions are satisfied:

a) There exists a recursive set N and an abstract machine $\mathfrak{A}$ such that

$$\mathfrak{A} = M_1^*/N .$$

b) There exist partially computable functions λ, μ such that

$$\mathfrak{A} \xrightarrow[\text{syn}]{\lambda,\, \mu} M_2^* .$$

Definition 5. A TM M_0 is called to be *universal* if the following conditions are satisfied.

a) There exists a recursive set N and an abstract machine $\mathfrak{A}$ such that

$$\mathfrak{A} = M_0^*/N .$$

b) There is a partially computable function ψ which associates to each gn. of a TM M a pair (λ^*, μ^*) of gn.'s of partially computable functions λ, μ such that

$$\mathfrak{A} \xrightarrow[\text{syn}]{\lambda,\, \mu} M^* .$$

Universality thus defined is general enough to cover the notions conceived by Turing Shannon, Minsky and Watanabe. In fact, this definition is too broad in a sense. We shall notice this point in the next section.

2. COMPARISON OF DEFINITIONS

Comparing our definition of simultability in the previous section with that of Shannon, we can notice that he imposed implicitly to our definition the following restrictions:

(I) $\mu(x) = 0$ if and only if $x = 0$.

In other words, the "simulator" stops at the same time as the machine to be simulated.

(II) The mapping

$$\mu : N_1' \cup \{0\} \to N_2 \cup \{0\}$$

is a bijection (1 to 1 and onto) and

$$\lambda = \mu^{-1}(N_2) .$$

This restriction excludes such an interpretation (simulation) illustrated below (Fig. 4).

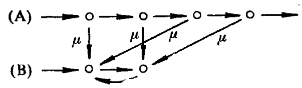


Fig. 4.

(A): Behavior of the "simulator"; an infinite sequence of different total states.
(B): Behaviour of the machine to be simulated (a short loop).

(III) By definition, a total state of a machine contains complete description of its tape. So under the conditions of Definition 4, a tape of the machine M_1 is converted to (interpreted as) a tape of M_2 in the manner specified indirectly by μ. Now the third restriction is the following:

The tape conversion specified by μ can be carried out by a simple "conversion table" of symbols, that is, independently of the internal states and of positions of the read-write head, a tape of M_1 is converted to that of M_2 block by block, one after another, replacing every symbol of M_1 by the corresponding symbols of M_2 according to the conversion table.

The third restriction is very important since, without this restriction, we cannot follow his proof of the proposition (S2): he considered there computation of one single irrational number assuming that the UTM should write down on its tape the digit numerals of the irrational number in order of the ordinary decimal (or binary) fraction.*

This assumption, however, was not employed by other scholars, for example Turing and Minsky. Thus Shannon's notion should be considered as too rigid. On the other hand, following to our definitions (Definitions 4–5), we can show a trivial example of UTM having one state and two symbols as shown in Fig. 5.

Roughly speaking, this machine M_0 simulates, the behavior of a given machine M starting with a total state w in the following manner:

Let $M^* = (N, f^*)$. Put $m = f^*$ and $n = w^*$ in Fig. 5. We put at first the head of M_0 on the block marked "S". Then the head travels simply to the right.

Obviously, the total state of the machine M at every step can be determined by the function f, the initial state w and the number of steps k of execution. More precisely

* However, no TM having one state can produce a non-periodic sequence of symbols on its tape. Thus he concluded the proposition (S2).

speaking, *the gn. of* $f^k(w)$ *is computed from the integers* f^*, w^* *and* k. So a powerful TM can realise the relevant interpretation μ.

This simple example shows definitely that the notion of universality becomes trivial if we do not impose any restriction to our definition described in the previous section. Thus the Definition 5 should be considered as too broad.

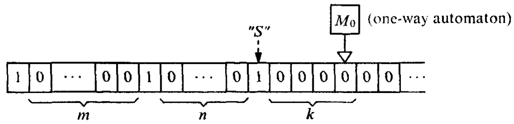


Fig. 5.

Disregarding some details, the relationship among the definitions by Shannon, Minsky and Watanabe seems to be illustrated as following Fig. 6.

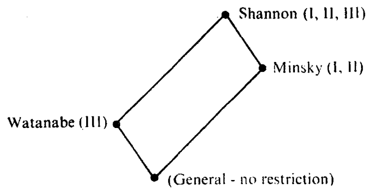


Fig. 6.

Minsky removed the restriction (III) while Watanabe omitted the restriction (II) and weakened the restriction (I) so as to admit so-called "dynamic stops" i.e. some short loops which can be easily recognized, instead of the complete stops ($f(w) = 0$). In consequence the UTM's considered in their proofs of the propositions (M5) and (W6) can not be accepted in Shannon's theory.

As for definitions proposed by Minsky and by Watanabe, the author prefers Minsky's since it seems mathematically more natural and less complicated than Watanabe's. Moreover, the original definition of Watanabe involves some informalities which I could not re-formulate rigorously.

In the next section, we shall discuss the universality of some simpe TM's following to Minsky's definition.

3. UNIVERSALITY OF A CERTAIN TYPE OF MACHINES

Fortunately, we could give a new proof of the statement (S2) without the third restriction (III). Besides, we proved the followings:

(N7) There is no UTM having two states and two symbols.
(N8) There is no UTM having three states and two symbols.

In the proofs of these propositions, we utilized two approaches, i.e. considerations on the decidability of the machine-stop (the halting problem) and on the number of possible cyclic behaviors of a machine.

a) Decidability of the machine-stop (the halting problem):

Under the restriction (I), a UTM must stop if and only if the machine being simulated stops. Then, since the halting problem of TM's is unsolvable, it is the case that

(A) *the halting problem for a* UTM *is unsolvable.*

So the proposition (S2) follows immediately to Proposition 1 as following.

Proposition 1. *The halting problem for a* TM *having one state is solvable.*

b) Number of periods of cyclic behaviors.

Under the restriction (II), a UTM should take the same total state again and again peirodically, if so does the machine being simulated. Consequently, it is the case that

(B) *a* UTM *can execute infinitely many periodic motions of different periods.*

So we have the proposition (N7) as a corollary of the following proposition.

Proposition 2. *A* TM *having two states and two symbols can execute cyclic behaviors of at most two different periods.*

The proposition (N8) was proved in the following way.

(*First step*) By dint of the remark (B), the types of TM's to be considered were reduced to less than ten.

(*Second step*) By dint of the remark (A), all TM's under consideration were shown to be non-universal.

Unfortunately, it takes too much space to give a precise description of these steps. So we shall give only the proofs of the propositions 1 and 2 in Appendix.

4. FURTHER PROBLEMS

We have so far discussed only four selected definitions in order to clarify the issue. However, there are other different definitions also interesting e.g. Turing's original definition and the definition given by M. Davis, etc. ... here further consideration is desirable.

As for the minimum value of the state-symbol product of UTM, the problem remains open: are there any UTM having two states and three symbols? What UTM can be the simplest? etc ...

Though these are of course very special sort of problems, their difficulty seems to call for new techniques which could be of general interest.

APPENDIX I

PROOF OF THE PROPOSITION 1

Let us consider the halting problem of a TM $\mathfrak{M}$ having one state, starting with a tape α as following Fig. 7.

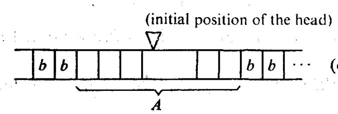


Fig. 7.

We assume that:

a) the blank code "b" is written on every block outside of the finite segment A (which can be known by the gn. of the given initial total state),

b) in the beginning, the head is put on a block in the segment A,

c) when the head comes to a block containing the blank code, it moves always to the left. (Therefore if the head goes out of the segment A to the left, it moves left for ever.)

Now, let us consider two preliminary experiments.

(A) Put the head of the machine $\mathfrak{M}$ on a tape α' as following Fig. 8.

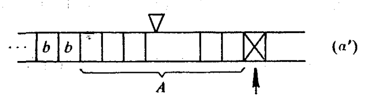


Fig. 8.

We assume that, with due modification of the machine $\mathfrak{M}$, the head moves left if it arrives at the tail of the tape in the right. Since the segment A is is assumed to be finite, we can distinguish the case among the followings after finite steps of execution of the machine $\mathfrak{M}$.

(A1) The head goes out of the segment A to the left after arriving k times at the tail of the tape α'.

(A2) The head eventually stops on a block in A after arriving k times at the tail block.

(A3) The head eventually repeats a periodic behavior for ever on a segment contained in A after arriving k times at the tail block.

(A4) The head eventually repeats a periodic behavior on a segment containing the tail block.

We shall call the number k *the index of the tape* α, which is defined to be ∞ in the case (A4).

(B) Let us consider the behavior of the machine $\mathfrak{M}$ starting with the following blank tape α'' (Fig. 9).

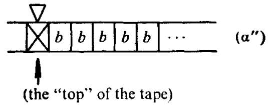


Fig. 9.

We now assume that the head is initially put on the top block of the tape on which it moves always to the right. Then the following cases are possible.

(B1) The head stops after arriving h times at the top block.

(B2) The head moves for ever but arrives at the top only a finite number h of times.

(B3) The head comes to the top block infinitely many times.

The number h is called *the index of the machine* $\mathfrak{M}$, which is defined to be ∞ in the case (B3).

If we can distinguish the case and know the exact value of h, then we can solve the halting problem of the machine $\mathfrak{M}$. In fact, the machine $\mathfrak{M}$ eventually stops if and only if one of the following conditions is satisfied:

a) $$h \leqq k \text{ and (A2) is the case}$$

or

b) $$h > k \text{ and (B1) is the case .}$$

Now let us consider how we can distinguish the case among (B1) $\sim$ (B3).

Since $\mathfrak{M}$ has only one state, the direction of the head is determined only by the symbol being read. So we can assume that we have the following "direction function".

$$\alpha\colon (\text{the set of symbols}) \to \begin{cases} 1 & (\text{left}) \\ 0 & (\text{stop}) \\ -1 & (\text{right}) \end{cases}$$

By assumption, $\alpha(b) = 1$.

Let g be the "next symbol function" of $\mathfrak{M}$ whose domain can also be considered as the set of symbols. We assume that $g(x) = x$ if $\alpha(x) = 0$.

Let $x_i = g^i(b)$, $x_0 = b$. Since the number of symbols is finite, there are integers $\gamma < s$ such that

a)
$$x_\gamma = x_s \,,$$

b)
$$x_i \neq x_j \quad \text{for} \quad i < j < s \,.$$

Definition.

$$c(k) = \sum_{i=0}^{k} d(x_i) \,.$$

Lemma 1.

$$c(k) \geqq 0 \quad \text{for all } k \quad \Leftrightarrow \quad c(0), \ldots, c(s) \geqq 0 \quad \text{and} \quad c(s) - c(\gamma) \geqq 0 \Leftrightarrow$$
$$\Leftrightarrow c(0), \ldots, c(s^3) \geqq 0 \,.$$

Let $\pi_P(i, t)$ be the number of arrivals of the head at the i-th block B_i (see Fig. 10), until the head comes to the block B_P for the t-th time.

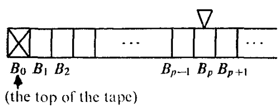


Fig. 10.

If the head can not come to the block B_P t times, then the value $\pi_P(i, t)$ is not defined.

Lemma 2. Suppose that the head comes to B_P at least t times.

a) If $t' \leqq t$, then $\pi_P(i, t')$ is defined for all $i \geqq 0$ and not greater than $\pi_P(i, t)$:

$$\pi_P(i, t') \leqq \pi_P(i, t) \,.$$

b) If $p' < p < p''$ and $\pi_{p-p'}(i, t)$ is defined, then

$$\pi_p(p'', t) = \pi_{p-p'}(p'' - p', t) \,.$$

c) If $t' = \pi_p(g, t) > 0$ and $p < g$, then $\pi_g(i, t')$ is also defined and

$$\pi_p(i, t) = \pi_g(i, t') \quad \text{for} \quad i > g \,.$$

Lemma 3. Suppose that the head comes to a block B_p at least t times.

a)
$$\pi_p(p, t) > \pi_p(p + 1, t) > \ldots > \pi_p(n, t) = 0 \quad \text{for some} \quad n > p \,.$$

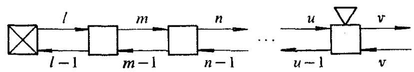
(Evidently, $\pi_p(m, t) = 0$ for $m > n$.)

b) If $c(k) \geqq 0$ for all k, then

$$\pi_p(1, t) > \pi_p(1, t) > \ldots > \pi_p(p, t),$$
$$2\pi_p(0, t) > \pi_p(1, t).$$

For proof see Fig. 11.

Fig. 11.



The letters $l, m, n, \ldots$ denotes the numbers of the motions of the head indicated by arrows. Obviously

$$\pi_p(0, t) = l - 1,$$
$$\pi_p(1, t) = l + m - 1,$$
$$\pi_p(2, t) = m + n - 1, \text{ etc.}$$

But if $c(k) \geqq 0$ for all k, we have

$$l - 1 \geqq m, \quad m - 1 \geqq n \quad \text{etc.},$$

which imply

$$2 . \pi_p(0, t) \geqq \pi_p(1, t)$$

and

$$\pi_p(1, t) > \pi_p(2, t) \text{ etc. ... b).}$$

Now, suppose that

$$0 < \pi_p(g, t) \leqq \pi_p(g + 1, t)$$

for some $g \geqq p$. Put

$$t_k = \pi_p(g + k, t).$$

By supposition, $0 < t_0 \leqq t_1$.

If $t_{k-1} \geqq t_0$, then

$$\begin{aligned} t_k &= \pi_{g+k-1}(g + k, t_{k-1}) && \text{(Lemma 2c))} \\ &> \pi_{g+k-1}(g + k, t_0) && \text{(Lemma 2a))} \\ &= \pi_g(g + 1, t_0) && \text{(Lemma 2b))} \\ &= \pi_p(g + 1, t) && \text{(Lemma 2c))} \\ &= t_1 \geqq t_0 . \end{aligned}$$

So $t_k \geqq t_0$ for all k.

Let N be the number of steps of the machine until it comes to B_p for the t-th time. Then evidently

$$N - \sum_{k=0}^{\infty} \pi_p(k, t) \geqq \sum_{k=0}^{\infty} t_k \geqq \sum t_0 = \infty .$$

But this is absurd.

Corollary 1. If $c(k) \geqq 0$ and $d(k) \neq 0$ for all k, then $h = \infty$ (B3).

Proof. By Lemma 3, we have

$$2\,\pi_p(0, t) > \pi_p(1, t) > \ldots > \pi_p(n, t) = 0$$

for some n. So, if

$$\pi_p(0, t) \leqq h < \infty ,$$

then $\pi_p(2h, t) = 0$ for all possible p and t and therefore the head moves only a finite number of times against the assumption $d(k) \neq 0$.

Corollary 2. If $c(k) \geqq 0$ for all k and $d(s) = 0$, then the machine eventually stops at the block B_1, after arriving $s + 1$ times at B_1 and

$$h = (s + c(s))/2$$

times at B_0.

Corollary 3. If $c(0), \ldots, c(k-1) \geqq 0$ and $c(k) < 0$ for some k, then the machine moves for ever but comes to the top block B_0 only $k/2$ times.

Proof. Obviously, $c(k-1) = 0$, and $d(x_k) = d(x_{k-1}) = -1$.

a) The head comes to B_0 at least $k/2$ times. Consider a machine $\mathfrak{M}'$ obtained by modifying $\mathfrak{M}$ as follows:

$$d(x_k) = 0 , \quad g(x_k) = x_k .$$

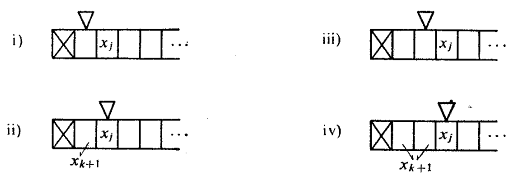


Fig. 12.

Then $\mathfrak{M}'$ moves just as $\mathfrak{M}$ until its stop and comes to B_0 $k/2$ times and to B_1 $k + 1$ times by Corollary 2.

b) The head comes to B_0 no more than $k/2$ times (see Fig. 12).

In the first stage i), the head is assumed to have arrived at B_1 for the $(k + 1)$-th time (Fig. 13).

(*) Since $c(k - 1) = 0$, $l - 1 = m = k/2$ and therefore the head moves $k/2$ times from B_2 to B_1.

Next the head moves to B_2, rewriting the symbol on B_1 from x_k to x_{k+1} (see ii)). Let x_j be the symbol on B_2. By Lemma 3b), $k + 1 > j$ (i.e. $k \geqq j$).

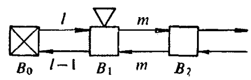


Fig. 13.

Now, by the remark (*), we have

$$d(j) = d(j + 1) = \ldots = d(k) = -1 .$$

So, the head never returns to B_1 until it comes to B_2 $k + 1$ times.

But when the head comes to B_2 for the $(k + 1)$-th time, the block B_3 contains the symbol x_j by Lemma 2. b) So the head repeats similar motions as from i) to iii), starting from the block B_1 and so on. It returns therefore no more to B_0.

By Lemma 1, we can determine by finite procedure if the conditions appeared in Corrollaries 1, 2 and 3 are satisfied or not. Thus the proof of the proposition has been completed.

APPENDIX II

PROOF OF THE PROPOSITION 2

When a machine repeats the same behavior periodically, its head of course repeats the same motion periodically on a finite segment of the tape. So a 1-way automaton can not be universal. In other words, a UTM $\mathfrak{M}$ must be capable of moving both left and right as well as of stopping. If $\mathfrak{M}$ has only two states A, B and two symbols 0, 1, then we can assume without loss of generality that the head of $\mathfrak{M}$ moves left *only* when it reads the symbol 0 at the state A.

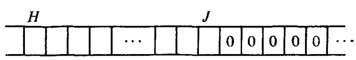


Fig. 14.

Now, let us consider a periodic motion of the machine. We assume that the head moves now periodically on a segment between two blocks H and J (Fig. 14).

We assumes that all blocks in the right of the block J contain the symbol 0. (This assumption does not affect the periodic motion of the machine under consideration.)

Table 1.

State	Symbol	New state	New symbol	Direction
A	0	U	u	left
A	1	V	v	right or stop
B	0	X	x	right or stop
B	1	Y	y	right or stop

Since the head turns to the left at the block J, J contains always the symbol 0. Therefore $u = 0$ in the Table 1. In other words, the symbol $u = 0$ remains in the block from which the head moves left. So, when the head arrives at the block H, all blocks in the right of H contain 0.

Now, the head then turns to the right and the periodic motion is continued. But the head moves right at most in two cases (see the above table.) So there are at most only two periodic motions of the head. Though the symbols outside of the segment $H - J$ are arbitrary, we can say that there are at most two periods of cyclic behaviors of the machine. This completes the proof of Proposition 2.

(Received June 14th, 1968.)

REFERENCES

[1] A. M. Turing: On Computable Numbers. Proc. London Math. Soc. *42* (1936).

[2] C. E. Shannon: A Universal Machine with Two Internal States. Automata Studies. Princeton 1956.

[3] M. A. Minsky: A 6-symbol 7-state Universal Turing Machine. MIT Lincoln Laboratory Report G-0027 (1960).

[4] P. C. Fisher: On Formalisms for Turing Machines. Journ. ACM *12* (1965).

[5] S. Watanabe: A 7-state 3-symbol Universal Turing Machine. Mimeographed, 1967.

O pojmu universálnosti Turingova stroje

A. Nozaki

Autor podává přehled a podrobné srovnání různých pojmů či typů universálního Turingova stroje (u Turinga, několik typů u Shannona, u Minskyho a u Watanabeho). Dále předkládá vlastní pojetí universálního Turingova stroje, který „simuluje" práci kteréhokoliv Turingova stroje. Je dokázána řada dílčích výsledků, které ilustrují vztahy mezi jednotlivými pojmy. Autor na základě své definice ukazuje, že některé jednoduché Turingovy stroje nejsou universální.

Akihiro Nozaki, Associate Professor, University of Tokyo, Tokyo, Japan.