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PARTIALLY ORDERED COMBINING STRUCTURES AND MULTIPLE SOURCES OF KNOWLEDGE EVIDENCES IN RULE-BASED EXPERT SYSTEMS

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The use of partially ordered combining structures for consulting systems is presented as a generalization of combining structures based on (fully) ordered abelian groups. This theoretical problem is related with both the construction of expert systems in which the knowledge base is the result of contributions from several experts (which might not agree completely), and with the simultaneous consultation of an expert system by several users concerning the same problem.

Some first theoretical results from Valdés [4] are given, which are relevant to the above mentioned problems, including the case in which the set of experts has some structure (i.e. they are grouped into "teams" or "schools"). However there are still many open problems arround this topic which deserves future attention.

An expert system prototype (VEXN) working with such principles was constructed for experimental purposes.

1. ORDERED ABELIAN GROUPS AND COMBINING STRUCTURES IN EXTENSIONAL EXPERT SYSTEMS

Hájek [2] formalizes the notions of extensional consulting systems. Such an expert system contains a knowledge base composed by *propositions* and *rules*. The former are labeled by natural numbers and the latter are structures of the form $A \Rightarrow C(w)$ with A (antecedent) being an elementary conjunction of literals, C (succedent) a proposition, and w the weight which represents the uncertainty (usually, but not obligatory, an element from $[-1, 1]$).

A loop free sequence of rules forms a tree from which the weight of any proposition in the knowledge base can be computed by the algorithm of *backward chaining*. Here the weight of a compound proposition is determined solely by the weights of the components and similarly the weight of a proposition which is a conclusion of several rules is determined only by the contribution of the corresponding rules. Thus these systems are extensional.

The effective computations in an expert system require the so called *combining functions*. These are NEG for derive the weight of the negation of a proposition, CONJ for that of an elementary conjuction, CTR for the contribution of a rule, and GLOB for the combination of contributions from different rules leading to the same conclusion. The latter is of most importance since as Hájek shows GLOB can be defined using a binary operation $\oplus$ associative and commutative which makes the set of non-extremal weights to an *ordered abelian group* (OAG) (fully ordered with respect to an odering relation $\leq$; for ordered groups see [1].

To each OAG G a combining structure Gc is associated by introducing the extremals $\top$ (truth) and $\bot$ (falsity) with the operations and ordering extended in the form

$$x \leqq \top, \quad x \geqq \bot, \quad (x \oplus \top = \top, \; x \neq \bot),$$

$$(x \oplus \bot = \bot, \; x \neq \top), \quad \top \oplus \bot = \text{undefined}, \text{ and } -\top = \bot .$$

Here $x \in Gc$.

Thus the combining functions are given by

$$\begin{aligned}
&\mathrm{NEG}(x) = -x \\
&\mathrm{CONJ}(x, y) = \min(x, y) \\
&\mathrm{CTR}(x, y) = \begin{cases} 0 & \text{if } x \leqq 0 \\ \min(x, y) & \text{if } x, y \geqq 0 \\ -\min(x, -y) & \text{if } x \geqq 0, \; y \leqq 0 \end{cases} \\
&\mathrm{GLOB}(x_1, x_2, \ldots, x_n) = x_1 \oplus x_2 \oplus \ldots \oplus x_n
\end{aligned}$$

This theory of combining functions based on OAG's has been axiomatized and further developed (see [3], [4]).

2. PARTIALLY ORDERED COMBINING STRUCTURES AND CONSULTING SYSTEMS

A natural generalization of the theory of combining structures based on fully ordered abelian groups is to consider partially ordered structures and accordingly, partially ordered groups.

Why to go to partially ordered combining structures?

There are several reasons for it:

- Multiple sources of knowledge in expert systems: In the practice of knowledge engineering, usually the knowledge base is built with the contributions of several experts which almost never agree completely about the weight they assign to the rules or even in their content themselves. What is actually being done now by the knowledge engineer is either to prefer one expert over the rest, or assign to the rule a "compromise" weight. In any case it is his subjectivity which decides, but he is not an expert in the given domain and therefore there is a "contamination" of the knowledge base. Moreover, the real multidimensionality of the knowledge base is collapsed to an "overall" unidimensional one, since there is only one criteria (weight) associated with each rule and the individuality of the different experts is lost.

  Also, several users might simultaneously consult the expert system for the same case, in which the same primary evidences might be seen differently according with each user. Again these propositions will have multiple weights.
- Another reason comes from the theoretical result concerning the representation of OAG's which are finitely generated as a Hahn product of Archimedean OAG's.

A natural way how to extend the theory is by considering n copies of the original OAG G and this leads to the case of *direct products* (powers) of the form $G^n = G \times G \times \ldots \times G$ n-times; an element of G^n is positive if all its coordinates are positive. The group operation is given by the coordinatewise application of the original group operation. Since G is an OAG, G^n will be a *lattice ordered group* (l.o. group or l. group) with elements given by *tuples* $\langle g_1, \ldots, g_n \rangle$ where $g_i \in G$ for each $1 \leqq i \leqq n$. We may construct a combining structure by adding two elements (extremals) $\top^n$, $\bot_n$ given by $\top^n = \langle \top, \ldots, \top \rangle$, $\bot_n = \langle \bot, \ldots, \bot \rangle$ n-times. Since the meaning of the extremals are truth and falsity, this means that truth is understood in the sence that all the experts coincide on it, which is intuitively clear.

The combining functions are given by the coordinatewise application of the corresponding functions in the combining structure constructed with G. We call this partially ordered combining structure a *vector combining structure* (which is lattice ordered), and it is a special case of a *lattice ordered combining structure* (LOCS). However at first glance it is not clear how to construct a LOCS starting from an arbitrary l.o. group.

The result of a consultation with a vector combining structure (VCS) will be a vector of weights for both propositions and the contributions of the rules. Thus, this can be interpreted as n consultations made in parallel in which each expert is consulted simultaneously but independently from the others. However, since the result is a vector of weights, in the case of VCS's the task of comparing and deciding with these might be difficult because incomparable elements may appear in the set of solutions, and it is necessary to compare them since decisions are expected from an expert system. Then either this task may be left to the user or a *linearization* must be considered.

A linearization must be understood as an order preserving mapping to a fully ordered combining structure trying to preserve as much structure as possible, first of all the group operation.

Now consider the case of a linearization of an l. group of the form G^n (direct product of n copies of the OAG G) into an OAG, in particular, onto G. Assume G Archimedean, i.e. $G \subseteq \mathbb{R}$ ($\mathbb{R}$ are the additive reals).

Define in G^n subsets L_a of the form ($a_i \neq 0$, $a_i \in \mathbb{R}$) as follows:

$$L_a = \{(x_1, \ldots, x_n) : \sum_{i=1}^{n} a_i x_i\,,\ x_i \in G,\ 1 \leqq i \leqq n\}$$

The main result reads as follows (for a proof see [4]).

Theorem. L_a is a subgroup of G^n, and in particular if the signs of all the a are the same, then G^n/L_a is o-isomorphic with the OAG G. (For all notions see again [1].)

This result provides a way how to construct a linearization by using subgroups of the kind L_a which can be interpreted as "weighted group sums" of the components of the elements of G^n. In other words, and "overall" opinion of the experts.

Examples of linearizations are:

a) Epimorphic mapping of G^n onto G or to an isomorphic copy of it by the formula $f(g_1, \ldots, g_n) = a_1 g_1 \oplus \ldots \oplus a_n g_n$ with the a's satisfying the conditions of the above theorem.

b) Linearization using the Hahn product (lexicographic product). In this case $(g_1, \ldots, g_n) \leqq (h_1, \ldots, h_n)$ iff for the first i such that $g_i \neq h_i$ we have $g_i \leqq h_i$. This results in a non-Archimedean OAG whenever $n \geqq 2$. This means that the experts are linearly ordered according to their credit, i.e. the ith expert decides if the expert of higher credit $(i - 1)$ cannot.

Thus, in this case we have an o-homomorphism of the l.o. group G^n onto a fully ordered group, H say, but the mapping is not a homomorphism of the corresponding combining structures since CONJ and therefore CTR are not preserved. This means that to run the consultation in the combining structure constructed with G^n and then linearize to G is not the same as to linearize G^n first and run the consultation in G.

A necessary goal in this direction is to obtain a classification of all linearizations, but this leads to the problem of finding all convex subgroups of G^n with G being any OAG, which seems to be an open question.

3. VEXN, AN EXPERT SYSTEM WORKING WITH PARTIALLY ORDERED COMBINING STRUCTURES

A small prototype expert system working with partially ordered combining structures called VEXN was implemented.

Here the number of experts must be set forth (n). Then a vector combining structure G^n is constructed after n copies of an Archimedean OAG (PROSPECTOR's G) and one is given to each expert when creating the knowledge base. The consultation will go on G^n and finally a linearization can be made according to the following ways:

a) Factorization of G^n by a L_a subgroup (given by an a-vector) representing the "credits" of the experts.

b) Linearization by a Hahn product (ranking the experts according to their credit).

c) A combination of both: m subsets of experts with m_i expeits each can be defined $(1 \leqq m_i \leqq n, \sum_{i=1}^{m} m_i = n)$, and a ranking over the subsets. Then

First: factorization od $G^n \to G^m$ using subgroups given by m a-vectors.

Second: linearization by a Hahn product of G^m according to the credit given to the subsets of the original m experts.

This last case accounts for the situation of n experts belonging to m "teams" or "schools". The first step produces an "overall" criteria representative of the team, and the second, the decision according to the relative rank (credit) of the different teams. Several variants of the above mentioned mechanisms are considered as well in VEXN's implementation allowing a rather detailed use of real multiexpert knowledge during a consultation. The case of a multiuser consultation runs along similar lines and the general case of a multiuser consultation of a multiexpert knowledge base is also considered (under some constraints).

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