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Kybernetika, Vol. 21 (1985), No. 6, 470--479

Persistent URL: http://dml.cz/dmlcz/125888

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STATE ESTIMATION IN DISCRETE-TIME DISTRIBUTED PARAMETER SYSTEMS UNDER INCOMPLETE PRIORI INFORMATION ABOUT THE SYSTEM

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The algorithm of the state estimation is presented for linear discrete-time distributed parameter systems in case of incomplete priori information concerning system parameters and statistic characteristics of noises. The algorithm is based on the correction of the covariance matrix of the estimation error taking into consideration a real, not theoretical, error. Digital simulation results confirm the algorithm stability and its practical utility.

1. INTRODUCTION

The basic state estimation method in lumped [1], [2] and distributed parameter systems (DPS) [3], [4] is the Kalman filter method. Numerous applications of this method in various branches of science, technology confirm a wide range of its possible use [1], [5]. Practical realizations of the Kalman filter (see [6], [7], [8]) show that the real estimation error expressed by the covariance matrix exceeds considerably the values calculated theoretically. The basic reasons for the divergence of the optimal filter may be, among other things, as follows: i) inaccuracy of the mathematical model of the system applied at filter synthesis, ii) calculation errors of digital systems, iii) lack of complete information concerning noise characteristics acting on the dynamic system and the measurement system. The above mentioned resons, except for calculation errors, are errors resulting from incomplete priori information about the real system.

The problem of filter estimates convergence for lumped parameter systems was solved by means of two various methods. The first one consisted in synthetizing adaptive filter algorithms which make it possible to obtain not only state vector estimates, but also parameter estimates [9] or statistical characteristics of noises [10] too. The drawback of such a solution consists in considerable complication of the algorithm filter calculation program. The adaptive Kalman filter for DPS [10] is similarly disadvantageous where characteristics of noises distributed in space are indentified.

From the calculation complexity point of view, the other methods based on "weighing" the covariance matrix are more advisable. These methods consist in correcting the covariance matrix or the filter gain matrix in case where the real values of these matrices differ from those calculated theoretically.

The aim of this paper is to propose a practical algorithm of the state estimation for multi-dimensional linear discrete-time DPS in case of incomplete priori data concerning both system parameters as well as the noises acting on the system. Applying the covariance matrix correction method the stability of the given algorithm was shown on the simple example of digital simulation.

2. PROBLEM STATEMENT

Consider a linear discrete-time stochastic DPS described by

$$(1) \qquad Y(x, k + 1) = \mathscr{L}_x Y(x, k) + A(x, k)\, U(x, k) + B(x, k)\, W(x, k)\,, \quad x \in \Omega\,, \quad k = 1, 2, \ldots, K$$

defined on the open spatial domain Ω of an r-dimensional Euclidean space E^r, $\Omega \subset E^r$, with smooth boundary $\partial\Omega$. Above $k = 1, 2, \ldots, K$ is the discrete time, x is the spatial coordinate r-dimensional vector, $Y(x, k + 1)$ is the n-dimensional state vector of the system, $U(x, k)$ is the p-dimensional control vector, $\mathscr{L}_x$ is a linear spatial $(n \times n)$ matrix differential operator, $A(x, k)$ and $B(x, k)$ are $(n \times p)$ and $(n \times q)$ known matrices. $W(x, k)$ is the Gaussian stochastic process with zero mean and the covariance matrix

$$(2) \qquad \mathsf{E}[W(x, k)\, W^{\mathrm{T}}(y, l)] = Q(x, y, k)\, \delta_{kl}\,, \quad x, y \in \Omega$$

where δ_{kl} is the Kronecker delta function, $\mathsf{E}[\cdot]$ denotes the expectation operator and "T" denotes the transpose of a matrix.

The initial and boundary conditions for (1) are given by

$$(3) \qquad Y(x, k)|_{k=0} = Y_0(x)\,, \quad x \in \Omega$$

$$(4) \qquad \beta_x Y(x, k) = 0\,, \qquad x \in \partial\Omega\,, \quad k = 1, 2, \ldots, K$$

where β_x is a linear spatial $(n \times n)$ matrix differential operator while $Y_0(x)$ is the Gausssian vector-function with mean $\hat{Y}_0(x)$ and the covariance matrix

$$(5) \qquad \mathsf{E}[(Y_0(x) - \hat{Y}_0(x))\,(Y_0(y) - \hat{Y}_0(y))^{\mathrm{T}}] = P_0(x, y)\,.$$

It is assumed that the equation (1) with boundary (4) and initial (3) conditions have the uniquely solution depends continuously on the initial, boundary and control data i.e. the problem is well posed in the sense of Hadamard.

The state $Y(x, k)$ is estimated from measurements which are taken at the select discrete N points x^j, $j = 1, 2, \ldots, N$ of the coordinate space $\bar{\Omega} = \Omega \cup \partial\Omega$ such as

$$(6) \qquad Z(k) = H(k)\, Y_N(k) + V(k)$$

where $Z(k)$ is the mN-dimensional observation vector, $Y_N(x)$ is the nN-dimensional state vector at the measured points, $H(k)$ is $(mN \times nN)$ known matrix which characterizes the measurement system, $V(k)$ is the Gaussian white noise with zero mean and the covariance matrix

$$\mathsf{E}[V(k)\, V^{\mathrm{T}}(l)] = R(k)\, \delta_{kl}\,. \tag{7}$$

It is also assumed that the stochastic processes $W(x, k)$, $V(k)$ and $Y_0(x)$ are mutually independent i.e.

$$\mathsf{E}[W(x, k)\, V^{\mathrm{T}}(l)] = 0\,, \quad \mathsf{E}[W(x, k)\, Y_0^{\mathrm{T}}(y)] = 0\,, \quad \mathsf{E}[V(k)\, Y_0^{\mathrm{T}}(y)] = 0 \tag{8}$$

We assume that both parameters of mathematical model (1), i.e. the matrices $A(x, k)$, $B(x, k)$ and parameters of the operator $\mathscr{L}_x$, as well as the covariance matrices $Q(x, y, k)$ and $R(k)$ are approximate values of the real system. The problem consists in the synthesis of a stable algorithm of the state estimation for the system (1)–(8) with incomplete priori data of its parameters and stochastic characteristics of noises.

3. PROBLEM SOLUTION

3.1. Optimal filter in case of complete priori data

Optimal algorithms filter for this case in the various manner by many authors were derived. So, Watanabe (see [11]) used Wiener-Hopf theory and the discrete-time innovation theory to derive discrete-time distributed filtering algorithm. Analogous results were obtained by Omatu (see [12]). Based on an unibiased and minimum variance estimation error criterion, the optimal filter was derived. According to this work [12] the optimal algorithm filter can be described by the set of equations

$$\begin{aligned}\hat{Y}(x, k+1 \mid k+1) = \mathscr{L}_x \hat{Y}(x, k \mid k) + A(x, k)\, U(x, k) + \\ + K_N(x, k+1)\,[Z(k+1) - H(k+1)\, \mathscr{L}_* Y(k \mid k)]\end{aligned} \tag{9}$$

$$\begin{aligned}K_N(x, k+1) = P_N(x, k+1 \mid k)\, H^{\mathrm{T}}(k+1)\,. \\ .\,[R(k+1) + H(k+1)\, P_{NN}(k+1 \mid k)\, H^{\mathrm{T}}(k+1)]^{-1}\end{aligned} \tag{10}$$

$$P(x, y, k+1 \mid k) = \mathscr{L}_x P(x, y, k \mid k)\, \mathscr{L}_y^{\mathrm{T}} + B(x, k)\, Q(x, y, k)\, B^{\mathrm{T}}(y, k) \tag{11}$$

$$\begin{aligned}P(x, y, k+1 \mid k+1) = \\ = P(x, y, k+1 \mid k) - K_N(x, k+1)\, P_N^{\mathrm{T}}(y, k+1 \mid k)\end{aligned} \tag{12}$$

with boundary conditions

$$\beta_x \hat{Y}(x, k+1 \mid k+1) = 0\,, \quad x \in \partial\Omega \tag{13}$$

$$\beta_x P(x, y, k+1 \mid k) = 0\,, \quad x \in \partial\Omega\,, \quad y \in \bar{\Omega} \tag{14}$$

$$P(x, y, k+1 \mid k)\, \beta_y^{\mathrm{T}} = 0\,, \quad y \in \partial\Omega\,, \quad x \in \bar{\Omega} \tag{15}$$

where $\hat{Y}(x, k+1 \mid k+1)$ is the vector estimate of $Y(x, k+1)$, $K_N(x, k+1)$ is the filter gain matrix at the measured points $x^j \in \bar{\Omega}$, $j = 1, 2, \ldots, N$ i.e.

$$K_N(x, k+1) = [K(x, x^1, k+1)\, K(x, x^2, k+1) \ldots K(x, x^N, k+1)] \tag{16}$$

while $K(x, x^j, k+1)$, $j = 1, 2, \ldots, N$ is an gain matrix. $P(\cdot)$ is a filtering error covariance matrix given by

$$P(x, y, k+1 \mid k+1) = E[\delta Y(x, k+1 \mid k+1)\, \delta Y^{T}(y, k+1 \mid k+1)] \tag{17}$$

$$P(x, y, k+1 \mid k) = E[\delta Y(x, k+1 \mid k)\, \delta Y^{T}(y, k+1 \mid k)] \tag{18}$$

$$P_N(x, k+1 \mid k) = [P(x, x^1, k+1 \mid k)\, P(x, x^2, k+1 \mid k) \ldots \ldots P(x, x^N, k+1 \mid k)] \tag{19}$$

$$P_{NN}(k+1 \mid k) = \begin{bmatrix} P(x^1, x^1, k+1 \mid k) & P(x^1, x^2, k+1 \mid k) & \ldots & P(x^1, x^N, k+1 \mid k) \\ \vdots & \vdots & \ldots & \vdots \\ P(x^N, x^1, k+1 \mid k) & P(x^N, x^2, k+1 \mid k) & \ldots & P(x^N, x^N, k+1 \mid k) \end{bmatrix} \tag{20}$$

and $\mathscr{L}_*[\cdot] = \operatorname{diag}[\mathscr{L}_{x^1}[\cdot]\, \mathscr{L}_{x^2}[\cdot] \ldots \mathscr{L}_{x^N}[\cdot]]$ is the discrete-space operator at the measured points. Above $\delta Y(x, k+1 \mid k+1) = Y(x, k) - \hat{Y}(x, k+1 \mid k+1)$ and $\delta Y(x, k+1 \mid k) = Y(x, k+1) - \hat{Y}(x, k+1 \mid k)$ are filtration error and one-step ahead prediction error, respectively.

Equations (10)–(12) can be solved off-line while filter realization (9)–(12), since their solution depends only on priori data of the system, i.e. matrices $A(x, k)$, $B(x, k)$, $H(k)$, $R(k)$, $Q(x, y, k)$ and operator $\mathscr{L}_x$ form. This solution does not depend on measurements data $Z(k)$ and it is a theoretical solution. Such dependence is not necessary in the case when mathematical model of system (1)–(8) corresponds completely to the real system. From the practical point of view this assumption is difficult to realize at least of two reasons. Firstly, while determining a mathematical model of the real system it is theoreticaly impossible to take into consideration all static properties of the system what always leads to certain simplifications. Secondly, a mathematical model corresponding to a real system is, as a number of applications show (see [13]), complicated and not convenient system at the synthesis of control systems. The activity of the control system designer consists in compromising between a precise and at the same time simple model.

3.2. Suboptimal algorithm in case of incomplete priori data

It follows from the above that while practically realizing the algorithm of the state estimation the problem of a greater or smaller indefinitness of priori data about the system almost always arises, and this turn is the reason of the filter estimates divergence.

The filtering covariance matrix may be corrected by introducing a definite correction coefficient $S(k)$. The correction coefficient is equal $S(k) = 1$ when the estimates are convergent and $S(k) > 1$ when they are divergent. To determine coefficient $S(k)$ we shall use the convergent condition of the optimal filter estimates expressed by an inequality (see [14])

$$(21) \qquad X^{\mathrm{T}}(k+1)\, X(k+1) \leqq \gamma \operatorname{Tr}\{\mathsf{E}[X(k+1)\, X^{\mathrm{T}}(k+1)]\}$$

where $X(k+1) = Z(k+1) - H(k+1)\, \hat{Y}_N(k+1 \mid k) = H(k+1)\, \delta Y_N(k+1 \mid k) + V(k+1)$ is the innovation process, γ is the coefficient ($\gamma \geqq 1$), $\operatorname{Tr}\{\cdot\}$ is the operator of matrix trace $\{\cdot\}$, $\delta Y_N(k+1 \mid k) = Y_N(k+1) - Y_N(k+1 \mid k)$ is the filtered error at the measured points.

It follows from the definition of the innovation process [15] that it is a stochastic process and its realizations are measurable. It can be proved in a simple way that it is the white noise with the zero mean and covariance matrix given by (see [12], [15])

$$(22) \qquad \mathsf{E}[X(k+1)\, X^{\mathrm{T}}(k+1)] = \\ = H(k+1)\, P_{NN}(k+1 \mid k)\, H^{\mathrm{T}}(k+1) + R(k+1)\,.$$

If we assume for convergent condition (21) that $\gamma = 1$, it will be the case of the greatest sensibility of criterion. In this case inequality (21) takes the form of an equality

$$(23) \qquad X^{\mathrm{T}}(k+1)\, X(k+1) = \operatorname{Tr}\{\mathsf{E}[X(k+1)\, X^{\mathrm{T}}(k+1)]\}$$

from which it follows that

$$(24) \qquad X(k+1)\, X^{\mathrm{T}}(k+1) = \mathsf{E}[X(k+1)\, X^{\mathrm{T}}(k+1)]\,.$$

It is necessary to pay attention to the fact that the left sides of conditions (21), (22) and (24) can be determined on the basis of measuring innovation process and the right sides – by using property (22). From (22) and (24) follows

$$(25) \qquad X(k+1)\, X^{\mathrm{T}}(k+1) = H(k+1)\, P_{NN}(k+1 \mid k)\, H^{\mathrm{T}}(k+1) + R(k+1)\,.$$

In the case when estimates are divergent the correction of the filtering covariance matrix may be made by multiplying it by the scalar coefficient $S(k)$, i.e.

$$(26) \qquad P(x, y, k+1 \mid k) = S(k+1)\, \mathscr{L}_x P(x, y, k \mid k)\, \mathscr{L}_y^{\mathrm{T}} + \\ + B(x, k)\, Q(x, y, k)\, B^{\mathrm{T}}(y, k)\,.$$

For calculating of the coefficient $S(k+1)$ rewritten the equation (26) for measured points only, i.e.

$$(27) \qquad P_{NN}(k+1 \mid k) = S(k+1)\, \mathscr{L}_* P_{NN}(k \mid k)\, \mathscr{L}_*^{\mathrm{T}} + B_N(k)\, Q_{NN}(k)\, B_N^{\mathrm{T}}(k)$$

where $B_N(k) = [B(x^1, k)\, B(x^2, k) \ldots B(x^N, k)]$

$$(28) \qquad Q_{NN}(k) = \begin{bmatrix} Q(x^1, x^1, k) & Q(x^1, x^2, k) & \ldots & Q(x^1, x^N, k) \\ \vdots & \vdots & \ldots & \vdots \\ Q(x^N, x^1, k) & Q(x^N, x^2, k) & \ldots & Q(x^N, x^N, k) \end{bmatrix}$$

Now substituting (27) to (25) gives

$$S(k+1)\,H(k+1)\,\mathscr{L}_*P_{NN}(k\mid k)\,\mathscr{L}_*^{\mathrm{T}}H^{\mathrm{T}}(k+1) = X(k+1)\,X^{\mathrm{T}}(k+1) - H(k+1)\times B_N(k)\,Q_{NN}(k)\,B_N^{\mathrm{T}}(k)\,H^{\mathrm{T}}(k+1) - R(k+1)\,. \tag{29}$$

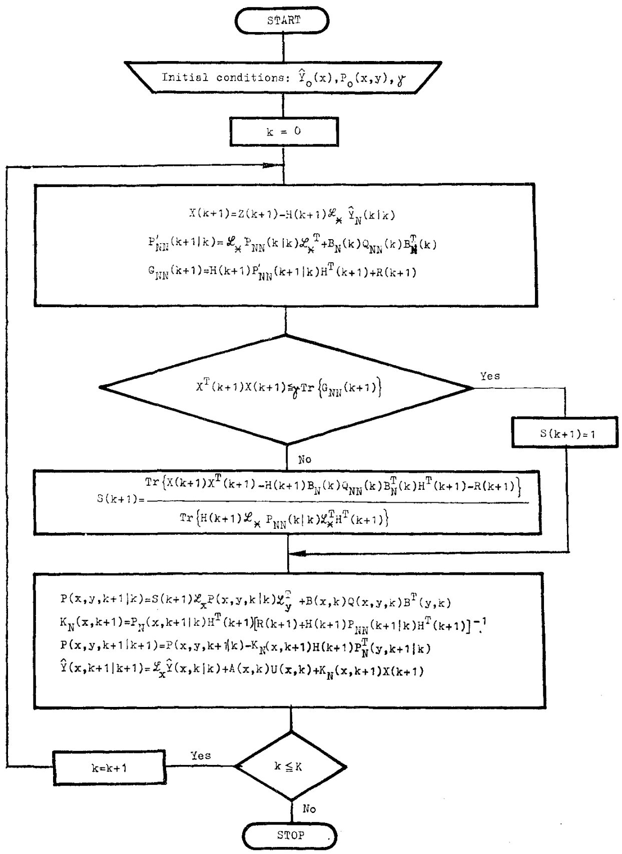

Fig. 1. Practical algorithm of the discrete-time distributed filter.

Since $S(k+1)$ is a scalar, then determining the traces of the right and left side of equality (29) we get

$$(30)\qquad S(k+1) = \\ = \frac{\operatorname{Tr}\{X(k+1)\,X^{\mathrm{T}}(k+1) - H(k+1)\,B_N(k)\,Q_{NN}(k)\,B_N^{\mathrm{T}}(k)\,H^{\mathrm{T}}(k+1) - R(k+1)\}}{\operatorname{Tr}\{H(k+1)\,\mathscr{L}_* P_{NN}(k \mid k)\,\mathscr{L}_*^{\mathrm{T}} H^{\mathrm{T}}(k+1)\}}$$

Finally, the algorithm of the discrete-time distributed filter with the correction of the filtering error covariance may be presented as on a block diagram Fig. 1. According to the block diagram in case of satisfying the convergence condition (21) the optimal filter algorithm is realized and $S(k+1) = 1$. In the other case the filtering covariance matrix $P_{NN}(k+1 \mid k)$, the filter gain $K_N(x, k+1)$ and the covariance $P(x, y, k + + 1 \mid k+1)$ are corrected. The covariance $P_{NN}(k+1 \mid k)$ is multiply by $S(k+1) > > 1$ and then $P(x, y, k+1 \mid k)$, $K_N(x, k+1)$ and $P(x, y, k+1 \mid k)$ are calculated.

4. EXAMPLE

As an example we shall consider a system described by a thermal conductivity equation in the form

$$(31)\qquad \frac{\partial Y(x,t)}{\partial t} = a^2 \frac{\partial^2 Y(x,t)}{\partial x^2} + c\,U(x,t) + b\,W(x,t)$$

with boundary and initial conditions

$$(32)\qquad Y(0,t) = Y(L,t) = 0\,, \quad 0 \leqq x \leqq L\,, \quad L = 1$$

$$Y(x,t)\big|_{t=0} = Y_0(x) = \sin(\pi x/L)$$

The measurement system is described by an equation

$$(34)\qquad Z(t) = Y_N(t) + V(t)\,, \quad N = 5\,.$$

The parameters and input deterministic signal $U(x, t)$ in equation (31) are: $a^2 = 1$, $b = 1{\cdot}5$, $c = 10$, and $U(x, t) = 1(t)$. Noise characteristics $W(x, t)$ and $V(t)$ correspond to the expressions: $R(t) = \operatorname{diag}[\sigma_V^2] = \operatorname{diag}[0{\cdot}032]$, $Q(x, y, t) = \sigma_W^2 = 5 \,.\, 10^{-3}$.

For exact derivation of the discrete-time model of the system (31) it is necessary to apply a rigorous approach, for example given by Watanabe (see [11]). Here will be consider some simple approach only. Using the difference scheme

$$(35)\qquad \frac{\partial Y(x,t)}{\partial t} \approxeq \frac{Y(x,k+1) - Y(x,k)}{\Delta t}$$

the discrete-time model of the system (31) may be described by

$$(36)\qquad Y(x,k+1) = \mathscr{L}_x Y(x,k) + \bar{c}U(x,k) + \bar{b}W(x,k)$$

where $\mathscr{L}_x[\cdot] = [\cdot] + \Delta t a^2\, \partial^2[\cdot]/\partial x^2$, $\bar{c} = \Delta t c$, $\bar{b} = \Delta t b$, $\Delta t = 0{\cdot}01$ is the sampling interval. Differential operator of the system (31) $\mathscr{L}'_x[\cdot] = a^2(\partial^2[\cdot]/\partial x^2)$ is a self-adjoint and has an eigenvalues λ_i and an eigenfunction $\psi_i(x)$ as follows: $\lambda_i = (i\pi a)^2$, $\psi_i(x) = \sqrt{(2)} \sin(i\pi x)$. The truncation number of the expansion coefficient was applied as $M = 5$.

Dynamic system (31), measurement system (34) and the filter (9)–(15) and (21), (27), (30) were simulated on the computer EC-1022 using Fourier's method. While the realizing the filter it was assumed that system parameters and noise characteristics

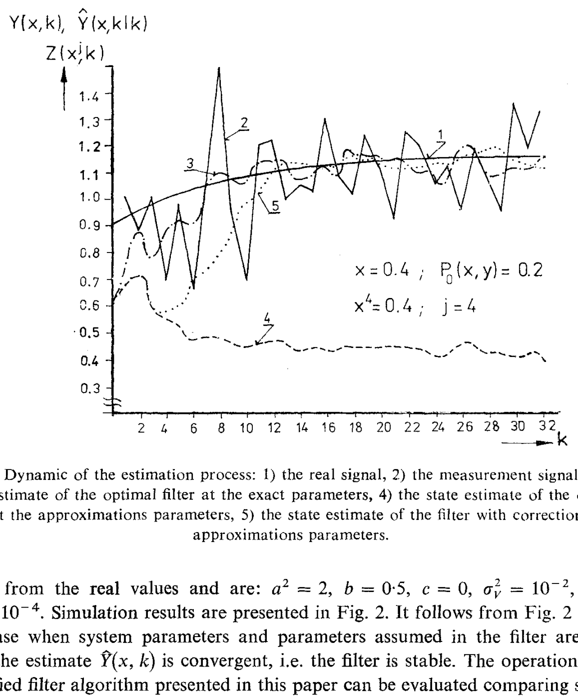

Fig. 2. Dynamic of the estimation process: 1) the real signal, 2) the measurement signal, 3) the state estimate of the optimal filter at the exact parameters, 4) the state estimate of the optimal filter at the approximations parameters, 5) the state estimate of the filter with correction at the approximations parameters.

differ from the real values and are: $a^2 = 2$, $b = 0{\cdot}5$, $c = 0$, $\sigma_V^2 = 10^{-2}$, $\sigma_W^2 = 8 \,.\, 10^{-4}$. Simulation results are presented in Fig. 2. It follows from Fig. 2 that in the case when system parameters and parameters assumed in the filter are equal then the estimate $\hat{Y}(x, k)$ is convergent, i.e. the filter is stable. The operation of the modified filter algorithm presented in this paper can be evaluated comparing simulation results (see signals 4 and 5 in Fig. 2). Owing to the application the correction method of covariance matrix, divergent filter estimates become convergent.

5. CONCLUSIONS

A synthesis of control systems for a great number of complex technological objects of power industry, (nuclear power), chemical, metallurgical, cement and others due to required control quality, should be based on the results of a control theory of stochastic DPS [16]. One of the basic requirements of control system synthesis is state or/and parameters estimation. The filter algorithm presented in this paper makes it possible to avoid basic difficulties connected with practical realization of the distributed filter. Reducing the requirements concerning the precise knowledge of the mathematical model and noise characteristics give the possibility to realize a stable algorithm. In the our algorithm the covariance matrix, similarly as in an optimal algorithm for linear systems, can be determined off-line and only its correction and the estimates should be determined on-line since they depend on the measurement data.

Another practical problems connected with the distributed filter realization for the DPS are: the state estimation and measurement location for the non-linear systems. The works in this direction will be presented later.

(Received January 11, 1984.)

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