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ON ALMOST QUASICONTINUOUS FUNCTIONS

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Summary. A function \( f: X \rightarrow Y \) is said to be almost quasicontinuous at \( x \in X \) if \( x \in \text{Cl} \text{Int} f^{-1}(V) \) for each neighbourhood \( V \) of \( f(x) \). Some properties of these functions are investigated.

Keywords: Almost quasicontinuity, \( \beta \)-continuity, Separate almost quasicontinuity

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Let \( X \) and \( Y \) be topological spaces. For a subset \( A \) of a topological space denote \( \text{Cl} A \) and \( \text{Int} A \) the closure and the interior of \( A \), respectively. The letters \( \mathbb{N}, \mathbb{Q} \) and \( \mathbb{R} \) stand for the set of natural, rational and real numbers, respectively.

A set \( A \) is called semi-open [8] (quasi-open [11]), if \( A \subset \text{Cl} \text{Int} A \), pre-open [10] (nearly open [18]), if \( A \subset \text{Int} \text{Cl} A \), \( \beta \)-open [1] (semi-preopen [2]), if \( A \subset \text{Cl} \text{Int} \text{Cl} A \), somewhat nearly open [18], if \( \text{Int} \text{Cl} A \neq \emptyset \).

Let \( f: X \rightarrow Y \) be a function and \( x \in X \). A function \( f \) is called quasicontinuous at \( x \) [9], if \( x \in \text{Cl} \text{Int} f^{-1}(V) \), almost continuous at \( x \) [5] (nearly continuous at \( x \) [18]), if \( x \in \text{Int} \text{Cl} f^{-1}(V) \), almost quasicontinuous at \( x \) [3], [15], if \( x \in \text{Cl} \text{Int} \text{Cl} f^{-1}(V) \), for each neighbourhood \( V \) of \( f(x) \).

A function \( f: X \rightarrow Y \) is quasicontinuous (almost continuous, almost quasicontinuous), if it is such at every point. A function \( f \) is called semi-continuous [8] (pre-continuous [10], \( \beta \)-continuous [1]), if \( f^{-1}(V) \) is semi-open (pre-open, \( \beta \)-open) for each open set \( V \) in \( Y \). A function \( f \) is somewhat continuous [6] (somewhat nearly continuous [18]), if \( \text{Int} f^{-1}(V) \neq \emptyset \) (\( f^{-1}(V) \) is somewhat nearly open) for each open \( V \) in \( Y \) such that \( f^{-1}(V) \neq \emptyset \). Evidently, \( f \) is pre-continuous iff \( f \) is almost continuous and \( f \) is semi-continuous iff \( f \) is quasicontinuous [14].

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The notion of almost quasicontinuity is a simultaneous generalization of almost continuity and of quasicontinuity. Properties of almost quasicontinuous functions are studied in [1], [3], [15], [16]. In this paper we shall show further properties of these functions. We also give answers to three Piotrowski's questions.

Immediately we see that \( f \) is almost quasicontinuous if and only if it is \( \beta \)-continuous. This is also true "pointwise".

**Theorem 1.** Let \( f: X \to Y \) and \( x \in X \). Then the following conditions are equivalent:

1. \( f \) is almost quasicontinuous at \( x \),
2. for each neighbourhood \( V \) of \( f(x) \) and each neighbourhood \( U \) of \( x \), \( f^{-1}(V) \cap U \) is not a nowhere dense set,
3. for each neighbourhood \( V \) of \( f(x) \) there is a \( \beta \)-open set \( U \) such that \( x \in U \) and \( f(U) \subset V \).

**Proof.** We shall prove (2) \( \Rightarrow \) (3). Other implications are obvious.

Let \( V \) be a neighbourhood of \( f(x) \). Then for each neighbourhood \( U \) of \( x \) there is a nonempty open set \( G_U \subset U \) such that \( G_U \subset \text{Cl} f^{-1}(V) \). Denote \( H_U = G_U \cap f^{-1}(V) \neq \emptyset \). Let \( H = \bigcup \{ H_U : U \text{ is a neighbourhood of } x \} \). Then \( x \in H \) and \( f(H) \subset V \). Let \( z \in \text{Cl} G_U \) and let \( T \) be an open neighbourhood of \( z \). Then \( T \cap G_U \) is a nonempty open set. Let \( u \in T \cap G_U \). Then \( u \in \text{Cl} f^{-1}(V) \) and hence \( \emptyset \neq (T \cap G_U) \cap f^{-1}(V) = H_U \cap T \). This yields \( z \in \text{Cl} H_U \) and \( \text{Cl} G_U \subset \text{Cl} H_U \). Since evidently \( \text{Cl} H_U \subset \text{Cl} G_U \), we have \( \text{Cl} G_U = \text{Cl} H_U \). Hence for each neighbourhood \( U \) of \( x \) we have \( H_U \subset G_U \subset \text{Int} \text{Cl} G_U = \text{Int} \text{Cl} H_U \subset \text{Int} \text{Cl} H \).

Let \( y \in H \). If \( y \neq x \), then there is a neighbourhood \( U \) of \( x \) such that \( y \in H_U \). Then \( y \in \text{Cl} \text{Int} \text{Cl} H \). If \( y = x \) and \( U \) is a neighbourhood of \( x \), then \( \emptyset \neq H_U \subset U \cap \text{Int} \text{Cl} H \) and hence \( x \in \text{Cl} \text{Int} \text{Cl} H \). Therefore \( H \) is a \( \beta \)-open set.

Evidently, every almost quasicontinuous function is somewhat nearly continuous. The converse is not true; however, we have

**Proposition 1.** A function \( f: X \to Y \) is almost quasicontinuous if and only if there is a base \( \mathcal{B} \) of the space \( X \) such that \( f|_B \) is somewhat nearly continuous for each \( B \in \mathcal{B} \).

**Proof.** Necessity follows from the obvious fact that the restriction of an almost quasicontinuous function to an open subspace is almost quasicontinuous.

Sufficiency. Let \( x \in X \), let \( U \) be an open neighbourhood of \( f(x) \) and let \( V \) be an open neighbourhood of \( x \). Let \( B \in \mathcal{B} \) be such that \( x \in B \subset U \). Then \( (f|_B)^{-1}(V) \neq \emptyset \) and hence \( \emptyset \neq \text{Int} \text{Cl} (f|_B)^{-1}(V) \subset \text{Int} \text{Cl} f^{-1}(V) \cap \text{Int} \text{Cl} B \). From this we get \( \text{Int} \text{Cl} f^{-1}(V) \cap B \neq \emptyset \) and hence \( x \in \text{Cl} \text{Int} \text{Cl} f^{-1}(V) \).
Proposition 1 shows that a relation between almost quasicontinuity and somewhat nearly continuity is similar to that between quasicontinuity and somewhat continuity (see [12]). Next proposition shows a similar relation between almost quasicontinuity and almost continuity and between quasicontinuity and continuity (see [11]).

**Proposition 2.** Let $X$ be a first countable Hausdorff space and let $Y$ be a first countable space. Let $x \in X$. Then $f: X \to Y$ is almost quasicontinuous at $x$ if and only if there is a semi-open set $A$ containing $x$ such that $f|_A$ is almost continuous at $x$.

**Proof.** Necessity. If $\{x\}$ is an open set, then we choose $A = \{x\}$. Let $\{x\}$ be not open, let $(V_n)$ be a nonincreasing base of neighbourhoods of $f(x)$ and $(U_n)$ a nonincreasing base of neighbourhoods of $x$. Then there is a nonempty open set $G_1 \subset U_1$ such that $G_1 \subset \text{Cl}f^{-1}(V_1)$. Evidently $G_1 \neq \{x\}$. Since $X$ is Hausdorff, there is $n_2 > 1$ such that $G_1 - \text{Cl}U_{n_2} \neq \emptyset$. Further there is an open nonempty set $G_2 \subset U_{n_2}$ such that $G_2 \subset \text{Cl}f^{-1}(V_2)$. In this way, we construct an increasing sequence $(n_k)$ of natural numbers (where $n_1 = 1$) and a sequence $(G_k)$ of nonempty open sets such that $G_k \subset U_{n_k}$, $G_k \subset \text{Cl}f^{-1}(V_k)$ and $G_k - \text{Cl}U_{n_{k+1}} \neq \emptyset$. Denote $A = \bigcup_{k=1}^{\infty} (G_k - \text{Cl}U_{n_{k+1}}) \cup \{x\}$. Then $A$ is a semi-open set containing $x$. Since for each $i \in \mathbb{N}$ we have $A \cap U_{n_i} \subset \text{Cl}f^{-1}(V_i)$, $f|_A$ is almost continuous at $x$.

Sufficiency. Let $U$ and $V$ be open neighbourhoods of $x$ and $f(x)$, respectively. Then there is an open neighbourhood $H$ of $x$ such that $A \cap H \subset \text{Cl}(f|_A)^{-1}(V) \subset \text{Cl}f^{-1}(V)$. Since $x \in \text{Cl}A \cap H \cap U$ is a nonempty open set and $G \subset U \cap \text{Cl}f^{-1}(V)$.

**Remark 1.** It is shown in [15] that almost quasicontinuous functions are closed with respect to uniform convergence. This is not true for pointwise convergence. In fact, every function $f: \mathbb{R} \to \mathbb{R}$ is a sum of two almost quasicontinuous functions and a limit of a sequence of almost quasicontinuous functions. By [4; p. 5] we can write $f = g + h$, where $g$ and $h$ are Darboux functions such that $g^{-1}(c)$ and $h^{-1}(c)$ are dense sets for each $c \in \mathbb{R}$. Similarly, we can write $f = \lim_{n \to \infty} f_n$, where $f_n$ are Darboux functions such that $f_n^{-1}(c)$ are dense sets for each $c \in \mathbb{R}$. Evidently, $g$, $h$, $f_n$ are almost quasicontinuous functions.

**Remark 2.** There is a Darboux function, which is not almost quasicontinuous. By [4; p. 13] there is a Darboux function $f$ which is zero on the complement of the Cantor set, but not identically zero. This function is not almost quasicontinuous.

A subset $A$ of $X$ is called $\beta$-closed [1] (semi-preclosed [2]), if $X - A$ is $\beta$-open, i.e. if $\text{Int} \text{Cl} \text{Int} A \subset A$. We say that a function $f: X \to Y$ has a $\beta$-closed graph if the
graph of \( f \), i.e. the set \( G(f) = \{(x, y) \in X \times Y : y = f(x)\} \) is a \( \beta \)-closed subset of the product \( X \times Y \).

**Proposition 3.** Let \( Y \) be a Hausdorff space and let \( f: X \to Y \) be an almost quasicontinuous function. Then \( f \) has a \( \beta \)-closed graph.

**Proof.** Let \((x, y) \in X \times Y - G(f)\). Then there are disjoint open sets \( A_{xy} \) and \( B_{xy} \) in \( Y \) such that \( f(x) \in A_{xy} \) and \( y \in B_{xy} \). The almost quasicontinuity of \( f \) gives that \( f^{-1}(A_{xy}) \) is a \( \beta \)-open set in \( X \). It is easy to see that \( f^{-1}(A_{xy}) \times B_{xy} \) is a \( \beta \)-open set in \( X \times Y \) and by [2] the set \( T = \bigcup \{ f^{-1}(A_{xy}) \times B_{xy} : (x, y) \in X \times Y - G(f)\} \) is \( \beta \)-open in \( X \times Y \). We see that \( X \times Y - G(f) = T \) and hence \( G(f) \) is \( \beta \)-closed. \( \square \)

Obviously, the converse assertion is not true. Denote by \( B_f \) the set of all almost quasicontinuity points of \( f \). We characterize this set.

**Lemma 1.** (See also [15].) Let \( Y \) be a second countable space. Let \( f: X \to Y \). Then \( X - B_f \) is a set of the first category.

**Lemma 2.** Let \( Y \) be a first countable Hausdorff space which has at least one accumulation point. Let \( A \subset X \) be a set such that \( X - A \) is a set of the first category. Then there is a function \( f: X \to Y \) such that \( B_f = A \).

**Proof.** We can write \( X - A = \bigcup_{n=1}^{\infty} A_n \), where \( A_n \) are nowhere dense pairwise disjoint sets. Let \( y_0 \) be an accumulation point of \( Y \) and let \( \{y_n : n \in \mathbb{N}\} \) be a one-to-one sequence converging to \( y_0 \) such that \( y_n \neq y_0 \) for each \( n \in \mathbb{N} \). Define a function \( f: X \to Y \) as

\[
f(x) = \begin{cases} 
y_n, & \text{for } x \in A_n, \\
y_0, & \text{for } x \in A.
\end{cases}
\]

We shall show that \( B_f = A \). Let \( x \in A \) and let \( V \) be a neighbourhood of \( f(x) = y_0 \). Then there is a finite set \( K \subset \mathbb{N} \) such that \( f^{-1}(V) = X - \bigcup_{i \in K} A_i \). Therefore \( f^{-1}(V) \) is dense in \( X \) and \( x \in B_f \).

Let \( x \in A_n \) for some \( n \in \mathbb{N} \). Let \( S \) and \( T \) be disjoint neighbourhoods of \( y_0 \) and \( y_n \), respectively. Then there is a finite set \( K \subset \mathbb{N} \) such that \( y_i \in S \) for each \( i \in \mathbb{N} - K \). Therefore \( T \cap f(X) \subset \bigcup_{i \in K} \{y_i\} \) and \( f^{-1}(T) \subset \bigcup_{i \in K} A_i \). This yields \( x \notin B_f \). \( \square \)

The condition \( Y \) is Hausdorff cannot be replaced by \( Y \) is \( T_1 \) as the following example shows.

**Example 1.** Let \( X = \mathbb{Q} \) with the usual topology. Let \( Y = \mathbb{N} \) and let a set \( S \subset Y \) be closed if \( S \) is a finite set or \( S = \mathbb{N} \). Then \( Y \) is a first countable \( T_1 \)-space.
without isolated points and $X - \emptyset$ is a set of the first category. Let $f : X \to Y$ be an arbitrary function. We shall show that $B_f \neq \emptyset$. We have two possibilities.

a) There is $y \in Y$ such that $f^{-1}(y)$ is not nowhere dense. Then there is a nonempty open set $G$ such that $G \subseteq \text{Cl} f^{-1}(y)$. Let $x \in G \cap f^{-1}(y)$, let $V$ be a neighbourhood of $f(x)$ and let $U$ be a neighbourhood of $x$. Then $f^{-1}(V) \cap U$ is dense in $G \cap U$ and hence $x \in B_f$.

b) For each $y \in Y$ the set $f^{-1}(y)$ is nowhere dense. Then for each nonempty open set $V$ in $Y$ the set $f^{-1}(Y - V)$ is nowhere dense and hence $G \cap f^{-1}(V)$ is nowhere dense for no nonempty open set $G$ in $X$. Therefore $B_f = X$.

**Theorem 2.** Let $X$ be a topological space and let $Y$ be a second countable Hausdorff space which has at least one accumulation point. Let $A \subseteq X$ be a set. Then $X - A$ is of the first category if and only if there is a function $f : X \to Y$ such that $A \subseteq B_f$.

Similarly as almost quasicontinuity we may define “almost cliquishness”.

**Definition 1.** Let $(Y,d)$ be a metric space. We say that a function $f : X \to Y$ is almost cliquish at $x \in X$, if for each $\varepsilon > 0$ and for each neighbourhood $U$ of $x$ there is a nonempty open set $G \subseteq U$ and a set $H$ such that $H$ is dense in $G$ and $d(f(y), f(z)) < \varepsilon$ for each $y, z \in H$. Denote by $Z_f$ the set of all almost cliquishness points of $f$. If $Z_f = X$, we say that $f$ is almost cliquish.

Easy we see that $Z_f$ is a closed set and $B_f \subseteq Z_f$. Hence by Lemma 1 we have

**Proposition 3.** Let $X$ be a Baire space and let $(Y,d)$ be a separable metric space. Then every function $f : X \to Y$ is almost cliquish.

We recall that a family $\mathcal{A}$ of nonempty open sets in $X$ is a pseudo-base [17] if every nonempty open subset of $X$ contains some member of $\mathcal{A}$. (The space $\mathbb{R}$ has a countable pseudo-base, but it is not second countable [17]). For a function $f : X \times Y \to Z$ the symbols $f_x, f^y$ denote its $x$-section or $y$-section, respectively, i.e. $f_x$ is the function defined on $Y$ such that $f_x(y) = f(x,y)$ for each $x \in X$ and analogically $f^y$.

We shall show that there is a function $f : \mathbb{R}^2 \to \mathbb{R}$, which is separately almost quasicontinuous but not almost quasicontinuous. However, the following statement is true

**Theorem 3.** Let $X$ be a Baire space, let $Y$ possess locally a countable pseudo-base and let $Z$ be an arbitrary topological space. Let $f : X \times Y \to Z$ be such that $f^y$ is quasicontinuous for each $y \in Y$ and $f_x$ is almost quasicontinuous with the exception of a set of the first category. Then $f$ is almost quasicontinuous.
Proof. Suppose that \( f \) is not almost quasicontinuous. Then there is a point \((a, b) \in X \times Y\) and open neighbourhoods \(G, U\) and \(V\) of \(f(a, b)\), \(a\) and \(b\), respectively, such that

\[
\text{Int} \text{Cl} f^{-1}(G) \cap (U \times V) = \emptyset.
\]

Without loss of generality we may assume that \(\{V_n : n \in \mathbb{N}\}\) is a countable pseudo-base in \(V\). The quasicontinuity of \(f^b\) at \(a\) gives

\[
A = \text{Int}(f^b)^{-1}(G) \cap U \neq \emptyset.
\]

Let \(T = \{x \in A : f_x \text{ is almost quasicontinuous}\}\) and

\[
T_n = \{x \in T : V_n \subset \text{Int} \text{Cl}(f_x)^{-1}(G)\}.
\]

We shall show that \(T = \bigcup_{n=1}^{\infty} T_n\). If \(x \in T\), then \(x \in A\) and hence \(f^b(x) \in G\).

Therefore \(b \in (f_x)^{-1}(G) \cap V\) and the almost quasicontinuity of \(f_x\) at \(b\) gives \(b \in \text{Cl} \text{Int} \text{Cl}(f_x)^{-1}(G)\) and this yields \(\text{Int} \text{Cl}(f_x)^{-1}(G) \cap V \neq \emptyset\). Hence there is \(n \in \mathbb{N}\) such that \(V_n \subset \text{Int} \text{Cl}(f_x)^{-1}(G)\) and \(x \in T_n\).

We shall prove that \(T_n\) is nowhere dense in \(A\) for each \(n \in \mathbb{N}\). Let \(n \in \mathbb{N}\) and let \(S \subset A\) be an open set. Then, in regard of (\(*)\), there is a nonempty open set \(K \subset S \times V_n\) such that \(K \cap f^{-1}(G) = \emptyset\). We may assume that \(K = K_1 \times K_2\), where \(K_1 \subset S\) and \(K_2 \subset V_n\) are nonempty open sets.

Let \(x \in K_1\) and \(y \in K_2\). Then \(f(x, y) \notin G\) and thus \(y \notin (f_x)^{-1}(G)\). This is true for each \(y \in K_2\) and therefore \(K_2 \cap (f_x)^{-1}(G) = \emptyset\). This yields \(K_2 \cap \text{Cl} \text{Int} \text{Cl}(f_x)^{-1}(G) = \emptyset\) and therefore \(V_n\) is not a subset of \(\text{Int} \text{Cl}(f_x)^{-1}(G)\). This is true for each \(x \in K_1\) and therefore \(K_1 \cap T_n = \emptyset\), i.e. \(T_n\) is nowhere dense in \(A\). Then \(T\) is of the first category, a contradiction. \(\square\)

Example 2. There is a function \(f : \mathbb{R}^2 \to \mathbb{R}\) such that

(i) functions \(f_x, f^y\) are continuous with the exception of a set of the first category,
(ii) functions \(f_x, f^y\) are almost continuous,
(iii) the function \(f\) is not somewhat nearly continuous.

Let \(\{t_n : n \in \mathbb{N}\}\) be a dense set in \(\mathbb{R}^2\) such that \(t_n = (p_n, q_n)\), where \(p_n\) and \(q_n\) are irrational numbers for each \(n \in \mathbb{N}\). Let \(\{u_n : n \in \mathbb{N}\}\), \(\{v_n : n \in \mathbb{N}\}\) be one-to-one sequences of all rational numbers. Denote

\[
P_n = \{(x, y) \in \mathbb{R}^2 : y = v_n\}\]

\[Q_n = \{(x, y) \in \mathbb{R}^2 : x = u_n\}.
\]

Since \(t_1 \notin P_1 \cup Q_1\), there is an open set \(V_1 = (a_1, b_1) \times (c_1, d_1)\) such that \(a_1, b_1, c_1, d_1\) are irrational numbers, \(t_1 \in V_1\) and \(V_1 \cap (P_1 \cup Q_1) = \emptyset\). Suppose that we
have open sets $V_1, \ldots, V_k$ such that $V_j = (a_j, b_j) \times (c_j, d_j)$, where $a_j, b_j, c_j, d_j$ are irrational numbers, $t_j \in V_j$ and $V_j \cap \left( \bigcup_{i=1}^{j} P_i \cup \bigcup_{i=1}^{j} Q_i \right) = \emptyset$ for each $j \in \{1, 2, \ldots, k\}$.

Since $t_{k+1} \notin \bigcup_{i=1}^{k+1} P_i \cup \bigcup_{i=1}^{k+1} Q_i$, there is an open set $V_{k+1} = (a_{k+1}, b_{k+1}) \times (c_{k+1}, d_{k+1})$, where $a_{k+1}, b_{k+1}, c_{k+1}, d_{k+1}$ are irrational numbers, such that $t_{k+1} \in V_{k+1}$ and $V_{k+1} \cap \left( \bigcup_{i=1}^{k+1} P_i \cup \bigcup_{i=1}^{k+1} Q_i \right) = \emptyset$.

Denote $T = \bigcup_{n=1}^{\infty} V_n$. Then $T$ is an open dense set and hence $\mathbb{R}^2 - T$ is a nonempty nowhere dense set. Define a function $f : \mathbb{R}^2 \to \mathbb{R}$ as

$$f(x, y) = \begin{cases} 1, & \text{for } (x, y) \in \mathbb{Q} \times \mathbb{Q} - T, \\ 0, & \text{otherwise.} \end{cases}$$

The function $f$ satisfies (i), (ii) and (iii).

In [18] there are three following questions:

Let $X$ be a Baire space, let $Y$ be a second countable space and let $Z$ be a metric space. Let $f : X \times Y \to Z$ be a function such that

(a) $f$ is separately somewhat continuous or

(b) $f$ is separately almost continuous or

(c) $f$ is separately somewhat nearly continuous.

Must $f$ be jointly somewhat nearly continuous?

The example 2 shows that the answer is negative in the cases (b) and (c). Now we shall show that the answer is positive in the case (a).

**Theorem 4.** Let $X$ be a Baire space, let $Y$ possess a countable pseudo-base and let $Z$ be arbitrary topological space. Let $f : X \times Y \to Z$ be such that $f^y$ is somewhat continuous for each $y \in Y$ and $f_x$ is somewhat continuous with the exception of a set of the first category. Then $f$ is somewhat nearly continuous.

**Proof.** Suppose that $f$ is not somewhat nearly continuous. Then there is an open set $G$ in $Z$ such that $f^{-1}(G) \neq \emptyset$ and $\text{Int Cl} f^{-1}(G) = \emptyset$. Let $\{V_n : n \in \mathbb{N}\}$ be a countable pseudo-base in $Y$. Let $(a, b) \in f^{-1}(G)$. Since $(f^b)^{-1}(G) \neq \emptyset$, the somewhat continuity of $f^b$ gives $A = \text{Int}(f^b)^{-1}(G) \neq \emptyset$. Denote

$T = \{x \in A : f_x \text{ is somewhat continuous}\}$ and

$T_n = \{x \in T : V_n \subset \text{Int}(f_x)^{-1}(G)\}$.

We shall show that $T = \bigcup_{n=1}^{\infty} T_n$. Let $x \in T$. Then $f^b(x) \in G$ and $b \in (f_x)^{-1}(G)$. This yields $\text{Int}(f_x)^{-1}(G) \neq \emptyset$ and hence there is $n \in \mathbb{N}$ such that $V_n \subset \text{Int}(f_x)^{-1}(G)$. Similarly as in Theorem 3 we can prove that $T_n$ is nowhere dense in $A$. \qed
References


Súhrn

O SKORO KVÁZISPOJITÝCH FUŇKCIÁCH

JÁN BORSÍK

Funkcia \( f: X \rightarrow Y \) je skoro kvázispojitá v \( x \in X \), ak \( x \in \text{Cl} \text{Int} \text{Cl} f^{-1}(V) \) pre každé okolie \( V \) bodu \( f(x) \). Vyšetrujú sa niektoré vlastnosti takýchto funkcií.

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