

## Book Reviews

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## BOOK REVIEWS

*Masahiro Shiota*: GEOMETRY OF SUBANALYTIC AND SEMIALGEBRAIC SETS. Progress in Mathematics, vol. 150, Birkhäuser, Basel 1997, xii+431 pages, ISBN 3-7643-4000-2, DM 188,-.

It is well known that real analytic sets in euclidean spaces do not have the nice properties one would expect. This fact had led to the definition of semianalytic sets, but unfortunately it turned out that even the semianalytic sets do not form an ideal family. The trouble was solved by A. M. Gabrielov and H. Hironaka who introduced the family of subanalytic sets in euclidean spaces. The book under review starts on a more general level. Namely, the authors introduce and study families  $\mathfrak{X}$  of subsets in euclidean spaces satisfying certain axioms. One example of such a family is the family of subanalytic sets. The smallest example is the family of semialgebraic sets. Other examples are the 0-minimal Tarski system generated by summation, multiplication and the exponential function, and the 0-minimal Tarski system generated by summation, multiplication and a finite number of Pfaffian functions. The main aim of the book is the investigation of these systems. Most of the results presented here are new. The axiomatic approach caused that the authors had often to develop new methods and find new proofs. In fact, they have found some new methods and proofs even for the families of subanalytic and semialgebraic sets. The prerequisites needed for reading the book are rather modest. The reader is assumed to have the basic knowledge of PL and differential topology (including Grassmann manifolds and characteristic maps of fiber bundles) and some basic knowledge of subanalytic and semialgebraic sets. The book is very carefully and clearly written. It will be good for postgraduate students. For specialists in the field it will be quite indispensable.

*Jiří Vanžura*, Brno

*Donald Greenspan*: PARTICLE MODELING. Birkhäuser, Boston, 1997, Series Modeling and Simulation in Science, Engineering & Technology, 288 pages, hardcover, DM 148,-.

The book is concerned with the modeling of various scientific and engineering phenomena on the basis of particle simulation. For the purpose of this review, particle modeling can be defined as the study of the dynamical behavior of solids and fluids in response to external forces, the solids and fluids being modeled as systems of atoms, or molecules, or aggregates of atoms and molecules. The dynamical equations treated in the book are systems of the second order nonlinear ordinary differential equations. Approximate (numerical) solution of initial value problems for these systems of equations is employed in the book.

The presentation can be divided into three parts. The first, rather short part contains some mathematical, physical and numerical considerations and serves as a basis for the remainder of the book. The author employs only two numerical methods, namely the leap frog method, which is basically a central difference, low order method which is efficient and easy to program, and a completely conservative method developed and studied by the author in the book.

The second part of the book deals with the development of intuition through extensive quantitative simulations and the analysis of their results. The third part is concerned with quantitative simulations of several basic scientific and engineering phenomena. These two parts of the book do not contain much mathematics but the results presented are quite

convincing and interesting. To show some topics studied, we quote the liquid drop formation, fall and collision, cavity flow, the turbulent and nonturbulent vortices, crack development in a stressed copper plate, or the computation of melting points. The last two chapters are devoted to Special Relativity (does not require a previous study of the subject) and to a speculative model of the diatomic molecular bond.

The book will be useful for researchers and graduate students in applied mathematics, computational physics, materials science, and mechanics. It is a source of information for all those interested in modeling physical phenomena.

*Petr Příkryl, Praha*

*I. Gohberg, Yu. Lyubich: NEW RESULTS IN OPERATOR THEORY AND ITS APPLICATIONS. (The Israel M. Glazman memorial volume), Operator Theory: Advances and Applications, vol. 98. Birkhäuser, Basel, 1997, 272 pages, hardcover, ISBN 3-7643-5775-4, DM 148,-.*

The volume contains 18 papers dedicated to the memory of Israel Glazman, the co-author (with N. Akhiezer) of the well-known book on linear operators in the Hilbert space and one of the outstanding personalities in the area of functional analysis in the former Soviet Union. The papers reflect the manifold topics that occurred in his work: spectral theory of bounded linear operators and linear differential operators in the Hilbert space (7 papers, by Alpay, Gohberg, Belitskii, Boutet de Monvel, Marchenko, Everitt, Markus, Levitan, Paneah, Sansuc and Tkachenko), invariant subspaces (2 papers by Lomonosov, Lyubich and Matsaev), differential equations (2 papers by Volpert and Zhitomirskii), analytic operator-valued functions (2 papers by Belyi, Tsekanovskii and Livšic), nonlinear functional analysis and fixed-point theorems (3 papers by Alber, Guerre-Delabriere, Ioffe, Schwartzman, Khatkevich, Reich and Shoikhet), approximation theory (Brudnyi) and mathematical statistics (Slivnyak). Apart from the research papers there is also a very nice introductory essay on Glazman's life and work. All the contributions are of good quality and, though not being a comprehensive survey of new results in OT and applications as the title might perhaps misleadingly suggest, they give a good picture of the current state of some problems in functional analysis and operator theory and its applications in particular.

*Miroslav Engliš, Praha*

*M. L. Gorbachuk, V. I. Gorbachuk: M. G. KREIN'S LECTURES ON ENTIRE OPERATORS. Birkhäuser, Basel, 1997, 232 pages, DM 138,-.*

The book is devoted to the study of entire Hermitian operators. The theory was developed by M. G. Krein and his students but most of the results were never published or they were only announced without proofs. Thus the theory has been hardly known outside Odessa.

The main part of the book contains the theory of entire operators with deficiency index  $(1, 1)$ . Applications to various classical problems of analysis, like the power moment problem or interpolation of functions, are also given. The presentation is based on the notes written by students of Krein's lecture held in 1961.

Further developments of the theory of entire operators with arbitrary defect indices are reflected in three appendices. It is proved that many differential operators are entire. This makes it possible to cover a number of systems of ordinary differential equations and partial differential equations of hyperbolic type.

The book is self-contained and the presentation is clear. It is intended for researchers as well as students interested in spectral theory of operators, complex analysis and differential equations.

*Vladimír Müller, Praha*

*Albrecht Böttcher, Yuri I. Karlovich: CARLESON CURVES, MUCKENHOUPT WEIGHTS, AND TOEPLITZ OPERATORS.* Progress in Mathematics vol. 154, Birkhäuser, Basel, 1997, xv+397 pages, ISBN 3-7643-5796-7, DM 118,-.

The book surveys recent results on Toeplitz operators with piecewise continuous symbols acting on weighted Lebesgue spaces on Carleson curves. It is a continuation of the classical theory and the contemporary state of the theory is due to the endeavour of many people, including important contributions of the authors themselves. The theory dealt with in the book is rather complicated in its nature and the methods used are very advanced and sophisticated. Nevertheless, the text is written in a clear way and proofs are sufficiently detailed so that the reader should have no difficulties to follow the exposition.

Let  $\Gamma$  be a simple, closed, and sufficiently smooth curve in the complex plane and let  $w$  be a weight function on  $\Gamma$ . Consider the Cauchy projection, the singular integral operator  $S$  on the weighted space  $L^2(\Gamma, w)$ , given by  $Sf(z) = (1/\pi i) \int_{\Gamma} f(\zeta)(\zeta - z)^{-1} d\zeta$ . Further, let  $P = (I + S)/2$ . For a bounded function  $a$  on  $\Gamma$  define the Toeplitz operator  $T_a$  with symbol  $a$  by  $T_a f = P(af)$ . For instance, the classical result (due to Gohberg) says that for sufficiently smooth  $\Gamma$ ,  $w$ , and  $a$ , the operator  $T_a$  is Fredholm if and only if  $a$  does not vanish at any point in  $\Gamma$ . Problems immediately occur if one admits a more general setting of the problem—less regular  $\Gamma$ , discontinuous  $a$ , etc. Considering Carleson curves and Muckenhoupt weights makes it possible to establish a rich theory, even in the context of  $L_p$  spaces,  $1 < p < \infty$ .

Recall that a curve  $\Gamma$  is a Carleson curve if  $\sup_{t \in \Gamma} \sup_{\varepsilon > 0} \varepsilon^{-1} |\Gamma(t, \varepsilon)| < \infty$ , where  $\Gamma(t, \varepsilon) = \Gamma \cap \{|z - t| < \varepsilon\}$  and  $|\Gamma(t, \varepsilon)|$  is its length. For  $1 < p < \infty$ , a weight function  $w$  is the Muckenhoupt (or the  $A_p$ ) weight if

$$\sup_{t \in \Gamma} \sup_{\varepsilon > 0} \left( \varepsilon^{-1} \int_{\Gamma(t, \varepsilon)} w(s)^p |ds| \right)^{1/p} \left( \varepsilon^{-1} \int_{\Gamma(t, \varepsilon)} w(s)^{-q} |ds| \right)^{1/q} < \infty.$$

The first three chapters of the book are devoted to properties of Carleson curves and Muckenhoupt weights. Chapters 4 and 5 deal with the problem of boundedness of  $S$  on  $L_p(\Gamma, w)$  and give very non-trivial characterizations in the spirit of the work done in the 1970's for classical operators on spaces with the Euclidean metric. Chapter 6 uses the localization and other relevant techniques for the study of the spectrum of a Toeplitz operator. From the classical result, which states that the essential spectrum of a Toeplitz operator consists of the essential range of the symbol and the line segments connecting the endpoints of jumps of the piecewise continuous symbol in question the exposition passes to a more general situation in Chapter 7, when the segments are replaced by circular arcs, logarithmic double spirals, horns, spiralic horns, further by more complicated spectral sets which have logarithmic double spirals as the boundary; and, eventually, due to certain instability properties, into the sets which the authors call leaves. The reader will certainly appreciate also the aesthetically pleasing pictures illustrating the theory. In Chapters 8 and 9 the authors study spectral properties of algebras of operators generated by the operator  $P$  and properties of curves with more complicated geometry. The concluding Chapter 10 gives a brief survey of the most recent efforts in the area. Authors tackle related results for Wiener-Hopf operators, Toeplitz operators on vector-valued spaces, use of the Mellin techniques, and some other problems that are currently subjects of intensive study.

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Bibliography, Index.

Miroslav Krbeč, Praha

Wolfgang Alt, Andreas Deutsch, Graham Dunn (eds.): DYNAMICS OF CELL AND TISSUE MOTION. Birkhäuser, Basel, 1997, x+336 pages, DM 118,-.

Motile dynamics at the cellular level—Cytoplasmic motion and cell shape, Dynamics and cell interaction with the environment, Dynamics of cell-cell interactions—Collective motion and aggregation and Dynamics within tissues—Morphogenesis and plant movement are the titles of the four parts of the present book.

The central problem of the book is the investigation of principles and mechanisms of the phenomena listed above. The mathematical models and methods are discussed from the mathematical as well as the biological or physical points of view. The book is of strictly interdisciplinary character and presents in some sense a challenge for mathematicians interested in biological applications.

Stefan Schwabik, Praha

*S. K. Jain, S. Tariq Rizvi (eds.): ADVANCES IN RING THEORY. Trends in Mathematics, Birkhäuser, Basel, 1997, 333 pages, DM 178,-.*

The content of the book is the collection of invited lectures delivered at the Ring Theory Section of the 23rd Ohio State-Denison Conference in May 1996. Here is the complete list of all 27 presented papers: Kasch Modules (*Albu, T.; Wisbauer, R.*), Compactness in Categories and Interpretations (*Lnh, P.N.; Wiegandt, R.*), A ring of Morita context in Which Each Right Ideal is Weakly Self-Injective (*Barthwal, S.; Jain, S.K.; Jhingan, S.R.; López-Permouth*), Splitting Theorems and a Problem of Müller (*Birkenmeier, G.F.; Kim J.Y.; Park J.K.*), Decompositions of D1 Modules (*Brown, R.A.; Wright, M.H.*), Right Cones in Groups (*Brungs, H.H.; Törner, G.*), On Extensions of Regular Rings of Finite Index by Central Elements (*Burgess, W.D.; Raphael, R.M.*), Intersections of Modules (*Dawns, J.*), Minimal Cogenerators over Osofsky and Camillo Rings (*Faith, C.*), Uniform Modules over Goldie Prime Serial Rings (*Guerriero, F.*), Co-Versus Contravariant Finiteness of Categories of Representations (*Huisgen-Zimmermann, B.; Smalø, S.O.*), Monomials and the Lexicographic Order (*Hulett, H.*), Rings over which Direct Sums of CS Modules are CS (*Huynh, D.V.; Müller, B.J.*), Exchange Properties and the Total (*Kasch, F.; Schneider, W.*), Local Bijective Gabriel Correspondence and Torsion Theoretic FBN Rings (*Kim, P.; Krause, G.*), Normalizing Extensions and the Second Layer Condition (*Kosler, K.A.*), Generators of Subgroups of Finite Index in  $GL_m(ZG)$  (*Lee, G.T.; Sehgal, S.K.*), Weak Relative Injective  $M$ -Subgenerated Modules (*Malik, S.; Vanaja, M.*), Direct Product and Power Series Formations Over 2-Primal Rings (*Marks, G.*), Localization in Noetherian Rings (*McConnell, M.; Sandomierski, F.L.*), Projective Dimension of Ideals in Von Neumann regular Rings (*Osofsky, B.L.*), Homological Properties of Color Lie Superalgebras (*Price, K.L.*), Indecomposable Modules over Artinian Right Serial Rings (*Singh, S.*), Nonsingular Extending Modules (*Smith, P.F.*), Right Hereditary, Right Perfect Rings are Semiprimary (*Tepley, M.L.*), On the Endomorphism Ring of a Discrete Module: A Theorem of Kasch (*Zelmanowitz, J.M.*), Nonsingular Rings with Finite Type Dimension (*Zhou, Y.*).

*Ladislav Bican, Praha*

*J.M. Ayerbe Toledano, T. Domínguez Benavides, G. López Acedo: MEASURES OF NONCOMPACTNESS IN METRIC FIXED POINT THEORY. Operator Theory: Advances and Applications, vol. 99, Birkhäuser, Basel, 1997, 224 pages, DM 138,-.*

Fixed point theorems provide an important tool in functional analysis, nonlinear analysis and differential equations. The best known examples of fixed point theorems are the Brower and Schauder theorems and the Banach contractive mapping principle.

The book is devoted to the systematic study of various generalizations of the above mentioned fixed point theorems. Usually the mapping is required to map a set into a "more compact" set. To state this precisely, various measures of noncompactness are considered. The typical examples are the Kuratowski, Hausdorff and the separation measure of noncompactness.

The book is well-written and easy to follow. It requires only the basic results of the real and functional analysis. Therefore it can be recommended to graduate students but can be useful also to researchers in the area.

*Vladimír Müller, Praha*

*M. Křížek, P. Neittaanmäki, R. Stenberg (eds.): FINITE ELEMENT METHODS: SUPERCONVERGENCE, POST-PROCESSING AND A POSTERIORI ESTIMATES.* Lecture Notes in Pure and Applied Mathematics, vol. 196, Marcel Dekker, New York, 1998, xii+348 pages, ISBN 0-8247-0148-8, USD 165,-.

The volume contains 23 papers presented at the conference with the same title, held at the University of Jyväskylä, Finland, July 1-4, 1996, as well as an extensive annotated bibliography on the subject with 600 references collected by Křížek and Neittaanmäki. The conference (actually the first devoted to superconvergence phenomena in finite element methods) aimed at bringing together specialists in numerical analysis, computational mathematics, and software developers who work in the field of superconvergence, post-processing and a posteriori estimates as well as other topics in mathematical physics.

Particularly interesting were the interactions between researchers and scientists from the west on the one hand, and from China on the other. The four papers in the volume presented by Chinese researchers in superconvergence give an excellent overview of their relatively less-known, but definitely not less interesting results achieved in the past decades, and moreover, in the English language.

Another interesting feature of the book is that the contributions can be subdivided into papers that approach the topics from an as general as possible viewpoint, trying to include as many as possible practical applications in their abstract framework, and papers that tackle very specific problems which allows for very detailed investigation of the occurring phenomena.

In spite of the specialized character of the topics presented, it can be very well read by anyone who has a knowledge of finite element methods and mathematical physics. Therefore the book has the potential of becoming one of the cornerstones in superconvergence literature.

*Jan Brandts, Praha*

*S. Dragomir, L. Ornea: LOCALLY CONFORMAL KÄHLER GEOMETRY.* Birkhäuser, Basel, 1998, 344 pages, DM 178,-.

From the theory of complex manifolds it is well known that the complex manifolds of complex dimension 1 (i.e. Riemann surfaces), the complex tori and complex projective spaces are Kähler manifolds. The simplest example of a compact complex manifold which is not a Kähler manifold (i.e. admits no Kähler metric), is the famous complex Hopf surface  $CH$ , more generally any complex Hopf manifold  $CH^n$  of (complex) dimension  $n$ . Surprisingly, it was discovered by W. M. Boothby (1954) that the Hopf surface  $CH$  admits a globally defined hermitian metric  $g$ , which is conformal to some local Kählerian metric in the neighborhood of each point, briefly, that  $g$  is a locally conformal Kähler (l.c.K.) metric. Subsequently, several other important examples of this phenomenon were discovered, and the definition of locally conformal Kähler manifolds, i.e. complex Hermitian manifolds which carry some l.c.K. metric, was given. However, the differential geometric study of l.c.K. manifolds has developed mainly since the seventies.

The monograph, written by specialists who actively contributed to the development in this field, presents in a systematic manner results, which were up to now accessible only in journals. The book is written for specialists in differential geometry and its reading is by no means easy. The literature contains 302 items.

*Jaroslav Fuka, Praha*

*Martino Bardi, Italo Capuzzo-Dolcetta:* OPTIMAL CONTROL AND VISCOSITY SOLUTIONS OF HAMILTON-JACOBI-BELLMAN EQUATIONS. Systems & Control: Foundations & Applications, Birkhäuser, Boston, 1997, xv+570 pages, ISBN 3-7643-3640-4, DM 218,-.

The book offers a comprehensive and self-contained presentation of the current state of the theory of viscosity solutions to first order partial differential equations of Hamilton-Jacobi type and its applications to optimal deterministic control and differential games. The concept of viscosity solutions that was initiated in early 80's in papers by M. G. Crandall, P. L. Lions and L. C. Evans, has been developed considerably in recent years. The book presents a profound introduction to the basic theory of viscosity solutions, at the same time stressing its connection to the most important applications.

The book is divided into seven chapters and two appendices. In Chapter I the infinite-horizon discounted regulator is discussed and its HJB equation is derived. Chapter II is devoted to the basic theory of continuous viscosity solutions of the HJB equations, especially in the case of convex (w.r.t. the gradient variable) Hamiltonian. Connections between the viscosity solutions and some different concepts of solutions are also discussed. In the next two chapters the basic theory of Chapter II is specialized and developed with reference to various optimal control problems with continuous value functions. Chapter V is devoted to discontinuous value functions; for example, various notions of discontinuous viscosity solutions are discussed and compared. Various limit problems are tackled in Chapters VI and VII while the last chapter is intended as an introduction to the theory of two-person zero sum differential games. Two appendices, written by M. Falcone and P. Soravia, respectively, deal with some additional topics important for the applications.

The book originated from the lecture notes of courses taught by the authors, which is reflected in the style of presentation. Each chapter is enriched with a section of bibliographical and historical notes. The book can be recommended to specialists in PDE's, control theory, differential games and related topics.

*Bohdan Maslowski, Praha*

*G. David, S. Semmes:* FRACTURED FRACTALS AND BROKEN DREAMS. Oxford Lecture Series in Mathematics and Its Applications 7, Oxford University Press Inc., New York, 1997, ISBN 0-1985-0166-8.

The poetic title hides a purely mathematical monograph providing a new concept for treating fractal sets in metric spaces. The technique known as geometric measure theory is used to generalize the familiar notion of self-similar sets: the authors introduce s.c. BPI spaces ("big pieces of itself") which have, roughly speaking, the property that within any two balls there are large enough subsets which are almost alike in the sense of bilipschitz equivalence. The book is full of further definitions, propositions and open problems connected with the authors' concept. As also clarified by the authors, the book is no systematic overview of a closed field of mathematics but it is open for further improvements and results and may serve as an interesting inspiration for mathematicians working on fractals.

*Jan Rataj, Praha*

*F. Di Biase:* FATOU TYPE THEOREMS. MAXIMAL FUNCTIONS AND APPROACH REGIONS. Birkhäuser, Boston, 1998, 166 pages, ISBN 3-7643-3976-4, DM 88,-.

In 1984 A. Nagel and E. M. Stein proved that in the Euclidean half-spaces (and in the unit disk) there are nontangential approach regions such that all harmonic functions satisfying certain growth conditions admit a boundary limit (with respect to this approach regions) for almost every point on the boundary. The authors of this book introduce a new method which shows that the Nagel-Stein phenomenon holds in great generality. Their technique rests on a process of discretization, leading to the discrete setting of trees. It is proved that the Nagel-Stein phenomenon holds under the following minimal assumptions: (1) the boundary of the domain is a space of the homogeneous type without atoms; (2) the shadow projected on the boundary by the natural approach regions is uniformly comparable to a boundary ball; (3) to each boundary ball there corresponds a point in the domain, that is close to the ball and whose shadow is uniformly comparable to the ball itself. This result covers, in particular, Euclidean half-spaces, NTA domains in  $\mathbb{R}^n$ , finite-type domains in  $\mathbb{C}^2$  and strongly pseudoconvex domains in  $\mathbb{C}^n$ .

This book should be a valuable resource for both mathematicians and graduate students.

*Dagmar Medková, Praha*

*Ram P. Kanwal:* GENERALIZED FUNCTIONS. THEORY AND TECHNIQUE. Birkhäuser, Basel, 1998, 456 pages, hardcover, 2nd edition, ISBN 3-7643-4006-1, DM 198,-.

The book represents a nice textbook to the theory of generalized functions and its applications. After an introductory chapter concerning the classical Dirac delta function and the delta sequences, the Schwartz-Sobolev theory of distributions is presented in 15 chapters:

In chap. 2 (THE SCHWARTZ-SOBOLEV THEORY OF DISTRIBUTIONS) some introductory definitions and notions are explained and some basic algebraic and analytic properties of distributions are described. Next three chapters (ADDITIONAL PROPERTIES OF DISTRIBUTIONS, DISTRIBUTIONS DEFINED BY DIVERGENT INTEGRALS AND DISTRIBUTIONAL DERIVATIVES OF FUNCTIONS WITH JUMP DISCONTINUITIES) deal with some more advanced properties of distributions (including the convergence of sequences of distributions). Chap. 6 is devoted to tempered distributions and Fourier analysis. Then, chap. 7 deals with the direct products and convolutions of distributions and chap. 8 with the Laplace transform. The second part of the book (chapters 9–15) is devoted to various applications of the theory of generalized functions: to ordinary differential equations, partial differential equations, boundary value problems, wave propagation, interplay between generalized functions and the theory of moment, impulse response of linear systems, probability and random processes, economics and microlocal theory. The book contains an extensive set of examples. Most of the material is easily accessible to senior undergraduate and graduate students in mathematical, physical and engineering sciences. In comparison with the first edition (Acad. Press, Orlando, FL, 1983) the second edition has been considerably strengthened in all aspects.

*Milan Tvrđý, Praha*