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PAIRWISE FUZZY IRRESOLUTE MAPPINGS

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Summary. In this paper the concepts of fuzzy irresolute and fuzzy presemiopen mappings due to Yalvac [12] are generalized to fuzzy bitopological spaces and their basic properties and characterizations are studied.

Keywords: fuzzy bitopological spaces, \((i, j)\)-fuzzy semiopen, \((i, j)\)-fuzzy semiclosed, \((i, j)\)-semineighbourhood, \((i, j)\)-semi-\(Q\)-neighbourhood

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1. INTRODUCTION

2. PRELIMINARIES

Let $X$ be a nonempty set and $I = [0,1]$. A fuzzy set in $X$ is a mapping from $X$ into $I$. The null fuzzy set 0 is the mapping from $X$ into $I$ which assumes only the value 0, and the fuzzy set 1 is a mapping from $X$ into $I$ which takes value 1 only. The union $\cup A_\alpha$ (intersection $\cap A_\alpha$) of a family $\{A_\alpha : \alpha \in A\}$ of fuzzy sets of $X$ is defined to be the mapping $\sup A_\alpha$ ($\inf A_\alpha$, respectively). A fuzzy set $A$ of $X$ is contained in a fuzzy set $B$ of $X$, denoted by $A \subseteq B$, if and only if $A(x) \leq B(x)$ for each $x \in X$. A fuzzy point $x_\beta$ in $X$ defined by

$$x_\beta(y) = \begin{cases} 
\beta & (\beta \in (0,1]) \text{ for } y = x \\
0 & \text{otherwise}
\end{cases} \quad (y \in X),$$

$x$ and $\beta$ are respectively called the support and the value of $x_\beta$. A fuzzy point $x_\beta \in A$ if $\beta \leq A(x)$. A fuzzy set $A$ is the union of all fuzzy points which belong to $A$. A fuzzy point $x_\beta$ is said to be quasi-coincident with the fuzzy set $A$, denoted by $x_\beta \approx A$, if and only if $\beta + A(x) > 1$. A fuzzy set $A$ is quasi-coincident with $B$, denoted by $A q B$, if and only if there exists $x \in X$ such that $A(x) + B(x) > 1$. $A \subseteq B$ if and only if $(A q B) \subseteq B$.

Let $f : X \to Y$ be a mapping. If $A$ is a fuzzy set of $X$, then $f(A)$ is a fuzzy set of $X$ defined by

$$f(A)(y) = \begin{cases} 
\sup A(x) & \text{if } f^{-1}(y) \neq 0 \\
x \in f^{-1}(y) & \text{for each } x \in X
\end{cases} \quad (y \in Y).$$

If $B$ is a fuzzy set of $Y$, then $f^{-1}(B)$ is a fuzzy set of $X$ defined by $f^{-1}(B)(x) = B(f(x))$ for each $x \in X$.

A family $\tau$ of fuzzy sets of $X$ is called a fuzzy topology [2] on $X$ if 0 and 1 belong to $\tau$ and $\tau$ is closed with respect to arbitrary union and finite intersection. The members of $\tau$ are called $\tau$-fuzzy open sets and their complements are $\tau$-fuzzy closed sets. For a fuzzy set $A$, the closure of $A$ (denoted by $\tau\text{-}\text{cl}(A)$) is the intersection of all $\tau$-fuzzy closed super-sets of $A$ and the interior of $A$ (denoted by $\tau\text{-}\text{int}(A)$) is the union of all $\tau$-fuzzy open subsets of $A$.

A mapping $f : (X, \tau) \to (X^*, \tau^*)$ is said to be fuzzy continuous (fuzzy open) if the inverse image (image) of every fuzzy open set in $X^*$ (of $X$) is fuzzy open in $X$ (in $X^*$).

Definition 2.1. A system $(X, \tau_1, \tau_2)$ consisting of a fuzzy set $X$ with two fuzzy topologies $\tau_1$ and $\tau_2$ on $X$ is called a fuzzy bitopological space [4].
Definition 2.2. A fuzzy set \( A \) in a fuzzy bitopological space \((X, n, \tau_1, \tau_2)\) is called:
(a) \((i,j)\)-fuzzy semiopen \([10]\) if there exists a \(\tau_i\)-fuzzy open set \(U\) such that \(U \subseteq A \subseteq \tau_i\text{-cl}(U)\);
(b) \((i,j)\)-fuzzy semiclosed \([10]\) if there exists a \(\tau_i\)-fuzzy closed set \(F\) such that \(\tau_i\text{-int}(F) \subseteq A \subseteq F\).

Definition 2.3. A fuzzy set \( A \) of a fuzzy bitopological space \((X, \tau_1, \tau_2)\) is said to be an \((i,j)\)-semineighbourhood \([10]\) (an \((i,j)\)-semi-Q-neighbourhood) \([10]\) of a fuzzy point \(x_0\) of \(X\) if there exists a \((i,j)\)-fuzzy semiopen set \(O\) such that \(x_0 \in O \subseteq A\) (resp. \(x_0 \notin O \subseteq A\)).

Definition 2.4. Let \((X, \tau_1, \tau_2)\) be a fuzzy bitopological space. The \((i,j)\)-semiclosure \([10]\) (denoted by \((i,j)\)-scl) and the \((i,j)\)-semiinterior \([10]\) (denoted by \((i,j)\)-sint) of a fuzzy set \(A\) are defined respectively as follows:
\[
(i,j)\text{-scl}(A) = \inf\{B : B \supseteq A, B \text{ is } (i,j)\text{-fuzzy semiclosed}\}
\]
\[
(i,j)\text{-sint}(A) = \sup\{B : B \subseteq A, B \text{ is } (i,j)\text{-fuzzy semiopen}\}
\]

Definition 2.5. A mapping \( f \) from a fuzzy bitopological space \((X, \tau_1, \tau_2)\) to a fuzzy bitopological space \((X^{*}, \tau_1^{*}, \tau_2^{*})\) is pairwise fuzzy continuous \([10]\) (pairwise fuzzy open) if \( f : (X, \tau_1) \rightarrow (X^{*}, \tau_1^{*}) \) and \( f : (X, \tau_2) \rightarrow (X^{*}, \tau_2^{*})\) are fuzzy continuous (fuzzy open).

Definition 2.6. A mapping \( f \) from a fuzzy bitopological space \((X, \tau_1, \tau_2)\) to a fuzzy bitopological space \((X^{*}, \tau_1^{*}, \tau_2^{*})\) is pairwise fuzzy semicontinuous \([10]\) if the inverse image of every \(\tau^{*}\)-fuzzy open set in \(X^{*}\) is \((i,j)\)-fuzzy semiopen in \(X\).

Throughout this paper \(i, j = 1, 2\) where \(i \neq j\) and if \(P\) is any fuzzy topological property then \(\tau_i\text{-}P\) \((\tau_j\text{-}P)\) denotes the property \(P\) with respect to the fuzzy topology \(\tau_i\) \((\tau_j)\).

3. PAIRWISE FUZZY IRRESOLUTE MAPPINGS

Definition 3.1. A mapping \( f \) from a fuzzy bitopological space \((X, \tau_1, \tau_2)\) to a fuzzy bitopological space \((X^{*}, \tau_1^{*}, \tau_2^{*})\) is pairwise fuzzy irresolute if the inverse image of every \((i,j)\)-fuzzy semiopen set in \(X^{*}\) is \((i,j)\)-fuzzy semiopen in \(X\).

Remark 3.1. The concepts of pairwise fuzzy irresolute and pairwise fuzzy continuous mappings are independent. Indeed,

Example 3.1. Let \(X = \{x, y, z\}, X^{*} = \{a, b\}\) and \(U \subseteq X, V \subseteq X^{*}\) be fuzzy sets defined as follows:
\[
U(x) = 0.3, \quad U(y) = 0.5, \quad U(z) = 0.4; \quad V(a) = 0.4, \quad V(b) = 0.6.
\]
Let \( \tau_1 = \{0, U, 1\}, \tau_2 = \{0, 1\} \) and \( \tau_1^* = \{0, V, 1\}, \tau_2^* = \{0, 1\} \). Then the mapping \( f: (X, \tau_1, \tau_2) \to (X^*, \tau_1^*, \tau_2^*) \) defined by \( f(x) = a, f(y) = f(z) = b \) is pairwise fuzzy irresolute but not pairwise fuzzy continuous.

**Example 3.2.** Let \( X = \{x, y\}, X^* = \{a, b\} \) and let \( U \subseteq X, V \subseteq X, W \subseteq X^* \) be fuzzy sets defined as follows:
\[
\begin{align*}
U(x) &= 0.4, & U(y) &= 0.7; \\
V(x) &= 0.2, & V(y) &= 0.3; \\
W(a) &= 0.4, & W(b) &= 0.7.
\end{align*}
\]
Let \( \tau_1 = \{0, U, 1\}, \tau_2 = \{0, V, 1\} \) and \( \tau_1^* = \{0, W, 1\}, \tau_2^* = \{0, 1\} \). Then the mapping \( f: (X, \tau_1, \tau_2) \to (X^*, \tau_1^*, \tau_2^*) \) defined by \( f(x) = a, f(y) = b \) is pairwise fuzzy continuous but not pairwise fuzzy irresolute.

**Remark 3.2.** Every pairwise fuzzy irresolute mapping is pairwise fuzzy semicontinuous but the converse need not be true since the mapping \( f: (X, \tau_1, \tau_2) \to (X^*, \tau_1^*, \tau_2^*) \) from Example 3.2 is pairwise fuzzy semicontinuous but not pairwise fuzzy irresolute.

**Theorem 3.1.** Let \( (X, \tau_1, \tau_2) \) and \( (X^*, \tau_1^*, \tau_2^*) \) be fuzzy bitopological spaces and \( f: (X, \tau_1, \tau_2) \to (X^*, \tau_1^*, \tau_2^*) \). Then the following conditions are equivalent:

(a) \( f \) is pairwise fuzzy irresolute;
(b) for every fuzzy point \( x_0 \) of \( X \) and every \((i,j)\)-fuzzy semiopen set \( B \) in \( X^* \) such that \( f(x_0) \in B \), there is an \((i,j)\)-fuzzy semiopen set \( A \) in \( X \) such that \( x_0 \in A \) and \( f(A) \subseteq B \);
(c) for every \((i,j)\)-fuzzy semiclosed set \( B \) in \( X^* \), \( f^{-1}(B) \) is \((i,j)\)-fuzzy semiclosed in \( X \);
(d) for every fuzzy point \( x_0 \) of \( X \) and every \((i,j)\)-semineighbourhood \( B \) in \( X^* \) of \( f(x_0) \), \( f^{-1}(B) \) is an \((i,j)\)-semineighbourhood of \( x_0 \) in \( X \);
(e) for every fuzzy point \( x_0 \) of \( X \) and every \((i,j)\)-semineighbourhood \( B \) in \( X^* \) of \( f(x_0) \) there is an \((i,j)\)-semineighbourhood \( O \) in \( X \) of \( x_0 \) such that \( f(O) \subseteq B \);
(f) for every fuzzy point \( x_0 \) of \( X \) and every \((i,j)\)-fuzzy semiopen set \( B \) such that \( f(x_0) \in B \) there is an \((i,j)\)-fuzzy semiopen set \( A \) in \( X \) such that \( x_0 \in A \) and \( f(A) \subseteq B \);
(g) for every fuzzy point \( x_0 \) of \( X \) and every \((i,j)\)-semi-Q'-neighbourhood \( B \) in \( X^* \) of \( f(x_0) \), \( f^{-1}(B) \) is an \((i,j)\)-semi-Q'-neighbourhood of \( x_0 \) in \( X \);
(h) for every fuzzy point \( x_0 \) of \( X \) and every \((i,j)\)-semi-Q'-neighbourhood \( B \) of \( f(x_0) \) in \( X^* \), there is an \((i,j)\)-semi-Q'-neighbourhood \( O \) of \( x_0 \) such that \( f(O) \subseteq B \);
(i) \( f((i,j)-\text{cl}(A)) \subseteq (i,j)-\text{cl}(f(A)) \), for every fuzzy set \( A \) of \( X \);
(j) \( (i,j)-\text{cl}(f^{-1}(B)) \subseteq f^{-1}((i,j)-\text{cl}(B)) \), for every fuzzy set \( B \) of \( X^* \);}
Proof. (a) $\Rightarrow$ (b). Let $x_{\beta}$ be a fuzzy point of $X$ and $B$ a fuzzy semiopen set in $X^*$ such that $f(x_{\beta}) \in B$. Then by (a) $A$ is an $(i,j)$-fuzzy semiopen set such that $x_{\beta} \in A$ and $f(A) \subseteq B$.

(b) $\Rightarrow$ (a). Let $B$ be an $(i,j)$-fuzzy semiopen set in $X^*$. Let $x_{\beta} \in f^{-1}(B)$. Then $f(x_{\beta}) \in B$. Now by (b) there is an $(i,j)$-fuzzy semiopen set $A$ in $X$ such that $x_{\beta} \in A$ and $f(A) \subseteq B$. Then $x_{\beta} \in A \in f^{-1}(B)$. Hence by Theorem 3.4 [10] $f^{-1}(B)$ is $(i,j)$-fuzzy semiopen in $X$.

(a) $\Leftrightarrow$ (c). Obvious.

(a) $\Rightarrow$ (d). Let $x_{\beta}$ be a fuzzy point of $X$ and let $B$ be an $(i,j)$-neighbourhood of $f(x_{\beta})$. Then there is an $(i,j)$-fuzzy semiopen set $O^*$ such that $f(x_{\beta}) \in O^* \subseteq B$. Now $f^{-1}(O^*)$ is $(i,j)$-fuzzy semiopen in $X$, because $f$ is fuzzy irresolute and $x_{\beta} \in f^{-1}(O^*) \subseteq f^{-1}(B)$. Thus $f^{-1}(B)$ is an $(i,j)$-semi-neighbourhood of $x_{\beta}$ in $X$.

(d) $\Rightarrow$ (e). Let $x_{\beta}$ be a fuzzy point of $X$ and let $B$ be an $(i,j)$-semi-neighbourhood of $f(x_{\beta})$. Then $O = f^{-1}(B)$ is an $(i,j)$-semi-neighbourhood of $x_{\beta}$ and $f(O) = f(f^{-1}(B)) \subseteq B$.

(e) $\Rightarrow$ (b). Let $x_{\beta}$ be a fuzzy point of $X$ and let $B$ be an $(i,j)$-fuzzy semiopen set containing $f(x_{\beta})$. Then $B$ is an $(i,j)$-semi-neighbourhood of $f(x_{\beta})$, so there is an $(i,j)$-semi-neighbourhood $O$ of $x_{\beta}$ of $X$ such that $x_{\beta} \in O$ and $f(O) \subseteq B$. Therefore there exists an $(i,j)$-fuzzy semiopen set $A$ in $X$ such that $x_{\beta} \in A \subseteq O$. Clearly $f(A) \subseteq f(O) \subseteq B$.

(a) $\Rightarrow$ (f). Let $x_{\beta}$ be a fuzzy point of $X$ and let $O^*$ be an $(i,j)$-fuzzy semiopen set in $X^*$ containing $f(x_{\beta})$. Then $A = f^{-1}(B)$, then $A$ is $(i,j)$-fuzzy semiopen in $X$ and $x_{\beta} \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.

(f) $\Rightarrow$ (g). Let $x_{\beta}$ be a fuzzy point of $X$ and let $B$ be an $(i,j)$-semi-Q-neighbourhood of $f(x_{\beta})$. Then there exists an $(i,j)$-fuzzy semiopen set $O^*$ in $X^*$ such that $f(x_{\beta}) \in O^* \subseteq B$. By hypothesis there is an $(i,j)$-fuzzy semiopen set $A$ in $X$ such that $x_{\beta} \in A$ and $f(A) \subseteq O^*$. Thus $x_{\beta} \in A \subseteq f^{-1}(O^*) \subseteq f^{-1}(B)$. Hence $f^{-1}(B)$ is an $(i,j)$-semi-Q-neighbourhood of $x_{\beta}$.

(g) $\Rightarrow$ (h). Let $x_{\beta}$ be a fuzzy point of $X$ and let $B$ be an $(i,j)$-semi-Q-neighbourhood of $f(x_{\beta})$ in $X^*$. Then $O = f^{-1}(B)$ is an $(i,j)$-semi-Q-neighbourhood of $x_{\beta}$ and $x_{\beta} \in O \subseteq f^{-1}(B) \subseteq B$.

(h) $\Rightarrow$ (f). Let $x_{\beta}$ be a fuzzy point of $X$ and let $B$ be an $(i,j)$-fuzzy semiopen set such that $f(x_{\beta}) \in B$. Then $B$ is an $(i,j)$-semi-Q-neighbourhood of $f(x_{\beta})$. So there is an $(i,j)$-semi-Q-neighbourhood $O$ of $x_{\beta}$ such that $f(O) \subseteq B$. Therefore there exists an $(i,j)$-fuzzy semiopen set $A$ in $X$ such that $x_{\beta} \in A \subseteq O$. Hence $x_{\beta} \in A \subseteq O \subseteq B$.

(f) $\Rightarrow$ (a). Let $O^*$ be an $(i,j)$-fuzzy semiopen set in $X^*$ and $x_{\beta} \in f^{-1}(O^*)$. Clearly $f(x_{\beta}) \in O^*$. Choose the fuzzy point $x_{\beta}^* = 1 - x_{\beta}$. Then $f(x_{\beta}^*) \in O^*$. And so by (f) there exists an $(i,j)$-fuzzy semiopen set $A$ such that $x_{\beta}^* \in A$ and $f(A) \subseteq O^*$. 277
Now \( x^2 \geq 0 \Rightarrow x^2 + A(x) = 1 - x_0 + A(x) > 1 \Rightarrow A(x) > x_0 \Rightarrow x_0 \in A \). Thus \( x_0 \in A < f^{-1}(O^*) \). Hence by Theorem 3.4 [10], \( f^{-1}(O^*) \) is \((i,j)\)-fuzzy semiopen in \( X \).

\((c) \Rightarrow (i)\). Suppose that \((c)\) holds. Let \( A \) be a subset of \( X \). Since \( A \subseteq f^{-1}(f(A)) \), then \( A \subseteq f^{-1}((i,j)\text{-scl } f(A)) \). Now \((i,j)\text{-scl}(f(A)) \) is \((i,j)\)-fuzzy semiopen so \( f^{-1}((i,j)\text{-scl } f(A)) \) is \((i,j)\)-fuzzy semiopen and contains \( A \). Consequently \((i,j)\text{-scl } f^{-1}(A) \subseteq f^{-1}((i,j)\text{-scl } f(A)) \), and so \((i,j)\text{-scl } f(A) \subseteq f^{-1}((i,j)\text{-scl } f(A)) \).

\((i) \Rightarrow (c)\). Suppose that \((i)\) holds for any subset \( A \) of \( X \). Let \( B \) be an \((i,j)\)-fuzzy semiopen subset of \( X^* \). Then \( f((i,j)\text{-scl } f^{-1}(B)) \subseteq (i,j)\text{-scl } f(f^{-1}(B)) \subseteq (i,j)\text{-scl } A = B \). Hence \((i,j)\text{-scl } f^{-1}(B) \subseteq (B) \) and \( f^{-1}(B) \) is \((i,j)\)-fuzzy semiopen in \( X \).

\((i) \Rightarrow (j)\). Let \( B \) be a fuzzy set of \( X^* \). Then \( f^{-1}(B) \) is a fuzzy set of \( X \). Therefore by \((i)\), \( f((i,j)\text{-scl } f^{-1}(B)) \subseteq (i,j)\text{-scl } f(f^{-1}(B)) \subseteq (i,j)\text{-scl } B \). Hence \((i,j)\text{-scl } f^{-1}(B) \subseteq (B) \).

\((j) \Rightarrow (i)\). Let \( B = f(A) \) where \( A \) is a subset of \( X \), and we know that \( A \subseteq B \Rightarrow (i,j)\text{-scl } A \subseteq (i,j)\text{-scl } B \) thus \((i,j)\text{-scl } A \subseteq (i,j)\text{-scl } f^{-1}(B) \subseteq f^{-1}((i,j)\text{-scl } f(A)) \). Therefore \( f((i,j)\text{-scl } A) \subseteq (i,j)\text{-scl } f(A) \).

\((a) \Rightarrow (k)\). Let \( B \) be an \((i,j)\)-fuzzy semiopen set in \( X^* \). Clearly \( f^{-1}((i,j)\text{-sint } B) \) is \((i,j)\)-fuzzy semiopen and we have

\[ f^{-1}((i,j)\text{-sint } B) \subseteq (i,j)\text{-sint } f^{-1}((i,j)\text{-sint } B) \subseteq (i,j)\text{-sint } f^{-1}(B) \].

\((k) \Rightarrow (a)\). Let \( B \) be any \((i,j)\)-fuzzy semiopen set in \( X^* \). Then \((i,j)\text{-sint } B = B \) and \( f^{-1}(B) - f^{-1}((i,j)\text{-sint } B) \subseteq (i,j)\text{-sint } f^{-1}(B) \). Hence we have \( f^{-1}(B) = (i,j)\text{-sint } f^{-1}(B) \). This shows that \( f^{-1}(B) \) is \((i,j)\)-fuzzy semiopen.

**Theorem 3.2.** Let \( f: (X, \tau_1, \tau_2) \rightarrow (X^*, \tau_1^*, \tau_2^*) \) and \( g: (X^*, \tau_1^*, \tau_2^*) \rightarrow (X^{**}, \tau_1^{**}, \tau_2^{**}) \) be two mappings. Then \( g \circ f \) is

- **(a)** pairwise fuzzy irresolute if \( f \) and \( g \) are pairwise fuzzy irresolute,
- **(b)** pairwise fuzzy semicontinuous if \( f \) is pairwise fuzzy irresolute and \( g \) is pairwise fuzzy semicontinuous.

**Proof.** Obvious.

**Theorem 3.3.** Let \( f: (X, \tau_1, \tau_2) \rightarrow (X^*, \tau_1^*, \tau_2^*) \) be a pairwise fuzzy semicontinuous and pairwise fuzzy open mapping. Then \( f \) is pairwise fuzzy irresolute.

**Proof.** Let \( B \) be an \((i,j)\)-fuzzy semiopen set in \( X^* \). Then there exists a \( \tau_1^* \)-fuzzy open set \( U \) such that \( U \subseteq B \subseteq \tau_1^*\text{-cl } U \). Therefore \( f^{-1}(U) \subseteq f^{-1}(B) \subseteq f^{-1}(\tau_1^*\text{-cl } U) \subseteq \tau_1\text{-cl } f^{-1}(U) \) because \( f \) is pairwise fuzzy open. Since \( f \) is pairwise fuzzy semicontinuous, \( f^{-1}(U) \) is \((i,j)\)-fuzzy semiopen in \( X \). Hence by Theorem 3.5 [10], \( f^{-1}(B) \) is \((i,j)\)-fuzzy semiopen in \( X \).
4. PAIRWISE FUZZY PRESEMIOPEN MAPPINGS

**Definition 4.1.** A mapping \( f \) from a fuzzy bitopological space \((X, \tau_1, \tau_2)\) to a fuzzy bitopological space \((X^*, \tau_1^*, \tau_2^*)\) is pairwise fuzzy presemiopen if the image of every \((i, j)\)-fuzzy semiopen set of \(X\) is \((i, j)\)-fuzzy semiopen in \(X^*\).

**Remark 4.1.** Every pairwise fuzzy presemiopen mapping is pairwise fuzzy semiopen, but the converse may be false. Indeed, 

**Example 4.1.** Let \( X = \{x, y\}, \quad X^* = \{a, b\} \) and \( U \subset X, \quad V \subset X^* \) and \( W \subset X^* \) be fuzzy sets defined as follows:

\[
U(x) = 0.5, \quad U(y) = 0.6, \quad V(a) = 0.5, \quad V(b) = 0.6, \quad W(a) = 0.2, \quad W(b) = 0.3.
\]

Let \( \tau_1 = \{0, U, 1\}, \quad \tau_2 = \{0, 1\}, \quad \tau_1^* = \{0, V, 1\}, \quad \tau_2^* = \{0, W, 1\} \). Then the mapping \( f: X \to X^* \) defined by \( f(x) = a \) and \( f(y) = b \) is pairwise fuzzy open and hence pairwise fuzzy semiopen but not pairwise fuzzy presemiopen.

Consider the following example.

**Example 4.2.** Let \( X = \{x, y\}, \quad X^* = \{a, b\} \) and \( S \subset X, \quad T \subset X, \quad U \subset X^* \) and \( V \subset X^* \) be fuzzy sets defined as follows:

\[
S(x) = 0.2, \quad S(y) = 0.3, \quad T(x) = 0.3, \quad T(y) = 0.2, \quad U(a) = 0.1, \quad U(b) = 0.2, \quad V(a) = 0.2, \quad V(b) = 0.1.
\]

Let \( \tau_1 = \{0, S, 1\}, \quad \tau_2 = \{0, T, 1\} \) and \( \tau_1^* = \{0, V, 1\}, \quad \tau_2^* = \{0, V, 1\} \). Then the mapping \( f: X \to X^* \) defined by \( f(x) = a \) and \( f(y) = b \) is pairwise fuzzy presemiopen but not pairwise fuzzy open.

**Remark 4.2.** Examples 4.1 and 4.2 show that the concepts of pairwise fuzzy open and pairwise fuzzy presemiopen mappings are independent.

**Theorem 4.1.** Let \( f: (X, \tau_1, \tau_2) \to (X^*, \tau_1^*, \tau_2^*) \) be a pairwise fuzzy presemiopen mapping. If \( B \) is a fuzzy set of \( X^* \) and \( A \) is an \((i, j)\)-fuzzy semiclosed set of \(X\) containing \( f^{-1}(B) \) then there exists an \((i, j)\)-fuzzy semiclosed set \( F \) of \(X^*\) containing \( B \) such that \( f^{-1}(F) \subseteq A \).

**Proof.** Let \( F = 1 - f(1 - A) \). Since \( f^{-1}(B) \subseteq A \), we have \( f(1 - A) \subseteq 1 - B \). Since \( f \) is pairwise fuzzy presemiopen then \( F \) is an \((i, j)\)-fuzzy semiclosed set of \(X^*\) and \( f^{-1}(F) = 1 - f^{-1}(f(1 - A)) = (1 - A) = A \). Thus \( f^{-1}(F) \subseteq A \). \( \square \)
Theorem 4.2. If $f: (X, \tau_1, \tau_2) \to (X^*, \tau_1^*, \tau_2^*)$ is pairwise fuzzy presemiopen then $f^{-1}((l, j)-\text{scl}(B)) \subseteq (l, j)-\text{scl} f^{-1}(B)$ for every fuzzy set $B$ of $X^*$.

Theorem 4.3. A mapping $f: (X, \tau_1, \tau_2) \to (X^*, \tau_1^*, \tau_2^*)$ is pairwise fuzzy presemiopen if and only if $f((l, j)-\text{sint}(A)) \subseteq (l, j)-\text{sint} f(A)$ for every fuzzy set $A$ of $X$.

Proof. Necessity: Suppose $f$ is fuzzy presemiopen then $f((l, j)-\text{sint}(A))$ is $(l, j)$-fuzzy semiopen in $Y$. Hence $f((l, j)-\text{sint}(A)) = (l, j)-\text{sint} f((l, j)-\text{sint}(A)) \subseteq (l, j)-\text{sint} f(A)$.

Sufficiency: Let $A$ be an $(l, j)$-fuzzy semiopen set of $X$, then by hypothesis $f((l, j)-\text{sint}(A)) \subseteq (l, j)-\text{sint} f(A)$. Hence $f(A)$ is $(l, j)$-fuzzy semiopen in $X^*$. □

Theorem 4.4. Let two mappings

\[ f: (X, \tau_1, \tau_2) \to (X^*, \tau_1^*, \tau_2^*) \quad \text{and} \quad g: (X^*, \tau_1^*, \tau_2^*) \to (X^{**}, \tau_1^{**}, \tau_2^{**}) \]

be pairwise fuzzy presemiopen. Then $g \circ f$ is pairwise fuzzy presemiopen.

References


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