Book Reviews

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BOOK REVIEWS

Gregory F. Lawler: INTERSECTIONS OF RANDOM WALKS. Probability and Its Applications, Birkhäuser, Boston 1996, 226 pages, ISBN 3-7643-3892-X, price DM 68,–.

Let X_i be independent random variables taking values in the d-dimensional lattice $\mathbb{Z}^d$ with $P\{X_i = e\} = 1/2d$ for any $e \in \mathbb{Z}^d$, $|e| = 1$. A simple random walk starting at x is a random process $\{S_n\}_{n\geqslant 0}$ such that $S_n = x + X_1 + \ldots + X_n$. The monograph under review is devoted to a systematic treatment of many results connected with the property of non-intersection of paths of simple random walks.

To make the book relatively self-contained the author recalls the necessary basic facts about simple random walks, including some less standard results (e.g. the discrete Harnack inequality), in the first chapter. Chapter 2 deals with the harmonic measure, while in the next three chapters estimates of the probability that the paths of independent random walks do not intersect are studied. One of the main problems may be described as follows: let (S_k^1), (S_k^2) be independent random walks starting at the same point, set

$$f(n) = P\{\omega; \{S_1^1(\omega), \ldots, S_n^1(\omega)\} \cap \{S_1^2(\omega), \ldots, S_n^2(\omega)\} = \emptyset\}.$$

We are interested in the asymptotic behaviour of the quantities $f(n)$ as n tends to infinity. In the sixth chapter, self-avoiding walks are considered. In contrast to the rest of the book, arguments given here are mostly heuristic. (Let us mention in this connection a recent monograph by N. Madras and G. Slade which has been published in the same series Probability and Its Applications and which collects rigorous results on the self-avoiding walk.) Besides the (strict) self-avoiding walk, several other models of random walks with self-repulsion are briefly discussed: the Domb-Joyce model, the (discrete) Edwards model, kinematically growing walks (given by transition kernels on the infinite random walk paths space). One of the latter models, the loop-erased walk, is then thoroughly investigated in the last chapter; in particular, the convergence of the (rescaled) loop-erased walk to the Brownian motion in dimensions $d \geqslant 4$ is established.

The author is one of the leading experts in the field and many of the presented results are due to him. The monograph was issued for the first time in the year 1991, in the new paperback edition some misprints were corrected and a very short appendix on recent results was amended. The book is well written and organized and remains a unique and very useful source of information about this important topic.

Jan Seidler, Praha

Neal Madras, Gordon Slade: THE SELF-AVOIDING WALK. Probability and Its Applications, Birkhäuser, Boston 1996, xiv+425 pages, ISBN 3-7643-3891-1, price DM 68,–.

An N-step self-avoiding walk is a path $(\omega(0), \ldots, \omega(N))$ on a d-dimensional lattice $\mathbb{Z}^d$ which does not visit the same site more than once, that is, $\omega(i) \neq \omega(j)$ for $i \neq j$. The self-avoiding walks have been studied in statistical physics and theoretical chemistry for a long time and have found applications e.g. in modelling long-chain polymer molecules; consequently, many results on the behaviour of the self-avoiding walks have been derived in a nonrigorous manner (in particular, by computer simulations). To obtain (at least a part of) those results with full mathematical rigor turned out to be a rather difficult task; the monograph under review aims at presenting what has been achieved in this direction.

To indicate the type of problems that are to be solved, let us mention one of them: Denote by Λ_N the set of all N-step self-avoiding walks beginning at the origin, let c_N be the cardinality of Λ_N and let $\langle\cdot\rangle$ stand for the expectation with respect to the uniform measure on Λ_N. The exact values of c_N and of the mean-square displacement $\langle|\omega(N)|^2\rangle$ are not known, but on the basis on nonrigorous calculations it is believed that

$$c_N \sim A\mu^N N^{\gamma-1}, \qquad \langle|\omega(N)|^2\rangle \sim DN^{2\nu}$$

hold true asymptotically as N tends to infinity, where A, D, μ, γ, ν are dimension-dependent constants. It can be shown easily that the limit $\lim_{N\to\infty} c_N^{1/N} = \mu$ exists (μ is called the connective constant), but its numerical value is not known. γ and ν are two of the so-called critical exponents, their existence (and values $\gamma = 1$, $\nu = \frac{1}{2}$) was established rigorously only in the case $d \geqslant 5$ by T. Hara and the second author by means of lace expansions, a method coming from mathematical physics. This is explained in Chapters 5 and 6 of the book; to avoid some technical difficulties, only the case of dimension d sufficiently large is treated. For $d = 2, 3, 4$ analogous results have not been proven yet, so the reader can find classical upper bounds on the growth of c_N's, due to Hammersley-Welsh and Kesten, in the third chapter. Some other topics covered here are: decay of the two-point function (Chapter 4), Kesten's pattern theorem and ratio limit theorems (Chapter 7), an in-depth analysis of Monte Carlo procedures for obtaining estimates of the connective constant and of critical exponents (Chapter 9).

As the authors say in Preface, they tried to make the book relatively self-contained and accessible to graduate students not only in mathematics, but also in physics and chemistry who are inclined to mathematics.

Jan Seidler, Praha

Malcolm Adams, Victor Guillemin: MEASURE THEORY AND PROBABILITY. Birkhäuser, Boston 1996, xiv+205 pages, ISBN 3-7643-3884-9, price DM 48,–.

The book under review provides a quite elementary introduction to measure theory. In their Preface, the authors argue in a very uncompromising manner that undergraduates should be taught this discipline from the perspective of probability theory, and indeed, their textbook presents the rudiments of measure theory with copious probabilistic motivations and applications.

The book is divided into three chapters. The first of them opens with a discussion of the strong law of large numbers for the coin tossing model, which indicates the necessity of considering σ-additive measures. These are then introduced formally and a construction of the Lebesgue measure in $\mathbb{R}^d$ is included. In the second chapter. the Lebesgue integral is constructed, the basic convergence theorems are established and the Fubini theorem is proven. The chapter closes with sections on independence, the strong law of large numbers (for bounded i.i.d. random variables) and on the discrete Dirichlet problem. Chapter 3 is named Fourier analysis: L^1 and L^2-spaces are introduced and basic facts about the geometry of Hilbert spaces and about Fourier series and integrals are treated. The theory developed here is applied to an investigation of the recurrence of one-dimensional random walks and to the proof of the central limit theorem. In the opposite direction, Kac's probabilistic proof of the Szegö theorem on Toeplitz matrices is given. The three appendices are devoted to metric spaces, L^p-spaces and Vitali's construction of a non-measurable subset of $\mathbb{R}$, respectively. Additional results are presented in a form of numerous exercises.

Jan Seidler, Praha

Laura Toti Rigatelli: EVARISTE GALOIS (1811–1832). Birkhäuser, Basel 1996, 168 pages, DM 38,–.

Evariste Galois (1811–1832) is well known to today's mathematicians. His classic theory of algebraic equations is fundamental in modern algebra. His short life and tragical death inspired many authors to write his biography. He has become a myth.

The book is the English translation from Italian of a book which appeared under the title *Matematica sulle barricate*. The first five chapters of this book represent a new critical biography of Evariste Galois. On the basis of an analysis and interpretation of a series archival documents and evidences from contemporary memoires and newspapers the author describes the life of Evariste Galois from his childhood to his tragical death. She presents a new version of the circumstances leading to Galois' death.

The next chapter The mathematical work of Evariste Galois was written especially for this new edition. This part gives a detailed description of Galois' mathematical works and of his contribution to the development of mathematics in the 19th century.

The third part contains the bibliography of Galois' works and biographical studies on Galois.

The book can be recommended to all mathematicians interested in the history of mathematics.

Martina Němcová, Praha

Michel Willem: MINIMAX THEOREMS. Birkhäuser, Boston 1996, 176 pages, DM 94,–.

The book is devoted to the critical points theory and its application to variational problems.

Starting from a deformation lemma, the so-called Palais-Smale sequences are constructed for various problems using basic topological arguments as the intermediate value theorem in the case of the Ambrosetti-Rabinowitz theorem, non retractibility of the ball onto the sphere, the Borsuk-Ulam theorem or the Kryszewski-Szulkin degree.

Several chapters deal with multiplicity results for semilinear elliptic problems. The material includes the fountain theorem as well as its dual version, a generalization of the Ljusternik-Schnirelman category etc.

Many applications are given to problems with lack of compactness including some still unpublished results as the treatment of solitary waves for the Kadomtsev-Petviashvili equation.

The final part of the book describes losses of compactness in variational problems due to the invariance by translations or dilations.

Many recent methods are described assuming only a basic knowledge of partial differential equations and the underlying Sobolev spaces.

The book may be a good texbook for graduate students as well as a reference material for those interested in the theory of nonlinear elliptic problems.

Eduard Feireisl, Praha