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A FUNCTORIAL CONSTRUCTION OF FIBRE BUNDLES WITH A STRUCTURAL GROUP

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The usual definition of a fibre bundle with a structural group can be formulated in terms of categories. The advantage of such a definition is that the functorial properties of fibre spaces with a structural group follow immediately.

Terminology: categories, direct products and functors will be used in the same sense as in [3].

Notations:

$\mathscr{S}p$ ... the category of topological spaces and continuous maps;

$\mathscr{B}un$ ... the category of topological bundles and bundle morphisms (see [2], Chap. II, § 3);

$\mathscr{S}p_G$ ... the category of G - spaces and G - morphisms (see [2], Chap. IV, § 1.3);

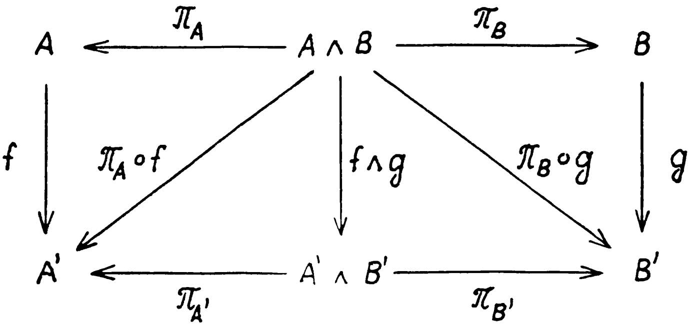
$\mathscr{P}_G$ ... the category of free perfect actions of a topological group G on topological spaces (see [1], Chap. III, § 4);

$\mathscr{I}_G$ ... the forgetful functor from $\mathscr{S}p_G$ to $\mathscr{S}p$ which assigns to every object of $\mathscr{S}p_G$ its action topological space and preserves morphisms;

$\mathscr{O}rb$ ... the functor from Sp_G to $\mathscr{S}p$ which assigns to each object A of $\mathscr{S}p_G$ the corresponding orbital decomposition A/G endowed with the induced topology and to each morphism $f: A \to A'$ the induced continuous map $f/G : A/G \to A'/G$;

$(A \wedge B,$
$\pi_A : A \wedge B \to A,$
$\pi_B : A \wedge B \to B)$ ... the direct product of objects A, B of $\mathscr{S}p_G$;

$\wedge$ ... the functor from the category $\mathscr{S}p_G \times \mathscr{S}p_G$ (the Cartesian product of $\mathscr{S}p_G$ with itself) to the category $\mathscr{S}p_G$ defined by: $(A, B) \to A \wedge B$ for each object (A, B) of $\mathscr{S}p_G \times \mathscr{S}p_G$ and $(f, g) \to f \wedge g$ for each morphism (f, g) of $\mathscr{S}p_G \times \mathscr{S}p_G$, where $f \wedge g$ is uniquely defined by the commutative diagram (0).



The definition of the functor $\wedge$ is correct since the direct product extsis for all objects of $\mathscr{S}p_G$.

Let $\mathscr{F}_G$ be the functor from the category $\mathscr{S}p_G \times \mathscr{S}p_G$ to the category $\mathscr{B}un$ defined by

$$(A, B)\mathscr{F}_G = ((A \wedge B)\mathscr{O}rb,\ (\pi_A)\mathscr{O}rb,\ (A)\mathscr{O}rb)$$

for each object (A, B) of $\mathscr{S}p_G \times Sp_G$ and

$$(f, g)\mathscr{F}_G = ((f \wedge g)\mathscr{O}rb,\ (f)\mathscr{O}rb)$$

for each morphism (f, g) of $\mathscr{S}p_G \times \mathscr{S}p_G$.

The definition of the functor $\mathscr{F}_G$ is justified by the commutativity of the diagram (0).

Since $\mathscr{P}_G$ is a subcategory of $\mathscr{S}p_G$, $\mathscr{F}_G$ can be restricted to $\mathscr{P}_G \times \mathscr{S}p_G$ and this restriction will be denoted by $\mathscr{A}sb_G$. Then the basis of $(A, B)\ \mathscr{A}sb_G$ is a Hausdorff space for each object (A, B) of $\mathscr{P}_G \times \mathscr{S}p_G$ (see [1], Chap. III, § 4.2); moreover $(A, B)\ \mathscr{A}sb_G$ is a fibre bundle over $(A)\mathscr{O}rb_G$ with a fiber $(B)\mathscr{I}_G$ and G as a structural group (see [2], Chap. IV, § 5). Therefore a fibre bundle with a structural group G can be defined by use of the functor $\mathscr{A}sb_G$.

Remark. It is possible to take the category of principal G - spaces with the same morphisms as in $\mathscr{S}p_G$ (see [2], Chap. III, § 2) instead of $\mathscr{P}_G$. In such a case the basis of $(A, B)\ \mathscr{A}sb_G$ is not necessarily a Hausdorff space.

REFERENCES

[1] BOURBAKI, N.: Topologie générale, 3rd ed. Paris Hermann, 1965.
[2] HUSEMOLLER, D.: Fibre bundles. New York, McGraw-Hill Book Co., 1966.
[3] LANG, S.: Algebra. Reading, Mass., Addison-Wesley P. Co., 1965.

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