Czechoslovak Mathematical Journal

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Czechoslovak Mathematical Journal, Vol. 46 (1996), No. 2, 309–316

Persistent URL: `http://dml.cz/dmlcz/127292`

Czechoslovak Mathematical Journal, 46 (121) 1996, Praha

HAUSDORFF COMPLETIONS OF QUASI-UNIFORM SPACES

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(Received June 28, 1994)

INTROCTUDION

It is an old question in the theory of quasi-uniform spaces which quasi-uniformities have a T_2-completion, cf. [6, p. 71]. In [14] the methods of nonstandard analysis have been used to derive necessary and sufficient conditions for the existence of a T_2-completion.

In this paper we give a standard proof of the following sufficient condition given in [14]: if $(X, \mathcal{V})$ is a quasi-uniform T_2-space containing a compatible *uniformity* $\mathcal{U}$ then X possesses a T_2-completion. More general, we prove here that $\mathcal{V}$ possesses a T_2-completion if and only if any compatible quasi-uniformity $\mathcal{W} \supset \mathcal{V}$ possesses a T_2-completion. It follows from the methods of proof that the finest uniformity and the finest quasi-uniformity on a completely regular T_2-space X have a T_2-completion exactly of the cardinality of the Stone-Čech compactification $\beta(X)$. A striking consequence of our main result is that every non-compact uniform T_2-space has a T_2-completion which is different from the usual uniform completion. All these results are contained in the first section.

It is a matter of fact that the construction of the T_2-completion in section 1 does not yield T_2-compactifications (except when the remainder is finite). In the second section we investigate a modified construction which is useful for locally compact spaces. In this case we can prove that every topological T_2-compactification satisfying a certain natural condition can be considered as a quasi-uniform T_2-compactification.

A *completion* of a quasi-uniform space $(X, \mathscr{V})$ is a complete quasi-uniform space $(Y, \mathscr{W})$ that has a dense subspace quasi-isomorphic to $(X, \mathscr{V})$. The induced topology of a quasi-uniform space $(X, \mathscr{V})$ is denoted by $\tau(\mathscr{V})$. Recall that two quasi-uniformities are *compatible* if they induce the same topology. A quasi-uniformity $\mathscr{V}$ is *point-symmetric* if for each $V \in \mathscr{V}$, $x \in X$ there exists a symmetric $U \in \mathscr{V}$ such that $U[x] \subset V[x]$. Throughout the paper we assume the following basic construction:

1.1 Definition. Let $(X, \mathscr{V})$ be a quasi-uniform space and $\mathscr{U}$ be a quasi-uniformity on a larger set S such that the restriction $\mathscr{U}|X$ is a compatible weaker quasi-uniformity than $\mathscr{V}$. We define a filter $\widehat{\mathscr{V}_{\mathscr{U}}}$ on $S \times S$ in the following way: for $V \in \mathscr{V}$, $U \in \mathscr{U}$ define

$$\widehat{V_U} := \bigcup_{x \in X} (\{x\} \times V[x]) \cup \bigcup_{y \in S \setminus X} \{y\} \times (\{y\} \cup (U[y] \cap X)).$$

By definition, $\widehat{\mathscr{V}_{\mathscr{U}}}$ is the filter generated by $\widehat{V_U}$ with $V \in \mathscr{V}$, $U \in \mathscr{U}$.

1.2 Proposition. *$\widehat{\mathscr{V}_{\mathscr{U}}}$ is a quasi-uniformity on S finer than $\mathscr{U}$, in particular $\tau(\mathscr{U}) \subset \tau(\widehat{\mathscr{V}_{\mathscr{U}}})$.*

Proof. It is easy to see that $\widehat{\mathscr{V}_{\mathscr{U}}}$ is a quasi-uniformity. For the second statement let $U \in \mathscr{U}$. Then $V := U \cap X$ is in $\mathscr{V}$. Now check that $\widehat{V_U} \subset U$. $\square$

Even in the case that $\mathscr{U}$ is a uniformity it may occur that $\tau(\widehat{\mathscr{V}_{\mathscr{U}}})$ is different from $\tau(\mathscr{U})$, cf. the proof of Proposition 1.8 or Theorem 1.12. However it is easy to see that $i\colon (X, \mathscr{V}) \to (S, \widehat{\mathscr{V}_{\mathscr{U}}})$ is a quasi-unimorphism.

The following theorem is our main result. It is a modification of a nonstandard construction of a T_2-completion given in [14, Theorem 3.3]. In contrast to that result we now *assume* the existence of a larger complete space $(S, \mathscr{U})$.

1.3 Theorem. *Let $(X, \mathscr{V})$ be a quasi-uniform space. If $\mathscr{U}$ is a complete quasi-uniformity on a larger space S such that $\mathscr{U}|X \subset \mathscr{V}$ are compatible then S is complete with respect to $\widehat{\mathscr{V}_{\mathscr{U}}}$.*

Proof. Let $\mathscr{F}$ be a $\widehat{\mathscr{V}_{\mathscr{U}}}$-Cauchy filter on S and let $U \in \mathscr{U}$. We now consider two cases: in the first one we assume that $G_F := F \cap X$ is non-empty for all $F \in \mathscr{F}$. Then $\{G_F \colon F \in \mathscr{F}\}$ generates a filter $\mathscr{G}$ on S and we claim that $\mathscr{G}$ is a $\mathscr{U}$-Cauchy filter: for $U \in \mathscr{U}$ there exists $V \in \mathscr{V}$ with $V \subset U \cap (X \times X)$. Since $\mathscr{F}$ is a $\widehat{\mathscr{V}_{\mathscr{U}}}$-Cauchy filter there exists $y \in S$ and $F \in \mathscr{F}$ such that $F \subset \widehat{V_U}[y] \subset U[y]$ (note that $V[y] \subset U[y]$ if $y \in X$). By $\mathscr{U}$-completeness $\mathscr{G}$ has an adherent point $z \in S$, i.e., that $G_F \cap U[z] \neq \theta$

for all $F \in \mathscr{F}$ and $U \in \mathscr{U}$. In the case of $z \in X$ we obtain $F \cap \widehat{V_U}[z] \neq \theta$ since $\mathscr{U}|X$ and $\mathscr{V}$ are compatible. If $z \in S \setminus X$ then obviously $F \cap \widehat{V_U}[z] \neq \theta$.

In the second case there exists $F_0 \in \mathscr{F}$ with $F_0 \cap X = \theta$. Since $\mathscr{F}$ is a $\widehat{\mathscr{V}_{\mathscr{U}}}$-Cauchy filter there exists $F \in \mathscr{F}$ with $F \subset \widehat{V_U}[y]$ for some $y \in S$. But y can not be in X; otherwise we would have $F \subset V[y]$ and therefore $F \cap F_0 \subset V[y] \cap F_0 \subset X \cap F_0 = \theta$, a contradiction. Since $y \notin X$ we obtain $F \subset U[y] \cap X \cup \{y\}$. Hence we obtain $F \cap F_0 = \{y\}$. Thus $\mathscr{F}$ is the ultrafilter consisting of all subsets $B \subset S$ with $y \in B$. Therefore $\mathscr{F}$ converges to y and the proof is complete. $\square$

1.4 Remark. A short review of the proof shows that $(S, \widehat{\mathscr{V}_{\mathscr{U}}})$ is convergence complete if $(S, \mathscr{U})$ is convergence complete.

1.5 Corollary. *Let $i = 1$ or $i = 2$. If $(X, \mathscr{V})$ possesses a T_i-completion then any compatible quasi-uniformity $\mathscr{W} \supset \mathscr{V}$ possesses a T_i-completion.*

Proof. Let $(S, \mathscr{U})$ be a T_i-completion of $(X, \mathscr{V})$. Since $\mathscr{U}|X = \mathscr{V} \subset \mathscr{W}$ induce the same topology $\widehat{\mathscr{W}_{\mathscr{U}}}$ is a complete quasi-uniformity in which $(X, \mathscr{W})$ is embedded. Now consider the closure of that subspace in S with respect to $\widehat{\mathscr{W}_{\mathscr{U}}}$. For the separation property just note that $\tau(\mathscr{U}) \subset \tau(\widehat{\mathscr{W}_{\mathscr{U}}})$ by Proposition 1.2. $\square$

1.6 Corollary. *Let $(X, \mathscr{V})$ be a quasi-uniform T_2-space. It there exists a compatible uniformity $\mathscr{W} \subset \mathscr{V}$ then $(X, \mathscr{V})$ possesses a T_2-completion.*

Proof. A uniform T_2-space $\mathscr{W}$ possesses a T_2-completion $(S, \mathscr{U})$. $\square$

1.7 Corollary. *Let $(X, \mathscr{V})$ be a non-compact uniform T_2-space. Then there exists a T_2-completion which is not a uniformity.*

Proof. It is a well-known fact that $\mathscr{V}$ contains a totally bounded uniformity $\mathscr{U}_0$. Then the completion $(S, \mathscr{U})$ of $(X, \mathscr{U}_0)$ is a T_2-compactification. Theorem 1.3 shows that $\widehat{\mathscr{V}_{\mathscr{U}}}$ is a T_2-completion of $(X, \mathscr{V})$. The next proposition shows that $\widehat{\mathscr{V}_{\mathscr{U}}}$ is not uniform on S. $\square$

For the second statement of the next proposition note that a locally compact Hausdorff space $(X, \mathscr{V})$ is an open subset in any (topological) Hausdorff extension S of $(X, \mathscr{V})$.

1.8 Proposition. *Let X be dense in the space $(S, \mathscr{U})$ and $X \neq S$. Then $\widehat{\mathscr{V}_{\mathscr{U}}}$ is not uniform and $\widehat{\mathscr{V}_{\mathscr{U}}} \neq \mathscr{U}$. If $(S, \mathscr{U})$ is a pointsymmetric Hausdorff space and if $(X, \tau(\mathscr{V}))$ is open in $(S, \tau(\mathscr{U}))$ then $(S, \widehat{\mathscr{V}_{\mathscr{U}}})$ is point-symmetric.*

Proof. Let $y \in S$ with $y \notin X$. Then we have $\widehat{V_U}^{-1}[y] = \{z \in S : y \in \widehat{V_U}[z]\} = \{y\}$. Hence the induced topology of $\widehat{\mathscr{V}_{\mathscr{U}}}^{-1}$ is discrete at $y \in S$. On the other side

$\widehat{V_U}[y] = \{y\} \cup (U[y] \cap X)$ is different from $\{y\}$ since y is in the $\tau(\mathscr{U})$-closure of X. It follows that $\widehat{\mathscr{V}_{\mathscr{U}}}$ is not uniform.

Recall that a quasi-uniformity $\mathscr{W}$ is point-symmetric iff $\tau(\mathscr{W}) \subset \tau(\mathscr{W}^{-1})$. Since $\tau(\widehat{\mathscr{V}_{\mathscr{U}}}^{-1})$ is discrete at $y \in S \setminus X$ we only need to consider the case $y \in X$. Let $\widehat{V_U}[y] = V[y]$ be a neighborhood. Since $\mathscr{U}$ (and therefore $\mathscr{V}$) is point-symmetric we can find symmetric $V_1 \in \mathscr{V}$, $U_1 \in \mathscr{U}$ with $V_1[y] \subset V[y]$ and $U_1[y] \subset U[y]$. Since X is an open subset we can assume that $U_1[y] \subset X$. It suffices to show that $\widehat{V_{1U_1}}^{-1}[y] \subset V[y]$. Let $x \in \widehat{V_{1U_1}}^{-1}[y]$. Then $(x, y) \in \widehat{V_{1U_1}}$. If x is in X then $y \in V_1[x]$ and, by symmetry of V_1, $x \in V_1[y] \subset V[y]$. If $x \in S \setminus X$ then $y \in (U_1[x] \cap X) \cup \{x\}$. Since $y \in X$ we have $y \neq x$, in particular $y \in U_1[x]$. The symmetry yields $x \in U_1[y] \subset X$, a contradiction. □

1.9 Corollary. *Let X be a completely regular Hausdorff space. Then the finest compatible uniformity and the finest compatible quasi-uniformity have a T_2-completion of the cardinality of $\beta(X)$.*

Proof. Let $\mathscr{V}$ be the filter considered in Corollary 1.9. Let $\mathscr{W}$ be the weak uniformity induced by the set $C^b(X, \mathbb{R})$ of all bounded continuous real-valued functions. Then $\mathscr{W}$ and $\mathscr{V}$ are compatible and trivially $\mathscr{W} \subset \mathscr{V}$. Moreover $\mathscr{W}$ is totally bounded and it is well known that the completion $\mathscr{U}$ of $\mathscr{W}$ is the Stone-Čech compactification $\beta(X)$. Now apply Theorem 1.3. □

1.10 Theorem. *Let $(X, \mathscr{V})$ be a completely regular quasi-uniform space. If $\mathscr{V}$ contains the Pervin quasi-uniformity $\mathscr{P}$ (with respect to τ) then $\mathscr{V}$ possesses a T_2-completion.*

Proof. $\mathscr{P} \subset \mathscr{V}$ contains a compatible uniformity, see the proof of Theorem 3.11 in [14]. □

The next two results show that the quasi-uniformity $\widehat{\mathscr{V}_{\mathscr{U}}}$ is almost never a compactification.

1.11 Proposition. *Assume that $\mathscr{V}$ is precompact. Then S is precompact with respect to $\widehat{\mathscr{V}_{\mathscr{U}}}$ iff $S \setminus X$ is finite.*

Proof. Choose $V \in \mathscr{V}$ and $U \in \mathscr{U}$. If $\widehat{\mathscr{V}_{\mathscr{U}}}$ is precompact there exists $y_1, \dots, y_n \in S$ with $S \subset \bigcup_{i=1}^{n} \widehat{V_U}[y_i]$. Since $\widehat{V_U} \subset X \cup \{y\}$ we obtain $S \subset X \cup \{y_1, \dots, y_n\}$. For the converse assume that $(X, \mathscr{V})$ is precompact. Hence there exists $x_1, \dots, x_m \in X$ with $X \subset V[x_1] \cup \dots \cup V[x_m]$. Let $S = X \cup \{y_1, \dots, y_n\}$. Then $S \subset \bigcup_{i=1}^{m} \widehat{V_U}[x_i] \cup \bigcup_{j=1}^{n} \widehat{V_U}[y_j]$. □

1.12 Theorem. *Let $(X,\mathscr{V})$ be a precompact quasi-uniform space and $(S,\mathscr{U})$ be a complete Hausdorff space such that $\mathscr{U}\big|X \subset \mathscr{V}$ are compatible. Then the following statements are equivalent for $\widehat{\mathscr{V}_{\mathscr{U}}}$:*

a) *S is precompact.*

b) *$S \setminus X$ is finite.*

c) *S is a Hausdorff compactification*

d) *S is regular.*

Proof. Obviously c) implies d). For the converse note at first that $(X,\mathscr{V})$ is precompact and dense in S. By Theorem 1.3 S is a complete space containing a dense precompact subspace X. Since S is regular a well-known Corollary in [6, p. 53] shows that S is compact. Proposition 1.11 yields the equivalence of a) and b) and c) $\Rightarrow$ a) is clear. For a) $\Rightarrow$ c) note that S is complete (Theorem 1.3) and precompact and therefore compact. □

2. Hausdorff compactifications

2.1 Definition. Let $\mathscr{U}$ and $\mathscr{V}$ as in Definition 1.1. Define

$$\widehat{V_U}(S) := \bigcup_{x\in X}(\{x\}\times V[x]) \cup \bigcup_{y\in S\setminus X}\{y\}\times U[y].$$

Let $\widehat{\mathscr{V}_{\mathscr{U}}}(S)$ be the filter generated by the sets $\widehat{V_U}(S)$ with $U \in \mathscr{U}, V \in \mathscr{V}$.

As before, $\widehat{\mathscr{V}_{\mathscr{U}}}(S)$ is a quasi-uniformity and we have $\mathscr{U} \subset \widehat{\mathscr{V}_{\mathscr{U}}}(S) \subset \widehat{\mathscr{V}_{\mathscr{U}}}$.

2.2 Proposition. *The quasi-uniformity $(X,\mathscr{V})$ is an open subspace of $(S,\widehat{\mathscr{V}_{\mathscr{U}}})$ and $\widehat{\mathscr{V}_{\mathscr{U}}}(S)$. In particular, if S is a compact regular space then X is locally compact.*

Proof. Let $x \in X$. Then $x \in \widehat{V_U}[x] = V[x] \subset X$. Hence X is open in S. The case $\widehat{\mathscr{V}_{\mathscr{U}}}(S)$ is similar. □

2.3 Proposition. *If $(X,\mathscr{V})$ is precompact and $(S,\mathscr{U})$ is hereditarily precompact then S is precompact with respect to $\widehat{\mathscr{V}_{\mathscr{U}}}(S)$.*

Proof. Let $\widehat{V_U}(S)$ be given with $V \in \mathscr{V}$ and $U \in \mathscr{U}$. Since S is precompact with respect to $\mathscr{V}$ and $S \setminus X$ is precompact with respect to $\mathscr{U}\big|(S\setminus X)$ we obtain $X \subset V[x_1] \cup \ldots \cup V[x_m]$ and $(S\setminus X) \subset U[y_1] \cup \ldots \cup U[y_n]$ for some $x_1,\ldots,x_m \in X$ and $y_1,\ldots,y_n \in S \setminus X$. Now observe that $\widehat{V_U}(S)[x_i] = V[x_i]$ and $\widehat{V_U}[y_i] = U[y_i]$ for $i = 1,\ldots,m$ and $j = 1,\ldots,n$. The proof is complete. □

The last proposition has an interesting consequence: Let $(X, \mathcal{V})$ be a precompact T_2-uniformity and let $(S, \mathcal{U})$ be the (unique) uniform Hausdorff completion of $\mathcal{V}$. Then $(S, \widehat{\mathcal{V}_{\mathcal{U}}}(S))$ is precompact and Hausdorff, cf. Proposition 1.2 and 2.3. If S is complete then S is a compact Hausdorff space and therefore X is locally compact by Proposition 2.2. Hence an analogue of Theorem 1.3 for $\widehat{\mathcal{V}_{\mathcal{U}}}(S)$ can only be expected for locally compact spaces. More precisely, we prove

2.4 Theorem. *If $(X, \tau(\mathcal{V}))$ is open in $(S, \tau(\mathcal{U}))$ and $(S, \mathcal{U})$ is a complete quasi-uniformity such that $\mathcal{U}|X \subset \mathcal{V}$ are compatible then S is complete with respect to $\widehat{\mathcal{V}_{\mathcal{U}}}(S)$.*

Proof. Let $\mathcal{F}$ be a $\widehat{\mathcal{V}_{\mathcal{U}}}(S)$-Cauchy filter. Case 1 in the proof of 1.3 can be treated as in 1.3. Hence we can assume that there exists $F_0 \in \mathcal{F}$ such that $F_0 \cap X = \theta$. It is clear that $\mathcal{F}$ is as well a $\mathcal{U}$-Cauchy filter. Hence there exists an adherent point $y_0 \in S$ by $\mathcal{U}$-completeness. It suffices to show that y_0 is an adherent point of $\mathcal{F}$. At first we consider the case $y_0 \in S \setminus X$. Let $\widehat{V_U}(S) = U[y_0]$ be a neighborhood of y_0 and let $F \in \mathcal{F}$. Then $F \cap \widehat{V_U}(S)[y_0] = F \cap U[y_0] \neq \theta$.

In the other case we have $y_0 \in X$. Since $(X, \tau(\mathcal{V}))$ is open in $(S, \tau(\mathcal{U}))$ we can find $U \in \mathcal{U}$ such that $U[y_0] \subset X$. Hence $F_0 \cap U[y_0] \subset F_0 \cap X = \theta$, a contradiction. Hence $y_0 \in X$ is impossible. □

2.5 Theorem. *If $(X, \mathcal{V})$ is locally compact and $(S, \mathcal{U})$ is compact Hausdorff such that $\mathcal{U}|X \subset \mathcal{V}$ are compatible then S is compact with respect to $\widehat{\mathcal{V}_{\mathcal{U}}}(S)$.*

Proof. Let $(T_x)_{x \in S}$ be an $\widehat{\mathcal{V}_{\mathcal{U}}}(S)$-open covering of S with $x \in T_x$. For $x \in S \setminus X$ there exists $U_x \in \mathcal{U}$ such that $U_x[x] \subset T_x$. For $x \in X$ there exists $V_x \in \mathcal{V}$ such that $x \in V_x[x] \subset (X \cap T_x)$ by local compactness. Since $\mathcal{U}|X$ and $\mathcal{V}$ are compatible there exists $U_x \in \mathcal{U}$ such that $x \in U_x[x] \subset V_x[x]$. Since $(U_x[x])_{x \in X}$ is a covering of S the $\mathcal{U}$-compactness implies that there exists a finite subcovering, say $\{U_{x_1}[x_1], \ldots, U_{x_n}[x_n]\}$. Then $\{T_{x_1}, \ldots, T_{x_n}\}$ is the desired finite subcovering. The proof is complete. □

Recall that a *topological T_2-compactification* K of the *topological space* X consists of compact T_2-space K and a topological embedding $i\colon X \to K$ such that $i(X)$ is dense in K. A *quasi-uniform T_2-compactification* of the *quasi-uniform space* $(X, \mathcal{V})$ is a compact quasi-uniform T_2-space $(K, \mathcal{V}_K)$ and a quasi-uniform embedding $i\colon X \to K$ such that $i(X)$ is dense in K. Clearly every quasi-uniform compactification of $(X, \mathcal{V})$ induces a topological compactification; but observe that this correspondence is in general *not* injective, cf. Proposition 3.48 in [6].

It is a natural question whether for every topological T_2-compactification K of the quasi-uniform space $(X, \mathcal{V})$ (seen as a topological space) there exists a quasi-uniformity $\mathscr{V}_K$ on K such that $(K, \mathscr{V}_K)$ is a quasi-uniform T_2-compactification of $(X, \mathscr{V})$. Since every compact T_2-space has a (unique) compatible uniformity $\mathscr{U}(K)$ which is the smallest compatible quasi-uniformity we obtain the following necessary condition for our problem:

$(*)$ The restriction of the associated uniformity $\mathscr{U}(K)$ to the subspace X is smaller than or equal to $\mathscr{V}$.

It is shown in [6, p. 69] that $(*)$ is also sufficient *provided* that $\mathscr{V}$ is totally bounded. We show that $(*)$ is sufficient provided that X is locally compact:

2.6 Theorem. *Let $(X, \mathscr{V})$ be a locally compact quasi-uniform Hausdorff space and K a topological compactification of $(X, \tau(\mathscr{V}))$. Then K is a quasi-uniform T_2-compactification of $(X, \mathscr{V})$ for a quasi-uniformity $\overline{\mathscr{V}}$ on K iff $(*)$ holds.*

Proof. Suppose that $(*)$ holds. Define $\mathscr{U} := \mathscr{U}(K)$ and $S := K$. Now Theorem 2.5 shows that $(K, \widehat{\mathscr{V}_{\mathscr{U}}}(S))$ is a compact space which contains $(X, \mathscr{V})$ as a quasi-uniform dense subspace. □

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