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OSCILLATORY PROPERTIES OF SOLUTIONS OF
THREE-DIMENSIONAL DIFFERENTIAL SYSTEMS
OF NEUTRAL TYPE

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Abstract. The purpose of this paper is to obtain oscillation criteria for the differential system

$$\begin{aligned} [y_1(t) - a(t)y_1(g(t))] &= p_1(t)f_1(y_2(h_2(t))) \\ y_2'(t) &= p_2(t)f_2(y_3(h_3(t))) \\ y_3'(t) &= -p_3(t)f_3(y_1(h_1(t))), \quad t \in \mathbb{R}_+ = [0, \infty). \end{aligned}$$

Keywords: differential system of neutral type, oscillatory (nonoscillatory) solution

MSC 2000: 34K15, 34K40

1. INTRODUCTION

In this paper we consider the neutral differential system of the form

$$(S) \quad \begin{aligned} [y_1(t) - a(t)y_1(g(t))] &= p_1(t)f_1(y_2(h_2(t))) \\ y_2'(t) &= p_2(t)f_2(y_3(h_3(t))) \\ y_3'(t) &= -p_3(t)f_3(y_1(h_1(t))), \quad t \in \mathbb{R}_+ = [0, \infty). \end{aligned}$$

The following conditions are assumed to hold throughout the paper:

- (a) $p_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$, $i = 1, 2, 3$ are continuous functions not identically equal to zero in every neighbourhood of infinity,

$$\int^{\infty} p_j(t) dt = \infty, \quad j = 1, 2;$$

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- (b) $a: \mathbb{R}_+ \rightarrow \mathbb{R}$ is a continuous function satisfying $|a(t)| \leq \lambda < 1$, where λ is a constant and $a(t)a(g(t)) \geq 0$ on \mathbb{R}_+ ;
- (c) $g: \mathbb{R}_+ \rightarrow \mathbb{R}$ is a continuous and increasing function, $g(t) < t$ on \mathbb{R}_+ and $\lim_{t \rightarrow \infty} g(t) = \infty$;
- (d) $h_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ are continuous functions and $\lim_{t \rightarrow \infty} h_i(t) = \infty$, $i = 1, 2, 3$;
- (e) $f_i: \mathbb{R} \rightarrow \mathbb{R}$ are continuous and nondecreasing functions, $uf_i(u) > 0$ for $u \neq 0$, $i = 1, 2, 3$.

The asymptotic properties of solutions of systems with deviating arguments or systems of neutral type are studied for example in the papers [1–12].

The purpose of this paper is to obtain oscillation criteria for the system (S). The paper is a generalization of the results obtained in the paper [12].

Let $t_0 \geq 0$. Denote

$$\tilde{t}_0 = \min \left\{ g(t_0), \inf_{t \geq t_0} h_i(t), i = 1, 2, 3 \right\}.$$

A function $y = (y_1, y_2, y_3)$ is a solution of the system (S) if there exists a $t_0 \geq 0$ such that y is continuous on $[\tilde{t}_0, \infty)$, $y_1(t) - a(t)y_1(g(t))$, $y_i(t)$, $i = 2, 3$, are continuously differentiable on $[t_0, \infty)$ and y satisfies (S) on $[t_0, \infty)$.

Denote by W the set of all solutions $y = (y_1, y_2, y_3)$ of the system (S) which exist on some ray $[T_y, \infty) \subset \mathbb{R}_+$ and satisfy

$$\sup \left\{ \sum_{i=1}^3 |y_i(t)| : t \geq T \right\} > 0 \quad \text{for any } T \geq T_y.$$

A solution $y \in W$ is nonoscillatory if there exists a $T_y \geq 0$ such that its every component is different from zero for all $t \geq T_y$. Otherwise a solution $y \in W$ is said to be oscillatory.

Denote

$$\begin{aligned} h_i^*(t) &= \min\{t, h_i(t)\}, \quad i = 1, 2, 3; \\ \gamma_i(t) &= \sup\{s \geq 0, h_i^*(s) \leq t\}, \quad t \geq 0, \quad i = 1, 2, 3; \\ \beta(t) &= \sup\{s \geq 0, g(s) \leq t\}, \quad t \geq 0; \\ \gamma(t) &= \max\{\gamma_1(t), \gamma_2(t), \gamma_3(t), \beta(t)\}. \end{aligned}$$

For any $y_1(t)$ we define $z_1(t)$ by

$$(1) \quad z_1(t) = y_1(t) - a(t)y_1(g(t)).$$

2. SOME BASIC LEMMAS

Lemma 1. ([6, Lemma 1]) *Let $y \in W$ be a solution of the system (S) with $y_1(t) \neq 0$ on $[t_0, \infty)$, $t_0 \geq 0$. Then y is nonoscillatory and $z_1(t)$, $y_2(t)$, $y_3(t)$ are monotone on some ray $[T, \infty)$, $T \geq t_0$.*

Lemma 2. ([6, Lemma 2]) *Let $y = (y_1, y_2, y_3) \in W$ be a nonoscillatory solution of the system (S) and let $\lim_{t \rightarrow \infty} |z_1(t)| = L_1$, $\lim_{t \rightarrow \infty} |y_i(t)| = L_i$, $i = 2, 3$. Then*

$$(2) \quad L_1 < \infty \quad \text{implies} \quad L_2 = L_3 = 0.$$

Lemma 3. ([6, Lemma 4]) *Let $y = (y_1, y_2, y_3) \in W$ be a nonoscillatory solution of the system (S) on $[t_0, \infty)$, $t_0 \geq 0$. Then there exist an integer $l \in \{1, 3\}$ and a $t_1 \geq t_0$ such that for $t \geq t_1$ either*

$$(3_1) \quad \begin{aligned} z_1(t)y_1(t) &> 0 \\ y_2(t)y_1(t) &< 0 \\ y_3(t)y_1(t) &> 0 \end{aligned}$$

or

$$(3_3) \quad \begin{aligned} z_1(t)y_1(t) &> 0 \\ y_i(t)y_1(t) &> 0, \quad i = 2, 3. \end{aligned}$$

Remark. The case $z_1(t)y_1(t) < 0$ on $[t_1, \infty)$ cannot occur (see [6, Lemma 4]).

We denote by N_1^+ or N_3^+ the set of all nonoscillatory solutions of (S) which satisfy (3₁) or (3₃), respectively. Denote by N the set of all nonoscillatory solutions of (S). Then by Lemma 3 we have

$$N = N_1^+ \cup N_3^+.$$

Lemma 4. ([6, Lemma 5])

I) *Let $y \in N_3^+$ on $[t_1, \infty)$. Then*

$$(4) \quad |y_1(t)| \geq (1 - \lambda)|z_1(t)| \quad \text{for large } t.$$

II) *Let $y \in N_1^+$ on $[t_1, \infty)$.*

i) *If $\lim_{t \rightarrow \infty} |z_1(t)| = L_1 > 0$, then there exists an a_0 : $0 < a_0 < 1$ such that*

$$(5) \quad |y_1(t)| \geq a_0|z_1(t)| \quad \text{for large } t;$$

ii) *if $\lim_{t \rightarrow \infty} z_1(t) = 0$ then $\liminf_{t \rightarrow \infty} |y_1(t)| = 0$, $\lim_{t \rightarrow \infty} y_i(t) = 0$, $i = 2, 3$.*

3. OSCILLATION THEOREMS

Theorem 1. *Let the following conditions be satisfied:*

$$(6) \quad xyf_i(xy) \geq Kxyf_i(x)f_i(y) \quad (0 < K = \text{const.}), \quad i = 1, 2, 3;$$

$$(7) \quad h_j(t), \quad j = 2, 3 \quad \text{are nondecreasing functions;}$$

$$(8) \quad h_3(h_2(h_1(t))) \leq t;$$

$$(9) \quad \int_{\gamma(0)}^{\infty} p_2(t)f_2 \left(\int_{h_3(t)}^{\infty} p_3(s) \, ds \right) dt = \infty;$$

$$(10) \quad \int_{\gamma(\gamma(0))}^{\infty} p_3(t)f_3 \left(\int_{\gamma(0)}^{h_1(t)} p_1(s)f_1 \left(\int_0^{h_2(s)} p_2(x) \, dx \right) ds \right) dt = \infty;$$

$$(11) \quad \int_0^{\alpha} \frac{dt}{f_3(f_1(f_2(t)))} < \infty, \quad \int_0^{-\alpha} \frac{dt}{f_3(f_1(f_2(t)))} < \infty,$$

for every constant $\alpha > 0$.

Then every solution $y \in W$ is either oscillatory or $\liminf_{t \rightarrow \infty} |y_1(t)| = 0$ and $\lim_{t \rightarrow \infty} y_i(t) = 0, i = 2, 3$.

Proof. Let $y \in W$ be a nonoscillatory solution of (S). Then $y \in N_1^+ \cup N_3^+$ on $[t_1, \infty)$.

A) Let $y \in N_1^+$ on $[t_1, \infty)$. Without loss of generality we suppose that $y_1(t) > 0$ for $t \geq t_1$. Then the function $z_1(t)$ is nonincreasing on $[\gamma(t_1), \infty)$ and $\lim_{t \rightarrow \infty} z_1(t) = L_1 < \infty$. From (2) we obtain

$$(12) \quad \lim_{t \rightarrow \infty} y_2(t) = \lim_{t \rightarrow \infty} y_3(t) = 0.$$

We shall prove that $\lim_{t \rightarrow \infty} z_1(t) = 0$. Let $\lim_{t \rightarrow \infty} z_1(t) = L_1 > 0$. Lemma 4 implies that there exist a $t_2 \geq \gamma(t_1)$ and a constant $C_1 = a_0 L_1$ such that $y_1(t) \geq C_1$ for $t \geq t_2$. From (e) we get

$$(13) \quad f_3(y_1(h_1(t))) \geq C_2, \quad t \geq t_3 = \gamma(t_2), \quad \text{where} \quad C_2 = f_3(C_1) > 0.$$

Integrating the third equation of (S) from t to ∞ and then using (13) we have

$$y_3(t) \geq C_2 \int_t^{\infty} p_3(s) \, ds, \quad t \geq t_3.$$

Then in view of (e), (6) and the last inequality we get

$$(14) \quad f_2(y_2(h_3(t))) \geq K f_2(C_2) f_2 \left(\int_{h_3(t)}^{\infty} p_3(s) \, ds \right), \quad t \geq t_4 = \gamma(t_3).$$

Integrating the second equation of (S) from t_4 to t and then using (14) we get

$$y_2(t) \geq y_2(t_4) + K f_2(C_2) \int_{t_4}^t p_2(z) f_2 \left(\int_{h_3(z)}^{\infty} p_3(s) ds \right) dz, \quad t \geq t_4.$$

By virtue of (9), the last inequality implies for $t \rightarrow \infty$ that $\lim_{t \rightarrow \infty} y_2(t) = \infty$, which contradicts (12). Therefore $\lim_{t \rightarrow \infty} z_1(t) = 0$ and from Lemma 4 we have $\liminf_{t \rightarrow \infty} |y_1(t)| = 0$.

B) Let $y \in N_3^+$ on $[t_1, \infty)$. Without loss of generality we suppose that $y_1(t) > 0$ on $[t_1, \infty)$. Integrating the second equation of (S) from t_5 to t we get

$$y_2(t) - y_5(t_5) = \int_{t_5}^t p_2(s) f_2(y_3(h_3(s))) ds, \quad t \geq t_5 = \gamma(t_1)$$

and

$$(15) \quad y_2(h_2(t)) \geq \int_{t_5}^{h_2(t)} p_2(s) f_2(y_3(h_3(s))) ds, \quad t \geq t_6 = \gamma(t_5).$$

Using (e), (6), (15) and the monotonicity of $f_2(y_3(h_3(s)))$ we get

$$f_1(y_2(h_2(t))) \geq K f_1(f_2(y_3(h_3(h_2(t)))) f_1 \left(\int_{t_5}^{h_2(t)} p_2(s) ds \right), \quad t \geq t_6.$$

Integrating the first equation of (S) from t_6 to t and then using the last inequality, we have

$$(16) \quad z_1(t) \geq K \int_{t_6}^t p_1(s) f_1(f_2(y_3(h_3(h_2(s)))) f_1 \left(\int_{t_5}^{h_2(s)} p_2(x) dx \right) ds, \quad t \geq t_6.$$

Using (8), (16) and the monotonicity of $f_1(f_2(y_3(t)))$ we get

$$(17) \quad z_1(h_1(t)) \geq K f_1(f_2(y_3(t))) \int_{t_6}^{h_1(t)} p_1(s) f_1 \left(\int_{t_5}^{h_2(s)} p_2(x) dx \right) ds, \\ t \geq t_7 = \gamma(t_6).$$

In view of Lemma 4 there exists a $t_8 \geq t_7$ such that

$$(18) \quad y_1(h_1(t)) \geq (1 - \lambda) z_1(h_1(t)), \quad t \geq t_9 = \gamma(t_8).$$

In view of (e), (6), (17) and (18) we have

$$(19) \quad f_3(y_1(h_1(t))) \geq C_3 f_3(f_1(f_2(y_3(t)))) f_3 \left(\int_{t_6}^{h_1(t)} p_1(s) f_1 \left(\int_{t_5}^{h_2(s)} p_2(x) dx \right) ds \right),$$

$t \geq t_9$ where $C_3 = K^2 f_3((1 - \lambda)K) > 0$.

Multiplying (19) by $\frac{p_3(t)}{f_3(f_1(f_2(y_3(t))))}$, using the third equation of (S) and then integrating from t_9 to t , we get

$$\int_t^{t_9} \frac{y_3'(z) dz}{f_3(f_1(f_2(y_3(z))))} \geq C_3 \int_{t_9}^t p_3(z) f_3 \left(\int_{t_6}^{h_1(z)} p_1(s) f_1 \left(\int_{t_5}^{h_2(s)} p_2(x) dx \right) ds \right) dz,$$

$t \geq t_9$. The last inequality for $t \rightarrow \infty$ gives a contradiction to (10) with (11). This case cannot occur. The proof of Theorem 1 is complete. \square

Theorem 2. *Suppose that (6)–(9) hold and in addition*

$$(20) \quad f_3(f_1(f_2(t))) = t;$$

$$(21) \quad \int_{\gamma(\gamma(0))}^{\infty} p_3(t) \left[f_3 \left(\int_{\gamma(0)}^{h_1(t)} p_1(s) \left(\int_0^{h_2(s)} p_2(x) dx \right) ds \right) \right]^{(1-\varepsilon)} dt = \infty,$$

where $0 < \varepsilon < 1$.

Then the conclusion of Theorem 1 holds.

Proof. Let $y \in W$ be a nonoscillatory solution of (S). Then $y \in N_1^+ \cup N_3^+$ on $[t_1, \infty)$. As in the proof of Theorem 1, we get two cases: A) and B). In the case A) we proceed in the same way as in the proof of Theorem 1. Consider now the case B). In this case the inequality (19) holds. Using (20), (19) implies

$$(22) \quad f_3(y_1(h_1(t))) \geq C_3 y_3(t) f_3 \left(\int_{t_6}^{h_1(t)} p_1(s) f_1 \left(\int_{t_5}^{h_2(s)} p_2(x) dx \right) ds \right), \quad t \geq t_9.$$

Raising (22) to $(1 - \varepsilon)$ th power we obtain

$$(23) \quad [C_3 y_3(t)]^{(1-\varepsilon)} \left[f_3 \left(\int_{t_6}^{h_1(t)} p_1(s) f_1 \left(\int_{t_5}^{h_2(s)} p_2(x) dx \right) ds \right) \right]^{(1-\varepsilon)} \leq [f_3(y_1(h_1(t)))]^{(1-\varepsilon)}, \quad t \geq t_9.$$

Lemma 4 together with (6) implies that there exist a $t_{10} \geq t_9$ and a constant $C_4 > 0$ such that

$$(24) \quad f_3(y_1(h_1(t))) \geq C_4, \quad t \geq t_{10}.$$

Now (24) implies

$$(25) \quad [f_3(y_1(h_1(t)))]^{(1-\varepsilon)} \leq C_5 f_3(y_1(h_1(t))), \quad t \geq t_{10},$$

where $C_5 = C_4^{-\varepsilon} > 0$.

Combining (23) with (25), we get

$$(26) \quad [C_3 y_3(t)]^{(1-\varepsilon)} \left[f_3 \left(\int_{t_6}^{h_1(t)} p_1(s) f_1 \left(\int_{t_5}^{h_2(s)} p_2(x) dx \right) ds \right) \right]^{(1-\varepsilon)} \\ \leq C_5 f_3(y_1(h_1(t))), \quad t \geq t_{10}.$$

Multiplying (26) by $p_3(t)[C_3 y_3(t)]^{(\varepsilon-1)}$, using the third equation of (S), integrating from t_{10} to t and then using the fact that $y_3(t)$ is positive and decreasing, we have

$$\int_{t_{10}}^t p_3(z) \left[f_3 \left(\int_{t_6}^{h_1(t)} p_1(s) f_1 \left(\int_{t_5}^{h_2(s)} p_2(x) dx \right) ds \right) \right]^{(1-\varepsilon)} dz \\ \leq C_5 (C_3)^{(\varepsilon-1)} (\varepsilon^{-1}) [y_3(t_{10})]^\varepsilon < \infty, \quad t \geq t_{10},$$

which contradicts (21). Therefore the case B) cannot occur.

The proof of Theorem 2 is complete. □

Theorem 3. *Suppose that (6), (9), (11) hold and in addition*

$$(27) \quad h_2(t) \geq t, \quad h_3(t) \leq t;$$

$$(28) \quad \int_{\gamma(\gamma(0))}^{\infty} p_3(t) f_3 \left(\int_{\gamma(0)}^{h(t)} p_1(s) f_1 \left(\int_0^s p_2(x) dx \right) ds \right) dt = \infty, \\ \text{where } h(t) = h_1^*(t) = \min\{t, h_1(t)\}.$$

Then the conclusion of Theorem 1 holds.

Proof. Let $y \in W$ be a nonoscillatory solution of (S) on $[t_1, \infty)$. Further, proceeding in the same way as in the proof of Theorem 2 we consider only the case B). Using (27) and the monotonicity of $f_1(y_2(t))$ on $[t_1, \infty)$ the first equation of system (S) implies

$$(29) \quad z_1'(t) \geq p_1(t) f_1(y_2(t)), \quad t \geq t_1.$$

Analogously to (29) we have

$$(30) \quad y_2'(t) \geq p_2(t) f_2(y_3(t)), \quad t \geq \gamma(t_1) \geq t_1.$$

Lemma 4 together with (e) and (6) implies that there exists a $t_2^* \geq \gamma(t_1)$ such that

$$(31) \quad f_3(y_1(h_1(t))) \geq C_6 f_3(z_1(h_1(t))), \quad t \geq t_2^*, \\ \text{where } C_6 = K f_3(1 - \lambda) > 0.$$

Using (31) and the monotonicity of $f_3(z_1(t))$ on $[t_2^*, \infty)$ the third equation of (S) implies

$$(32) \quad y_3'(t) \leq -C_6 p_3(t) f_3(z_1(h(t))), \quad t \geq t_2^*.$$

In view of (29), (30), (32) we modify the system (S) to the form

$$(S^*) \quad \begin{aligned} z_1'(t) &\geq p_1(t) f_1(y_2(t)) \\ y_2'(t) &\geq p_2(t) f_2(y_3(t)) \\ y_3'(t) &\leq -C_6 p_3(t) f_3(z_1(h(t))), \quad t \geq t_2^*. \end{aligned}$$

System (S*) yields

$$(33) \quad z_1(t) \geq \int_{t_2^*}^t p_1(s) f_1(y_2(s)) \, ds, \quad t \geq t_2^*$$

and

$$(34) \quad y_2(s) \geq \int_{t_2^*}^s p_2(x) f_2(y_3(x)) \, dx, \quad s \geq t_2^*.$$

In view of (e), (6) and the monotonicity of $f_2(y_3(x))$ on $[t_2^*, \infty)$, from (34) we have

$$(35) \quad f_1(y_2(s)) \geq K f_1(f_2(y_2(s))) f_1\left(\int_{t_2^*}^s p_2(x) \, dx\right), \quad s \geq t_2^*.$$

Combining (33) with (35) we get

$$(36) \quad z_1(t) \geq K \int_{t_2^*}^t p_1(s) f_1(f_2(y_3(s))) f_1\left(\int_{t_2^*}^s p_2(x) \, dx\right) \, ds, \quad t \geq t_2^*.$$

Using (e), (6) and the monotonicity of $f_1(f_2(y_3(s)))$ on $[t_2^*, \infty)$ we obtain

$$(37) \quad \begin{aligned} f_3(z_1(h(t))) &\geq C_7 f_3(f_1(f_2(y_3(t)))) f_3\left(\int_{t_2^*}^{h(t)} p_1(s) f_1\left(\int_{t_2^*}^s p_2(x) \, dx\right) \, ds\right), \\ t \geq t_3^* = \gamma(t_2^*), \quad &\text{where } C_7 = K^2 f_3(K) > 0. \end{aligned}$$

Multiplying (37) by $\frac{C_6 p_3(t)}{f_3(f_1(f_2(y_3(t))))}$, integrating from t_3^* to t , using the third inequality of (S*) and (11) we get

$$\begin{aligned} C_6 C_7 \int_{t_3^*}^t p_3(z) f_3\left(\int_{t_2^*}^{h(z)} p_1(s) \left(\int_{t_2^*}^s p_2(x) \, dx\right) \, ds\right) \, dz \\ \leq \int_{y_3(t)}^{y_3(t_3^*)} \frac{dz}{f_3(f_1(f_2(z)))} < \infty, \quad t \geq t_3^*, \end{aligned}$$

which contradicts (28) and therefore the case B) cannot occur. The proof of Theorem 3 is complete. \square

Theorem 4. Suppose that (6), (9), (20), (27) hold and in addition

$$(38) \int_{\gamma(\gamma(0))}^{\infty} p_3(t) \left[f_3 \left(\int_{\gamma(0)}^{h(t)} p_1(s) f_1 \left(\int_0^s p_2(x) dx \right) ds \right) \right]^{(1-\varepsilon)} dt = \infty, \quad 0 < \varepsilon < 1,$$

where $h(t) = h_1^*(t)$.

Then the conclusion of Theorem 1 holds.

We can prove Theorem 4 analogously to Theorem 2 and Theorem 3.

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