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STRONG RETRACTS OF UNARY ALGEBRAS

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Abstract. This paper introduces the notion of a strong retract of an algebra and then focuses on strong retracts of unary algebras. We characterize subuniverses of a unary algebra which are carriers of its strong retracts. This characterization enables us to describe the poset of strong retracts of a unary algebra under inclusion. Since this poset is not necessarily a lattice, we give a necessary and sufficient condition for the poset to be a lattice, as well as the full description of the poset.

Keywords: inflations of algebras, retracts of algebras, unary algebras

MSC 2000: 08A60, 08A35

1. Introduction

The motivation for the study of retracts of algebras undoubtedly came from topology. Plenty of results concerning retracts of various classes of algebras such as semigroups, groups, lattices, boolean algebras have already been obtained. In papers [14, 11, 2] the classes of ordered sets and graphs closed under the formation of direct product and retractions were studied. In [2] a structure theory for ordered sets based on the constructions of direct product and retraction is proposed. This idea was later developed in [16] for an arbitrary type of structures (a set equipped with operations and relations). In [5, 6, 7, 8, 9] retracts (and the so-called retract varieties) of monounary algebras were studied.

On the other hand, it seems that the study of inflations began with semigroups. An explicit definition of inflation in the case of an arbitrary universal algebra can be found in [10] (under the name "nilpotent extension"). In the paper the author proved that this construction is strongly connected with normal identities (see also [13, 4, 12, 3, 15]). An identity $p \approx q$ is said to be normal if both p and q are the same variable or if operation symbols occur in both p and q. A variety is normal if it satisfies only normal identities. Let $N(V)$ denote the smallest normal variety containing the class of algebras V. If V is a variety, then an algebra $\mathcal{B}$ belongs to $N(V)$ if and only if $\mathcal{B}$ is an inflation of an algebra $\mathcal{A}$ from V ([10]; see also [4, 12, 15]).

As can be easily seen from the definition, inflation is a special kind of retract extension. We adopt the following definition: if an algebra $\mathcal{A}$ is an inflation of an algebra $\mathcal{B}$, then $\mathcal{B}$ is a strong retract of $\mathcal{A}$. In this paper we focus on strong retracts of unary algebras. First we characterize strong retracts of unary algebras using some special congruences of those algebras. Since the poset of strong retracts of a unary algebra is not necessarily a lattice, we give a necessary and sufficient condition for the poset to be a lattice. Afterwards, we give the full description of the poset of strong retracts of a unary algebra.

2. Characterization of strong retracts of unary algebras

Our basic notation is adopted from [1].

Definition 1. Let $\mathcal{A}$ and $\mathcal{B}$ be algebras of the same type $\mathcal{F}$. $\mathcal{B}$ is an *inflation* of $\mathcal{A}$ if $A \subseteq B$ and there is a mapping $\varphi\colon B \to A$ such that φ is the identity on A (i.e. $\varphi\big|_A = 1_A$) and for all $b_1, \dots, b_n \in B$ and all $f \in \mathcal{F}_n$ $(n \geqslant 0)$ we have

$$f^{\mathcal{B}}(b_1, \dots, b_n) = f^{\mathcal{A}}(\varphi(b_1), \dots, \varphi(b_n)).$$

It is easy to see that if $\mathcal{B}$ is an inflation of $\mathcal{A}$, then $\mathcal{A}$ is a subalgebra and a homomorphic image of $\mathcal{B}$ (namely, the mapping φ is a homomorphism from $\mathcal{B}$ onto $\mathcal{A}$). Let us recall that a subalgebra $\mathcal{A}$ is a *retract* of an algebra $\mathcal{B}$ (and that $\mathcal{B}$ is a *retract extension* of $\mathcal{A}$) if there is a homomorphism $\varphi\colon B \to A$ such that $\varphi\big|_A = 1_A$. The mapping φ is often referred to as the *retraction* of $\mathcal{B}$ onto $\mathcal{A}$. So, inflation is a special kind of retract extension.

Definition 2. If an algebra $\mathcal{B}$ is an inflation of an algebra $\mathcal{A}$, then $\mathcal{A}$ is called a *strong retract* of $\mathcal{B}$. The corresponding retraction $\varphi\colon B \to A$ is referred to as a *strong retraction* of $\mathcal{B}$ (onto $\mathcal{A}$).

According to the definition above, a strong retract of $\mathcal{B}$ is an algebra of the same type. Since we are interested in nonempty subsets of B that are carriers of strong retracts, such subsets will also be referred to as strong retracts of $\mathcal{B}$. Therefore, by "A is a retract of $\mathcal{B}$" we mean that A is a carrier of a retract of $\mathcal{B}$.

In the sequel, we are interested in strong retracts of unary algebras (i.e. algebras with one or more unary operations).

Let $\mathcal{A}$ be a unary algebra of type $\mathcal{F}$. As usual, the interpretation $f^{\mathcal{A}}$ of the operational symbol $f \in \mathcal{F}$ will also be denoted by f. Therefore, if $\mathcal{A}$ is a unary algebra of type $\mathcal{F}$, we will denote it by $\mathcal{A} = \langle A, F\rangle$, where $F = \mathcal{F}^{\mathcal{A}}$. Also, let $F(A)$ denote the set $\{f(a)\colon f \in F, a \in A\}$. Finally, let us note that instead of $\langle A, \{f\}\rangle$ we simply write $\langle A, f\rangle$.

Let $\mathcal{A} = \langle A, F\rangle$ be a unary algebra. If X is a nonempty subset of A, one can easily verify that a mapping $\varphi\colon A \to X$ is a strong retraction of $\mathcal{A}$ if and only if

- $\varphi\big|_X = 1_X$,
- $F(A) \subseteq X$ and
- for all $a \in A$ and all $f \in F$, $f(a) = f(\varphi(a))$.

Definition 3. For a unary algebra $\mathcal{A} = \langle A, F\rangle$, let $\theta_{\mathcal{A}}$ be a binary relation on A defined by

$$\langle x, y\rangle \in \theta_{\mathcal{A}} \text{ if and only if } (\forall f \in F) f(x) = f(y).$$

An equivalence class E of $\theta_{\mathcal{A}}$ is called *trivial* if $|E| = 1$.

Relation $\theta_{\mathcal{A}}$ is obviously a congruence of $\mathcal{A}$. Also, if φ is a strong retraction of $\mathcal{A}$, then $\langle a, \varphi(a)\rangle \in \theta_{\mathcal{A}}$ for all $a \in A$.

The following characterization of strong retracts of a unary algebra is crucial for the rest of the paper.

Theorem 1. *Let $\mathcal{A} = \langle A, F\rangle$ be a unary algebra and let X be a nonempty subset of A. X is a strong retract of A if and only if*

(i) *$F(A) \subseteq X$, and*

(ii) *for every equivalence class E of $\theta_{\mathcal{A}}$, $X \cap E \neq \emptyset$.*

Proof. $\Rightarrow$: (i) is obvious. To prove (ii), let $E = a/\theta_{\mathcal{A}}$, for some $a \in A$, be an equivalence class of $\theta_{\mathcal{A}}$. Let φ be the strong retraction corresponding to X. Then $\varphi(a) \in E$ and $\varphi(a) \in X$.

$\Leftarrow$: For every equivalence class $E \in A/\theta_{\mathcal{A}}$ choose arbitrary $r_E \in X \cap E$ (we assume AC) and define $\varphi\colon A \to X$ as follows:

$$\varphi(x) = \begin{cases} x, & x \in X \\ r_E, & x \notin X \text{ but } x \in E \text{ for some } E \in A/\theta_{\mathcal{A}}. \end{cases}$$

Then φ is a strong retraction onto X: $\varphi\big|_X = 1_X$, $F(A) \subseteq X$ and $f(x) = f(\varphi(x))$ for all $x \in A$ and all $f \in F$. □

Corollary 1. *Let $\mathcal{A} = \langle A, F\rangle$ be a unary algebra.*

(i) *If $X \subseteq A$ is a strong retract of $\mathcal{A}$ and $X \subseteq X' \subseteq A$, then X' is also a strong retract of $\mathcal{A}$.*

(ii) *An arbitrary union of strong retracts of $\mathcal{A}$ is a strong retract of $\mathcal{A}$.*

(iii) *The intersection of strong retracts of $\mathcal{A}$ is not necessarily a strong retract of $\mathcal{A}$.*

P r o o f. (i) and (ii) are obvious. To show (iii), we present a simple example. Let $\mathcal{A} = \langle A, f \rangle$ be a monounary algebra where $A = \{0, 1, 2, 3\}$ and $f = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 \end{pmatrix}$. Then $X_1 = \{0, 1, 2\}$ and $X_2 = \{0, 1, 3\}$ are strong retracts of $\mathcal{A}$, while $X_1 \cap X_2 = \{0, 1\}$ is not. □

3. Posets of strong retracts of unary algebras

Definition 4. For a unary algebra $\mathcal{A}$, let $R^*(\mathcal{A})$ be the set of all strong retracts of $\mathcal{A}$ and let $\mathcal{R}^*(\mathcal{A}) = \langle R^*(\mathcal{A}), \subseteq \rangle$ be the corresponding poset.

Corollary 1 implies that the poset of strong retracts of a unary algebra is not necessarily a lattice. We shall give a necessary and sufficient condition for a unary algebra $\mathcal{A}$ under which the poset $\mathcal{R}^*(\mathcal{A})$ is a lattice.

Theorem 2. *Let $\mathcal{A} = \langle A, F \rangle$ be a unary algebra. $\mathcal{R}^*(\mathcal{A})$ is a lattice if and only if $F(A)$ intersects every nontrivial equivalence class of $\theta_{\mathcal{A}}$. If $\mathcal{R}^*(\mathcal{A})$ is a lattice, it is isomorphic to $\langle \mathcal{P}(S), \subseteq \rangle$ for some set S.*

P r o o f. $\Rightarrow$: Let $E \in A/\theta_{\mathcal{A}}$ be a nontrivial equivalence class of $\theta_{\mathcal{A}}$ such that $E \cap F(A) = \emptyset$. Choose distinct elements $a, b \in E$ and consider $X_a = (A \setminus E) \cup \{a\}$ and $X_b = (A \setminus E) \cup \{b\}$. X_a and X_b are strong retracts of $\mathcal{A}$ while $X_a \cap X_b$ is not (because $(X_a \cap X_b) \cap E = \emptyset$). If $\mathcal{R}^*(\mathcal{A})$ were a lattice, then the greatest lower bound $X_a \wedge X_b$ of X_a and X_b would be included in $X_a \cap X_b$ and Corollary 1, (i), would imply that $X_a \cap X_b \in R^*(\mathcal{A})$, a contradiction.

$\Leftarrow$: Suppose that $F(A)$ intersects every nontrivial class of $\theta_{\mathcal{A}}$ and let $B = F(A) \cup \{x\colon \{x\} \in A/\theta_{\mathcal{A}}\}$. According to Theorem 1, $X \subseteq A$ is a strong retract of $\mathcal{A}$ if and only if $B \subseteq X$. Therefore, $R^*(\mathcal{A}) = \{X \subseteq A\colon B \subseteq X\}$, and that is a lattice under set-theoretic operations.

We can also conclude that if $\mathcal{R}^*(\mathcal{A})$ is a lattice, it is isomorphic to $\mathcal{P}(A \setminus B)$. □

Our next aim is to give a full description of $\mathcal{R}^*(\mathcal{A})$. The key role in the description is played by the fact that for every unary algebra $\mathcal{A}$ there is a "canonical" monounary algebra $\mathcal{C}$ such that $\mathcal{R}^*(\mathcal{A}) \cong \mathcal{R}^*(\mathcal{C})$, where the isomorphism is an isomorphism between posets.

Let us recall some well-known definitions. Two posets $\mathcal{S}_1 = \langle S_1, \leqslant_1 \rangle$ and $\mathcal{S}_2 = \langle S_2, \leqslant_2 \rangle$ are *isomorphic* if there is a bijection $\psi\colon S_1 \to S_2$ such that $x \leqslant_1 y \Leftrightarrow \psi(x) \leqslant_2 \psi(y)$ for all $x, y \in S_1$. In this case we write $\mathcal{S}_1 \cong \mathcal{S}_2$. If $\mathcal{S}_i$, $i \in I$, is a family of posets, the *direct product* $\prod\limits_{i \in I} \mathcal{S}_i$ of the family is the poset $\left\langle \prod\limits_{i \in I} S_i, \leqslant \right\rangle$ where "$\leqslant$" is defined componentwise, i.e. for $x, y \in \prod\limits_{i \in I} S_i$, $x \leqslant y \Leftrightarrow (\forall i \in I) x_i \leqslant_i y_i$.

Let $\mathcal{A}_i = \langle A_i, f_i \rangle$ be a family of pairwise disjoint monounary algebras. The *disjoint sum* of the family, denoted by $\bigoplus_{i\in I} \mathcal{A}_i$, is a monounary algebra $\mathcal{A} = \langle A, f \rangle$ where $A = \bigcup_{i\in I} A_i$ and $f(x) = f_i(x)$ for $x \in A_i$.

Proposition 1. *Let $\mathcal{A}_i$, $i \in I$, be a family of pairwise disjoint monounary algebras. Then $\mathcal{R}^*\Big(\bigoplus_{i\in I} \mathcal{A}_i\Big) \cong \prod_{i\in I} \mathcal{R}^*(\mathcal{A}_i)$.*

Proof. Consider $\varphi\colon R^*\Big(\bigoplus_{i\in I} \mathcal{A}_i\Big) \to \prod_{i\in I} R^*(\mathcal{A}_i)$ defined by

$$\varphi(X) = \langle X \cap A_i \colon\ i \in I \rangle.$$

It is a routine to check that φ is well defined and that it is an isomorphism between the two posets. $\square$

It turns out that there are very simple monounary algebras which capture the structure of strong retracts of unary algebras. Those are referred to as "canonical" monounary algebras.

Definition 5. A monounary algebra $\mathcal{A} = \langle A, f \rangle$ is said to be an *almost constant monounary algebra* if there are elements $a, b \in A$ such that for all $x \in A \setminus \{a, b\}$, $f(x) = a$ and $f(a) = f(b) = b$. (Note that a and b are not necessarily distinct elements of A.) If $a = b$ then f is a constant unary operation. In that special case, the algebra $\mathcal{A}$ is said to be a *constant monounary algebra.*

Lemma 1. *If $\mathcal{C}$ is an almost constant monounary algebra, then there is a nonempty set S such that $\mathcal{R}^*(\mathcal{C}) \cong \langle \mathcal{P}(S), \subseteq \rangle$ or $\mathcal{R}^*(\mathcal{C}) \cong \langle \mathcal{P}(S) \setminus \{\emptyset\}, \subseteq \rangle$.*

Proof. Let $\mathcal{C} = \langle C, g \rangle$ and put $S = C \setminus g(C)$. $\square$

Definition 6. A monounary algebra is said to be *canonical* if it is a disjoint sum of almost constant monounary algebras.

Theorem 3. *For every unary algebra $\mathcal{A}$ there is a canonical monounary algebra $\mathcal{C}$ such that $\mathcal{R}^*(\mathcal{A}) \cong \mathcal{R}^*(\mathcal{C})$.*

Proof. Let $\mathcal{A} = \langle A, F \rangle$ be a unary algebra and let $P = \{p_\infty\} \cup \{p_E \colon E \in A/\theta_{\mathcal{A}}$ and $E \cap F(A) = \emptyset\} \cup \{q_E \colon E \in A/\theta_{\mathcal{A}}$ and $E \cap F(A) = \emptyset\}$ be a set of new distinct objects (i.e. $P \cap A = \emptyset$). Consider $\mathcal{C} = \langle C, g \rangle$ where $C = (A \setminus F(A)) \cup P$ and

$g\colon C \to C$ is defined by

$$g(s) = \begin{cases} p_\infty, & s \in E \in A/\theta_{\mathcal{A}} \text{ and } E \cap F(A) \neq \emptyset, \\ p_\infty, & s = p_\infty \\ q_E, & s \in E \in A/\theta_{\mathcal{A}} \text{ and } E \cap F(A) = \emptyset, \\ p_E, & s = p_E \text{ or } s = q_E. \end{cases}$$

Observe that

(1) $\mathcal{C}$ is a canonical monounary algebra,

(2) $C/\theta_{\mathcal{C}} = \{E \in A/\theta_{\mathcal{A}}\colon E \cap F(A) = \emptyset\} \cup \{\{p_\infty\} \cup \{s \in A \setminus F(A)\colon s \in E \in A/\theta_{\mathcal{A}} \text{ and } E \cap F(A) \neq \emptyset\}\} \cup \{\{p_E, q_E\}\colon E \in A/\theta_{\mathcal{A}} \text{ and } E \cap F(A) = \emptyset\}$,

(3) $g(C) = P$,

(4) $A \setminus F(A) = C \setminus g(C)$ and

(5) for each $E \subseteq A \setminus F(A)$ we have $E \in A/\theta_{\mathcal{A}} \Leftrightarrow E \in C/\theta_{\mathcal{C}}$.

We shall prove that $\psi\colon R^*(\mathcal{A}) \to R^*(\mathcal{C})$ given by $\psi(X) = (X \setminus F(A)) \cup P$ is an isomorphism of posets $\mathcal{R}^*(\mathcal{A})$ and $\mathcal{R}^*(\mathcal{C})$.

To prove that ψ is well defined, consider a strong retract X of $\mathcal{A}$. We have $g(C) = P \subseteq \psi(X)$ according to the definition of ψ. Let $E \in C/\theta_{\mathcal{C}}$ be arbitrary. If $P \cap E \neq \emptyset$, then $\psi(X) \cap E \neq \emptyset$, too. If, on the other hand, $P \cap E = \emptyset$, then $E \subseteq C \setminus g(C) = A \setminus F(A)$. Due to observation (5), $E \in A/\theta_{\mathcal{A}}$. Since X is a strong retract of $\mathcal{A}$, $X \cap E \neq \emptyset$. However, $E \subseteq A \setminus F(A)$. Therefore, $(X \setminus F(A)) \cap E \neq \emptyset$, i.e. $\psi(X) \cap E \neq \emptyset$.

One can similarly prove that ψ is surjective: it suffices to note that for a strong retract Y of $\mathcal{C}$, $(Y \setminus P) \cup F(A)$ is a strong retract of $\mathcal{A}$ and $\psi((Y \setminus P) \cup F(A)) = Y$.

Since ψ is trivially injective, ψ is a bijection. Moreover, in the previous paragraph we saw that $\psi^{-1}(Y) = (Y \setminus P) \cup F(A)$. Both ψ and ψ^{-1} are monotone. Therefore, ψ is the required isomorphism. □

Theorem 4. *A poset $\mathcal{Q}$ is isomorphic to the poset of strong retracts of a unary algebra if and only if $\mathcal{Q} \cong \prod_{i \in I} \mathcal{S}_i$ where each $\mathcal{S}_i$ is either $\langle \mathcal{P}(S), \subseteq \rangle$ for some S, or $\langle \mathcal{P}(S) \setminus \{\emptyset\}, \subseteq \rangle$ for some nonempty S.*

Proof. $\Rightarrow$: Let $\mathcal{Q}$ be a poset isomorphic to $\mathcal{R}^*(\mathcal{A})$ for some unary algebra $\mathcal{A}$ and let $\mathcal{C}$ be a canonical monounary algebra such that $\mathcal{R}^*(\mathcal{A}) \cong \mathcal{R}^*(\mathcal{C})$. Let $\{\mathcal{M}_i\colon i \in I\}$ be the set of all components of $\mathcal{C}$. Then $\mathcal{C} = \bigoplus_{i \in I} \mathcal{M}_i$ and $\mathcal{R}^*(\mathcal{C}) \cong \prod_{i \in I} \mathcal{R}^*(\mathcal{M}_i)$ (Proposition 1). Lemma 1 completes the proof.

$\Leftarrow$: Suppose $\mathcal{Q} \cong \prod_{i \in I} \mathcal{S}_i$ where each $\mathcal{S}_i$ is either $\langle \mathcal{P}(S_i), \subseteq \rangle$ or $\langle \mathcal{P}(S_i) \setminus \{\emptyset\}, \subseteq \rangle$ for some nonempty sets S_i, $i \in I$. Without loss of generality we can assume that S_i's are pairwise disjoint. To each S_i, $i \in I$, we shall assign a monounary algebra as follows:

- if $\mathcal{S}_i = \langle \mathcal{P}(S_i), \subseteq \rangle$, choose new object a_i, define $f_i\colon S_i \cup \{a_i\} \to S_i \cup \{a_i\}$ by $f_i(x) = a_i$ for all $x \in S_i \cup \{a_i\}$ and put $\mathcal{M}_i = \langle S_i \cup \{a_i\}, f_i \rangle$;
- if $\mathcal{S}_i = \langle \mathcal{P}(S_i) \setminus \{\emptyset\}, \subseteq \rangle$, choose new objects a_i and b_i, $a_i \neq b_i$, define $f_i\colon S_i \cup \{a_i, b_i\} \to S_i \cup \{a_i, b_i\}$ by

$$f_i(x) = \begin{cases} a_i, & x \in S_i \\ b_i, & x \in \{a_i, b_i\}, \end{cases}$$

  and put $\mathcal{M}_i = \langle S_i \cup \{a_i, b_i\}, f_i \rangle$.

It is easy to verify that $\mathcal{R}^*(\mathcal{M}_i) = \mathcal{S}_i$, $i \in I$. If $\mathcal{A} = \bigoplus\limits_{i\in I} \mathcal{M}_i$, then $\mathcal{R}^*(\mathcal{A}) \cong \prod\limits_{i\in I} \mathcal{R}^*(\mathcal{M}_i) \cong \prod\limits_{i\in I} \mathcal{S}_i \cong \mathcal{Q}$. □

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