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HYPERCONTRACTIVITY OF SOLUTIONS TO HAMILTON-JACOBI EQUATIONS

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Dedicated to Ivo Vrkoč on the occasion of his 70th birthday

Abstract. We show that solutions to some Hamilton-Jacobi Equations associated to the problem of optimal control of stochastic semilinear equations enjoy the hypercontractivity property.

Keywords: Hamilton-Jacobi equation, stochastic semilinear equation, invariant measure, Log-Sobolev inequality, hypercontractivity

MSC 2000: 60H15

1. Introduction

Let us consider a stochastic evolution equation on separable Hilbert space H:

$$\begin{cases} \mathrm{d}Y(s) = \big(AY(s) + F(Y(s))\big)\,\mathrm{d}s + \mathrm{d}W(s), \\ Y(0) = x \in H. \end{cases} \tag{1.1}$$

We assume that A is a generator of a strongly continuous semigroup $\boldsymbol{S} = (S(t))$ on H and W is a standard cylindrical Wiener process on H defined on a stochastic basis $(\Omega, \mathscr{F}, (\mathscr{F}_t), \mathbb{P})$. Under some conditions on $\boldsymbol{S}$ and the nonlinear mapping $F\colon H \to H$ (see below for details) there exists a unique solution $Y(\cdot, x)$ to (1.1) for each $x \in H$.

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Let $P_t\varphi(x) = \mathbb{E}\varphi\big(Y(t,x)\big)$ be the transition semigroup of the process Y. Assume that ν is an invariant measure of this semigroup, that is

$$\int_H P_t\varphi(x)\nu(\mathrm{d}x) = \int_H \varphi(x)\nu(\mathrm{d}x), \quad \varphi \in C_b(H).$$

Then the semigroup (P_t) may be extended to a C_0-semigroup of contractions on the space $L^p(H,\nu)$ for each $p \in [1,\infty)$. If the function φ is sufficiently regular then the function $u(t,x) = P_t\varphi(x)$ is a solution of the Backward Kolmogorov Equation

$$\begin{cases} \dfrac{\partial u}{\partial t}(t,x) = \dfrac{1}{2}\operatorname{tr}\big(QD^2u(t,x)\big) + \big\langle Ax + F(x), Du(t,x)\big\rangle, \\ u(0,x) = \varphi(x). \end{cases}$$

Moreover, the generator N of the semigroup (P_t) in $L^p(H,\nu)$ is an extension of the differential operator

$$N_0\varphi(x) = \frac{1}{2}\operatorname{tr}\big(QD^2\varphi(x)\big) + \big\langle Ax + F(x), D\varphi(x)\big\rangle,$$

for smooth cylindrical functions, see for example [7], [3], [4], [11]. If the generator N of (P_t) in $L^p(H,\nu)$ satisfies, for $p > 1$, the Logarithmic Sobolev Inequality

$$\int_E \varphi^p \log\varphi^p \,\mathrm{d}\nu \leqslant \alpha(p)\big\langle(\lambda(p) - N)\varphi, \varphi^{p-1}\big\rangle + \|\varphi\|_p^p \log\|\varphi\|_p^p, \tag{1.2}$$

where $\|\cdot\|_p$ stands for the norm in $L^p(H,\nu)$ then the semigroup (P_t) has the so-called hypercontractivity property:

$$\|P_t\varphi\|_{q(t)} \leqslant \mathrm{e}^{m(t)}\|\varphi\|_p, \tag{1.3}$$

where $q(t) > p$. It is well known, see for example [1], that this property yields the existence of the spectral gap for the generator N in $L^2(H,\nu)$.

The aim of this paper is to show that the analogous hypercontractive estimate (1.3) holds for solutions to the following Hamilton-Jacobi Equation (HJE):

$$\begin{cases} \dfrac{\partial u}{\partial t}(t,x) = \dfrac{1}{2}\operatorname{tr}\big(QD^2u(t,x)\big) + \big\langle Ax + F(x), Du(t,x)\big\rangle \\ \qquad\qquad\qquad -G\big(Q^{1/2}Du(t,x)\big) + f(x), \\ u(0,x) = \varphi(x), \end{cases} \tag{1.4}$$

where the Hamiltonian $G\colon H \to \mathbb{R}$ is specified below. It is well known that equation (1.4) is related to the problem of optimal control of the stochastic evolution equation

$$\begin{cases} \mathrm{d}X(s) = \big(AX(s) + F(X(s)) - \alpha(s)\big)\,\mathrm{d}s + \mathrm{d}W(s), \\ X(t) = x \in H, \quad t \leqslant s \leqslant T, \end{cases} \tag{1.5}$$

where the control α belonging to a set of admissible controls $\mathscr{A}$ is an H-valued process. The cost functional to be minimised is

$$J(t,x,\alpha) = \int_t^T \big(f(X^\alpha(s,x)) + g(\alpha(s))\big)\,\mathrm{d}s + \varphi\big(X^\alpha(T,x)\big),$$

where X^α stands for the solution of (1.5) corresponding to the control α. Then the Hamiltonian G is defined as

$$G(p) = \sup_a \{\langle p, a\rangle - g(a)\}, \tag{1.6}$$

and the optimal cost

$$u(t,x) = \inf_{\alpha\in\mathscr{A}} \mathbb{E} J(t,x,\alpha)$$

satisfies (under some technical conditions) the HJE (1.4).

We will formulate now the main assumptions of this paper. Let us consider a linear equation

$$\begin{cases} \mathrm{d}Z = AZ\,\mathrm{d}t + \mathrm{d}W, \\ Z(0) = x \in H. \end{cases} \tag{1.7}$$

Let

$$Q_t = \int_0^t S(s)S^*(s)\,\mathrm{d}s.$$

If $\mathrm{tr}(Q_t) < \infty$ for all $t > 0$, then the process

$$Z(t,x) = S(t)x + \int_0^t S(t-s)\mathrm{d}W(s), \quad x \in H, \tag{1.8}$$

defines a solution to (1.7) in H and moreover $Z(t,x) \sim N\big(S(t)x, Q_t\big)$. We will assume a stronger condition.

Hypothesis 1.1. *We have*

$$\mathrm{tr}(Q_\infty) < \infty.$$

Let ν be a probability measure on H. If (P_t) is a C_0 semigroup in $L^p(H,\nu)$ then the domain of its generator in $L^p(H,\nu)$ will be denoted by $\mathrm{dom}_p(N)$.

Hypothesis 1.2. *The function $F\colon\ H \to H$ is Lipschitz continuous. There exists a nondegenerate probability measure ν on H such that (P_t) is a strongly continuous semigroup in $L^p(H,\nu)$. Moreover,*

$$\int_H |x|^2 \nu(\mathrm{d}x) < \infty.$$

Finally we will need assumptions on G and f.

Hypothesis 1.3. *$G\colon\ H \to \mathbb{R}$ is Lipschitz and there exists $c > 0$ such that*

$$G(x) \geqslant -c, \quad x \in H.$$

We have $\varphi, f \in L^p(H,\mu)$. Moreover, $\varphi, f, g \geqslant 0$, where g is conjugate to G (see (1.6)*).*

Let us note that G satisfies Hypothesis 1.3 if the admissible controls α take values in a bounded subset of H. An important case of quadratic Hamiltonian is excluded by this condition.

In the sequel we denote by $C_b(H)$ the space of bounded continuous functions on H and $C_b^1(H)$ stands for the space of bounded continuous functions with bounded and continuous Fréchet derivatives.

Let P_n be an orthogonal projection in H such that $\dim \operatorname{im}(P_n) = n$ and $\operatorname{im}(P_n) \subset \operatorname{dom}(A^*)$. We define the space

$$\mathscr{F}C_0^2(A^*) = \{\varphi \in C_0^2(H)\colon\ \varphi = f \circ P_n,\ \ n \geqslant 0,\ \ f \in C_0^2(\mathbb{R}^n)\}.$$

In the notation $f \circ P_n$ above we identify $P_n x$ with the the vector

$$(\langle x, h_1\rangle, \ldots, \langle x, h_n\rangle) \in \mathbb{R}^n,$$

where $h_1, \ldots, h_n$ generate the space $\operatorname{im}(P_n)$.

2. Preliminaries on HJB equation

For $\varphi \in C_b^1(H)$ we denote by $D\varphi$ the Fréchet derivative of φ. Then

$$\|D\varphi\|_p^p = \int_H |D\varphi(x)|^p \nu(\mathrm{d}x) < \infty.$$

Note that we use the same notation for the norms in space $L^p(H,\nu)$ of real-valued functions and in the space $L^p(H,\nu;H)$ of vector-valued functions.

Hypothesis 2.1. *The operator* $\big(D, C_b^1(H)\big)$ *is closable in* $L^p(H,\nu)$.

Let $W^{1,p}(H,\nu)$ be the Sobolev space defined as the closure of $C_b^1(H)$ in the norm

$$\|\varphi\|_{1,p} = \big(\|\varphi\|_p^p + \|D\varphi\|_p^p\big)^{1/p}.$$

If Hypothesis 2.1 is satisfied then $W^{1,p}(H,\nu)$ is a continuously imbedded subspace of $L^p(H,\nu)$.

A solution to (1.4) is defined as a function $u \in W^{1,p}(H,\mu)$ such that

$$u(t,\varphi) = P_t\varphi + \int_0^t P_{t-s}\big(f - G\big(Du(s,\varphi)\big)\big)\,\mathrm{d}s. \tag{2.1}$$

In [2] and [11] this equation was studied in the space $C_b(H)$. If φ and f are continuous functions of polynomial growth then the solution to (2.1) exists by a result in [10]. The existence and uniqueness of solutions for $\varphi, f \in L^2(H,\nu)$ in case of degenerate noise was proved in [9] under the assumption that ν is an invariant measure for the system (1.1).

Theorem 2.2. *Assume Hypotheses* 1.1, 1.2, 1.3 *and* 2.1. *Then for each* $p \in (1,\infty)$ *there exists a unique solution to equation* (2.1).

P r o o f. Under the present assumptions the proof is rather standard so it is only sketched here. Note first that the law of the process Y is absolutely continuous with respect to the Gaussian law of the process Z. Therefore, by [13] the mapping F is ν-a.s. Gateaux differentiable and $DF \in L^\infty(H,\nu)$. Next, by the result in [8] the function $P_t\varphi$ is Gateaux differentiable on H and the Bismut-Elworthy formula holds: for $\varphi \in C_b(H)$

$$\langle DP_t\varphi(x), h\rangle = \frac{1}{t}\mathbb{E}\bigg(\varphi\big(Y(t,x)\big)\int_0^t \langle \zeta^h(s,x), \mathrm{d}W(s)\rangle\bigg), \tag{2.2}$$

where $\zeta(t,x) = DY(t,x)$ is well defined and for any $h \in H$ the process $\zeta^h(t,x) = \zeta(t,x)h$ satisfies an equation

$$\begin{cases} \dfrac{\mathrm{d}\zeta^h}{\mathrm{d}t}(t,x) = \big(A + DF\big(Y(t,x)\big)\big)\zeta^h(t,x), \\ \zeta(0,x) = h. \end{cases}$$

It is easy to see that (2.2) yields

$$\|DP_t\varphi\|_p \leqslant \frac{c}{\sqrt{t}}\|\varphi\|_p, \quad t \leqslant T,$$

first for $\varphi \in C_b(H)$ and then for $\varphi \in L^p(H,\nu)$ by approximations. Next, let us define an operator

$$\mathscr{K}\colon\ C\big(0,T;W^{1,p}(H,\nu)\big) \to C\big(0,T;W^{1,p}(H,\nu)\big)$$

by the formula

$$\mathscr{K}v(t) = P_t\varphi + \int_0^t P_{t-s}\big(f - G(Dv(s))\big)\,\mathrm{d}s.$$

It easy to check that $\mathscr{K}$ is a strict contraction for T small enough and therefore the existence of a unique solution (2.1) follows from the Banach Fixed Point Theorem.

□

We will formulate two rather standard lemmas which will be useful in the sequel.

Lemma 2.3.

(a) *For each $\varphi \in \mathscr{F}C_0^2(A^*)$ we have $\varphi \in \operatorname{dom}_p(N)$ and*

$$N\varphi(x) = \frac{1}{2}\operatorname{tr}\big(QD^2\varphi(x)\big) + \langle x, A^*D\varphi(x)\rangle + \langle F(x), D\varphi(x)\rangle. \tag{2.3}$$

(b) *The space $\mathscr{F}C_0^2(A^*)$ is dense in $L^p(H,\nu)$ for each $p \in [1,\infty)$. Moreover, if $\varphi \in L^p(H,\nu)$ and $\varphi \geqslant 0$ then there is a sequence $(\varphi_n) \subset \mathscr{F}C_0^2(A^*)$ such that $\varphi_n \to \varphi$ in $L^p(H,\nu)$ and $\varphi_n \geqslant 0$ for all $n \geqslant 1$.*

Lemma 2.4. *Let $(f_n), (G_n), (\varphi_n) \subset C_b^2(H) \cap \operatorname{dom}_p(N)$ be such that $f_n \to f$, $G_n \to G$ and $\varphi_n \to \varphi$ in $L^p(H,\nu)$ and let u_n be the corresponding solution to (2.1). Then the following holds.*

(a) *For each $n \geqslant 1$ and $t > 0$ we have $u_n(t) \in \operatorname{dom}_p(N)$, the function $t \to u_n(t)$ is in $C^1(0,T,H)$ and*

$$\begin{cases} \dfrac{\mathrm{d}u_n(t)}{\mathrm{d}t} = Nu_n(t) + f_n - G_n\big(Du_n(t)\big), \\ u_n(0) = \varphi_n. \end{cases} \tag{2.4}$$

(b) *We have*

$$\lim_{n\to\infty}\sup_{t\leqslant T}\|u_n-u\|_{1,p}=0.$$

3. Hypercontractivity

Hypothesis 3.1. *The Logarithmic Sobolev Inequality holds for the generator N in $L^p(H,\nu)$, $p>1$, that is, for each $p\in(1,\infty)$ there exist $\alpha>0$ and $\lambda\geqslant 0$ such that for all $\varphi\in\operatorname{dom}_p(N)$, with $\varphi>0$,*

$$\int_E \varphi^p\log\varphi^p\,\mathrm{d}\nu\leqslant\alpha(p)\langle(\lambda(p)-N)\varphi,\varphi^{p-1}\rangle+\|\varphi\|_p^p\log\|\varphi\|_p^p, \tag{3.1}$$

ν-a.s., where

$$\alpha(p)=\frac{p^2}{4(p-1)}\alpha,\quad \lambda(p)=\frac{4(p-1)}{p^2}\lambda. \tag{3.2}$$

Theorem 3.2. *Assume that Hypotheses* 1.1, 1.2, 1.3 *and* 3.1 *hold. If $\lambda>0$ then*

$$\|u(t,\varphi)\|_{p(t)}\leqslant \mathrm{e}^{\lambda\alpha t/p}\|\varphi\|_p+\frac{p}{\lambda\alpha}(\mathrm{e}^{\lambda\alpha t/p}-1)\|f+c\|_p, \tag{3.3}$$

where

$$p(t)=1+(p-1)\mathrm{e}^{4t/\alpha}. \tag{3.4}$$

If $\lambda=0$ then

$$\|u(t,\varphi)\|_{p(t)}\leqslant\|\varphi\|_p+t\|f+c\|_p. \tag{3.5}$$

P r o o f. Assume first that f, G and φ satisfy the assumptions of Lemma 2.4, hence (2.4) holds with u_n replaced by u. We start with an argument which is well known in the theory of the Logarithmic Sobolev Inequality for diffusions, see for example [12] or [1]. Let

$$F(t)=\|u(t,\varphi)\|_{p(t)}=\left(\int_H\big(u(t,\varphi)\big)^{p(t)}\,\mathrm{d}\nu\right)^{1/p(t)}.$$

Then

$$\frac{\mathrm{d}}{\mathrm{d}t}\big(p(t)\log F(t)\big) = p'(t)\log F(t) + p(t)\frac{F'(t)}{F(t)}$$
$$= \frac{\mathrm{d}}{\mathrm{d}t}\log\Big(\int_H \big(u(t,\varphi)\big)^{p(t)}\,\mathrm{d}\nu\Big)$$
$$= \frac{1}{(F(t))^{p(t)}}\int_H \frac{\partial}{\partial t}\big((u(t,\varphi))^{p(t)}\big)\,\mathrm{d}\nu.$$

Hence,

$$p'(t)(F(t))^{p(t)}\log F(t) + p(t)\,(F(t))^{p(t)-1}\,F'(t)$$
$$= \int_H \big(u(t,\varphi)\big)^{p(t)}\Big(p'(t)\log u(t,\varphi) + p(t)\frac{1}{u(t,\varphi)}\frac{\partial}{\partial t}u(t,\varphi)\Big)\,\mathrm{d}\nu$$
$$= \int_H \big(u(t,\varphi)\big)^{p(t)-1}\Big(p'(t)u(t,\varphi)\log u(t,\varphi) + p(t)\frac{\partial}{\partial t}u(t,\varphi)\Big)\,\mathrm{d}\nu.$$

Therefore,

$$p^2(t)(F(t))^{p(t)-1}F'(t)$$
$$= p'(t)\Big(\int_H \big(u(t,\varphi)\big)^{p(t)}\log\big(u(t,\varphi)\big)^{p(t)}\,\mathrm{d}\nu - (F(t))^{p(t)}\log(F(t))^{p(t)}\Big)$$
$$+ p^2(t)\int_H \big(u(t,\varphi)\big)^{p(t)-1}\frac{\partial}{\partial t}u(t,\varphi)\,\mathrm{d}\nu.$$

Then, taking (2.4) into account we obtain

$$p^2(t)(F(t))^{p(t)-1}F'(t)$$
$$= p'(t)\Big(\int_H \big(u(t,\varphi)\big)^{p(t)}\log\big(u(t,\varphi)\big)^{p(t)}\,\mathrm{d}\nu - (F(t))^{p(t)}\log(F(t))^{p(t)}\Big)$$
$$+ p^2(t)\int_H \big(u(t,\varphi)\big)^{p(t)-1}\big(f - G\big(Du(t,\varphi)\big)\big)\,\mathrm{d}\nu$$
$$+ p^2(t)\int_H \big(u(t,\varphi)\big)^{p(t)-1}Nu(t,\varphi)\,\mathrm{d}\nu.$$

Since (3.1) yields

$$p^2(t)(F(t))^{p(t)-1}F'(t) \leqslant -p'(t)(F(t))^{p(t)}\log\big(F(t)^{p(t)}\big)$$
$$+ p'(t)\Big(\alpha(p(t))\big\langle(\lambda(p(t)) - N)u(t,\varphi), \big(u(t,\varphi)\big)^{p(t)-1}\big\rangle$$
$$+ \|u(t,\varphi)\|_{p(t)}^{p(t)}\log\|u(t,\varphi)\|_{p(t)}^{p(t)}\Big)$$
$$+ p^2(t)\int_H \big(u(t,\varphi)\big)^{p(t)-1}(f + c)\,\mathrm{d}\nu$$
$$+ p^2(t)\int_H \big(u(t,\varphi)\big)^{p(t)-1}Nu(t,\varphi)\,\mathrm{d}\nu,$$

we find that

$$p^2(t)(F(t))^{p(t)-1}F'(t) \leqslant \big(p^2(t) - p'(t)\alpha(p(t))\big)\int_H \big(u(t,\varphi)\big)^{p(t)-1} N u(t,\varphi)\,\mathrm{d}\nu$$
$$+ p'(t)\alpha(p(t))\lambda(p(t))\|u(t,\varphi)\|_{p(t)}^{p(t)}$$
$$+ p^2(t)\int_H \big(u(t,\varphi)\big)^{p(t)-1}(f+c)\,\mathrm{d}\nu.$$

It follows from Hypothesis (3.1) that

$$p^2(t) - p'(t)\alpha(p(t)) \geqslant 0$$

and since $\langle \varphi^{p-1}, L\varphi\rangle \leqslant 0$ we obtain

$$p^2(t)(F(t))^{p(t)-1}F'(t) \leqslant p'(t)\alpha(p(t))\lambda(p(t))\|u(t,\varphi)\|_{p(t)}^{p(t)}$$
$$+ p^2(t)\int_H \big(u(t,\varphi)\big)^{p(t)-1}(f+c)\,\mathrm{d}\nu.$$

Taking into account that

$$\left|\int_H \big(u(t,\varphi)\big)^{p(t)-1}(f+c)\,\mathrm{d}\nu\right| \leqslant \|u(t,\varphi)\|_p^{(p-1)/p}\|f+c\|_p,$$

we find that

$$F'(t) \leqslant \frac{p'(t)\alpha(p(t))\lambda(p(t))}{p^2(t)}F(t) + \|f+c\|_p.$$

By (3.2) and (3.4)

$$\int_0^t \frac{p'(s)\alpha(p(s))\lambda(p(s))}{p^2(t)} \leqslant \frac{\lambda\alpha}{p},$$

and the Gronwall Inequality yields

$$F(t) \leqslant \mathrm{e}^{\lambda\alpha t/p}F(0) + \frac{p}{\lambda\alpha}(\mathrm{e}^{\lambda\alpha t/p} - 1)\|f+c\|_p,$$

which in turn implies (3.3). For arbitrary φ, G and f (3.3) follows from Lemma 2.4. The last part of the theorem is obtained by an obvious modification of the above argument. □

Example 3.3. Let $F = 0$ and assume that Hypothesis 1.1 is satisfied. Then $\nu = N(0, Q_\infty)$ is the unique invariant measure for (P_t). Since $\ker(Q_\infty) = \{0\}$ we find that ν is nondegenerate and clearly

$$\int_H |x|^p \nu(\mathrm{d}x) < \infty,$$

for all $p \geqslant 0$. By the result in [5] the Logarithmic Sobolev Inequality holds for (P_t) with $\lambda = 0$, and $\alpha = \alpha_0$, where α_0 is the smallest $c > 0$ such that

$$\int_0^\infty |S^*(t)x|^2 \, \mathrm{d}t \leqslant c|x|^2, \quad x \in H.$$

It follows that

$$\|u(t,\varphi)\|_{p(t)} \leqslant \|\varphi\|_p + t\|f + c\|_p,$$

where

$$p(t) = 1 + (p-1)\mathrm{e}^{4t/\alpha}.$$

Example 3.4. Let

$$\beta = \sup_{x \in H} |F(x)| < \infty,$$

and assume that Hypothesis 1.1 is satisfied. Similarly as in the previous example we will assume that $\nu = N(0, Q_\infty)$. By the result in [3] for any $\varepsilon \in (0,1)$ the Logarithmic Sobolev Inequality holds for (P_t) with

$$\alpha = \frac{\alpha_0}{1-\varepsilon} \quad \text{and} \quad \lambda = \frac{\beta^2}{2}\frac{1}{\varepsilon}.$$

Hence (3.1) holds with these constants. Let us note that invoking [4] or [3] we can show in this case that (3.1) is satisfied for any bounded Borel function $F\colon H \to H$.

Example 3.5. Assume that Hypotheses 1.1 and 1.3 hold and moreover, for a certain $\omega > 0$

$$\|S(t)\| \leqslant \mathrm{e}^{-\omega t},$$

and $F - k$ is m-dissipative for a certain $k \in (0, \omega)$. Then by the results in [7] there exists a unique invariant measure ν for equation (1.1) and if ν is nondegenerate then Hypothesis 1.2 is satisfied as well. Using similar arguments as in [6] one may show that Hypothesis 2.1 is satisfied and the Logarithmic Sobolev Inequality holds for the generator N of the semigroup (P_t) in $L^p(H,\nu)$ with $\lambda = 0$ and $\alpha = \omega - k$. Hence (3.1) holds.

References

[1] *D. Bakry*: L'hypercontractivité et son utilisation en théorie des semigroupes. Lectures on Probability Theory (Saint-Flour, 1992). Lecture Notes in Math. Vol. 1581. Springer, Berlin, 1994, pp. 1–114.

[2] *P. Cannarsa and G. Da Prato*: Some results on nonlinear optimal control problems and Hamilton-Jacobi equations in infinite dimensions. J. Funct. Anal. *90* (1990), 27–47.

[3] *A. Chojnowska-Michalik*: Transition semigroups for stochastic semilinear equations on Hilbert spaces. Dissertationes Math. (Rozprawy Mat.) *396* (2001), 1–59.

[4] *A. Chojnowska-Michalik and B. Goldys*: Existence, uniqueness and invariant measures for stochastic semilinear equations on Hilbert spaces. Probab. Theory Related Fields *102* (1995), 331–356.

[5] *A. Chojnowska-Michalik and B. Goldys*: Nonsymmetric Ornstein-Uhlenbeck generators. Infinite Dimensional Stochastic Analysis (Amsterdam, 1999). R. Neth. Acad. Arts Sci., Amsterdam, 2000, pp. 99–116.

[6] *G. Da Prato, A. Debussche and B. Goldys*: Invariant measures of non-symmetric dissipative stochastic systems. To appear in Probab. Theory Related Fields.

[7] *G. Da Prato and J. Zabczyk*: Stochastic Equations in Infinite Dimensions. Cambridge University Press, Cambridge, 1992.

[8] *G. Da Prato and J. Zabczyk*: Ergodicity for Infinite Dimensional Systems. Cambridge University Press, Cambridge, 1996.

[9] *B. Goldys and F. Gozzi*: Second order parabolic HJ Equations in Hilbert Spaces: L^2 Approach. Submitted.

[10] *B. Goldys and B. Maslowski*: Ergodic control of semilinear stochastic equations and the Hamilton-Jacobi equation. J. Math. Anal. Appl. *234* (1999), 592–631.

[11] *F. Gozzi*: Regularity of solutions of a second order Hamilton-Jacobi equation and application to a control problem. Comm. Partial Differential Equations *20* (1995), 775–826.

[12] *L. Gross*: Logarithmic Sobolev inequalities. Amer. J. Math. *97* (1975), 1061–1083.

[13] *R. Phelps*: Gaussian null sets and differentiability of Lipschitz maps on Banach Spaces. Pacific J. Math. *77* (1978), 523–531.

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