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ON A PROBLEM CONCERNING k-SUBDOMINATION NUMBERS OF GRAPHS

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Abstract. One of numerical invariants concerning domination in graphs is the k-subdomination number $\gamma_{kS}^{-11}(G)$ of a graph G. A conjecture concerning it was expressed by J. H. Hattingh, namely that for any connected graph G with n vertices and any k with $\frac{1}{2}n < k \leqslant n$ the inequality $\gamma_{kS}^{-11}(G) \leqslant 2k - n$ holds. This paper presents a simple counterexample which disproves this conjecture. This counterexample is the graph of the three-dimensional cube and $k = 5$.

Keywords: k-subdomination number of a graph, three-dimensional cube graph

MSC 2000: 05C69

In [2] the following conjecture from [1] is presented:

For any connected graph G of order n and any k with $\frac{1}{2}n < k \leqslant n$, $\gamma_{kS}^{-11}(G) \leqslant 2k - n$.

A problem is suggested to settle this conjecture. By a simple counterexample we shall show that this conjecture is false.

We start by defining basic concepts.

Let G be a graph with the vertex set $V(G)$, $|V(G)| = n$. Let $v \in V(G)$. The closed neighbourhood $N_G[v]$ of the vertex v in the graph G is the set consisting of the vertex v and of all vertices which are adjacent to v in G.

If f is a mapping of $V(G)$ into a certain set of numbers and $S \subseteq V(G)$, then we denote $f(S) = \sum\limits_{x \in S} f(x)$. The weight $w(f)$ of f is the number $w(f) = f(V(G)) = \sum\limits_{x \in V(G)} f(x)$.

Let k be an integer, $1 \leqslant k \leqslant n$. Let $f\colon V(G) \to \{-1, 1\}$. The function f is called a signed k-subdominating function (shortly a signed kSF) of G, if $f(N_G[v]) \geqslant 1$ for at least k vertices v of G. The minimum of $w(f)$ taken over all signed kSF's of G is the signed k-subdomination number $\gamma_{kS}^{-11}(G)$ of G.

Now we define some auxiliary notation. Let $f\colon V(G) \to \{-1,1\}$. Then $V_f^+ = \{x \in V(G) \mid f(x) = 1\}$, $V_f^- = \{x \in V(G) \mid f(x) = -1\}$, $W_f^+ = \{x \in V(G) \mid f(N_G[x]) \geqslant 1\}$. The subgraphs of G induced by the sets V_f^+, V_f^- will be denoted by G_f^+, G_f^-.

Now we are able to disprove the conjecture. A simple counterexample is the graph Q_3 of the three-dimensional cube and $k = 5$.

Theorem. *Let Q_3 be the graph of the three-dimensional cube, let $k = 5$. Then $\gamma_{kS}^{-11}(Q_3) = 4$.*

P r o o f. Suppose that $\gamma_{5S}^{-11}(Q_3) < 4$. Let f be a 5SF such that $w(f) = \gamma_{5S}^{-11}(Q_3)$. We have $|V_f^+| + |V_f^-| = 8$, $\gamma_{5S}^{-11}(Q_3) = |V_f^+| - |V_f^-|$ and thus $\gamma_{5S}^{-11}(Q_3)$ must be even and $\gamma_{5S}^{-11}(Q_3) \leqslant 2$. Then $|V_f^+| = \frac{1}{2}(\gamma_{5S}^{-11}(Q_3) + n) \leqslant 5$ and $|V_f^-| \geqslant 3$. We shall investigate the possibilities for the functions $f\colon V(G) \to \{-1,1\}$ with $|V_F^-| = 3$. (The functions with $|V_f^-| > 3$ are obtained from them by changing some values from 1 to -1.) These functions are of three types. The functions of the same type can be transferred into each other by automorphisms of Q_3. In the first type G_f^- is a path of length 2 (with two edges). In the second type G_f^- has two connected components, one isomorphic to K_2, the other to K_1. In the third type G_f^- consists of three isolated vertices. These types are illustrated in Figs. 1, 2, 3. The vertices

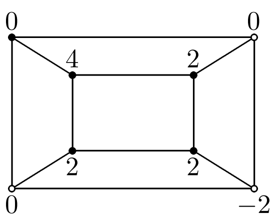

Fig. 1

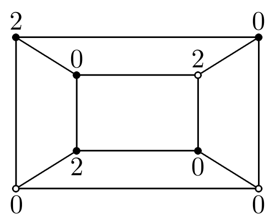

Fig. 2

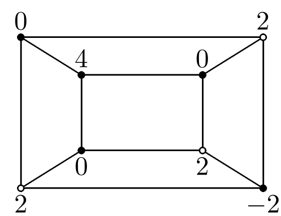

Fig. 3

of V_f^- are depicted as white circles and the vertices of V_f^+ by black circles. At each vertex v the value $f(N_{Q_3}[v])$ is written. We see that in all of the types $|W_f^+| \leqslant 4$ and thus no function $f\colon V(G) \to \{-1,1\}$ with $|V_f^-| = 3$ is a 5SF in Q_3. Evidently this holds also if $|V_f^-| \geqslant 3$. We have proved that $\gamma_{5S}^{-11}(Q_3) \geqslant 4$. Now take a function $f\colon V(G) \to \{-1,1\}$ such that $|V_f^-| = 6$ and G_f^+ contains a circuit of length 6 (Figs. 4, 5). Such a function is a 5SF and thus $\gamma_{5S}^{-11}(Q_3) = 4$. □

The assertion of Theorem disproves the conjecture, because in this case $2k - n = 2$.

But in [2], beside this conjecture there is another conjecture which is weaker and analogous; instead of any connected graph it is spoken in it about any tree. Our result does not exclude the validity of that conjecture.

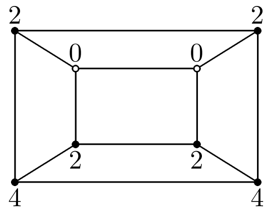

Fig. 4

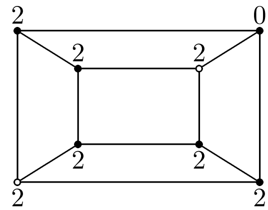

Fig. 5

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