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CORRECTION TO THE PAPER "EXISTENCE OF SOLUTIONS FOR THE DIRICHLET PROBLEM WITH SUPERLINEAR NONLINEARITIES"*

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5. Example

Consider the problem

$$x''(t) + W_x(t, x(t)) = 0 \quad \text{a.e. in } [0,T], \qquad (5.1)$$
$$x(0) = 0 = x(T)$$

where $W(t,\cdot)$, $t \in [0,T]$, is a convex, Frechet continuously differentiable function, $W(\cdot,x)$ is a measurable function for $x \in \mathbb{R}^n$, $W_x(\cdot,0)$ is continuous in $[0,T]$. Moreover W satisfies the following growth condition:

there exist $0 < \beta_1$, $0 < \beta_2$, $q_1 > 1$, $q > 2$, $k_1 \leqslant 0 \leqslant k_2$ such that for $t \in [0,T]$ and $x \in \mathbb{R}^n$

$$k_1 + \frac{\beta_1}{q_1}|x|^{q_1} \leqslant W(t,x) \leqslant \frac{\beta_2}{q}|x|^q + k_2.$$

In the notation of the paper we have $L(t,x') = \frac{1}{2}|x'|^2$ and $V(t,x) = W(t,x)$. It is easily seen that assumptions (H) and (H1) are satisfied. Therefore what we have to do is to construct a nonempty set X defined in Section 1. To this effect let us take any $k > 0$ and let $\overline{X}$ denote the same as in Section 1 with the new L and V. We assume the hypotheses

(H1′) $T^2\Big(\beta_2^{\frac{1}{q-1}}\Big(\dfrac{q}{q-1}\Big)(k + k_2 - k_1) + 1\Big)^{q-1} \leqslant k.$

(H2) $W_x(0,0) \neq 0$, or $W_x(T,0) \neq 0$.

We shall show that the set

$$\tilde{X} = \big\{v \in \overline{X} \colon\ 0 < \|v\|_{L^\infty} \leqslant k,\ v' \in A\big\}$$

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is the set X which we are looking for. That means: we must prove that for each function $x \in \tilde{X}$ the function

$$w\colon\ t \to \int_0^t \int_0^s W_x(\tau, x(\tau))d\tau + at = w_0(t) + at \tag{5.2}$$

belongs to $\tilde{X}$ for $a = -T^{-1}w_0(T)$. We easily observe that $w \in A_{0,0}$ and w' is absolutely continuous. Note, that in view of our assumption on W we get the estimation

$$\|W_x(\cdot, x(\cdot))\|_{L^\infty} \leqslant \left(\beta_2^{\frac{1}{q-1}}\left(\frac{q}{q-1}\right)\left(\|x(\cdot)\|_{L^\infty} + k_2 - k_1\right) + 1\right)^{q-1}.$$

Therefore

$$\|w_0\|_{L^\infty} \leqslant \frac{T^2}{2}\left(\beta_2^{\frac{1}{q-1}}\left(\frac{q}{q-1}\right)\left(\|x(\cdot)\|_{L^\infty} + k_2 - k_1\right) + 1\right)^{q-1}.$$

Hence, as $x \in \tilde{X}$, we have

$$\|w_0\|_{L^\infty} \leqslant \frac{T^2}{2}\left(\beta_2^{\frac{1}{q-1}}\left(\frac{q}{q-1}\right)(k + k_2 - k_1) + 1\right)^{q-1}$$

and, by (H1$'$), $\|w\|_{L^\infty} \leqslant \|w_0\|_{L^\infty} + |w_0(T)| \leqslant k$. Moreover, by (H2) w is not identically zero. Actually, if $w(t) \equiv 0$ for some $x \in X$ then $W_x(t, x(t)) = 0$ for all $t \in [0, T]$. Taking into account (H2), the latter equality is in contrary to boundary values of x ($x(0) = 0$ and $x(T) = 0$). Thus

$$0 < \|w\|_{L^\infty} \leqslant k. \tag{5.3}$$

It is obvious that if we take k_3 sufficiently large then

$$\int_0^T W(t, z(t))\,\mathrm{d}t \leqslant \frac{1}{4}\int_0^T \left|z'(t)\right|^2\,\mathrm{d}t + k_3$$

for all z satisfying (5.3). Therefore $w \in \tilde{X}$, and we can put $X = \tilde{X}$. It is also clear that the set $X = \tilde{X}$ is nonempty. Thus all assumptions of Theorem 4.2 are satisfied, so we come to the following theorem with $L = \frac{1}{2}|x'|^2$.

Theorem 5.1. *There exists a pair* $(\bar{x}, \bar{p} + d_{\bar{p}})$ *which is a solution to* (5.1) *such that* $\bar{x} \neq 0$ *and*

$$J(\bar{x}) = \min_{x\in X} J(x) = \min_{p\in X^d}\max_{d\in\mathbb{R}^n} J_D(p, d) = J_D(\bar{p}, d_{\bar{p}}).$$

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