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Czechoslovak Mathematical Journal, Vol. 56 (2006), No. 1, 99–108

Persistent URL: <http://dml.cz/dmlcz/128056>

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WEAK EDGE-DEGREE DOMINATION IN HYPERGRAPHS

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(Received May 22, 2003)

Abstract. In this paper we extend the notion of weak degree domination in graphs to hypergraphs and find relationships among the domination number, the weak edge-degree domination number, the independent domination number and the independence number of a given hypergraph.

Keywords: hypergraph, weak degree domination number, independent domination number, graph theory

MSC 2000: 05C

0. INTRODUCTION

For all terminology and notation in hypergraph theory we refer the reader to C. Berge [8]. All hypergraphs considered in this paper are simple, finite and loop-free.

Given a hypergraph $H = (X, E)$ and $D \subseteq X$, we call D a *dominating set* (or simply a *domset*) of H if for every vertex $y \in X - D$ there exist $x \in D$ and $e \in E$ such that $x, y \in e$ (cf., [5]) and a *weakly edge-degree dominating* (or briefly, WEDD-) *set* of H if for every vertex $y \in X - D$ there exists $x \in D$ such that (i) $x, y \in e$ for some $e \in E$ (i.e., D is a domset of H) and (ii) $|E_x| \leq |E_y|$, where E_a denotes the set of edges containing the vertex a . The *domination* (*weak edge-degree domination*) *number* $\gamma(H)$ (respectively, $\gamma_w(H)$) of H is then defined as the least cardinality of a domset (WEDD-set) of H . Further, let $\gamma_i(H)$ and $\beta_0(H)$ denote respectively the least and the largest cardinality of a maximal independent (or, strongly stable as in [8], p. 448) set of H . We have the well-known inequality

$$(1) \quad \gamma(H) \leq \gamma_i(H) \leq \beta_0(H).$$

What can one say about $\gamma_w(H)$ with regard to (1)? We have examples of hypergraphs H for which each of the possibilities (i) $\gamma_w(H) < \gamma_i(H)$, (ii) $\gamma_w(H) = \gamma_i(H)$ and (iii) $\gamma_w(H) > \gamma_i(H)$ may occur (see Fig. 1 for illustrative examples). In this paper we give a class of hypergraphs in which for every hypergraph H , $\gamma_w(H)$ lies between $\gamma(H)$ and $\gamma_i(H)$, as well as a class in which for every hypergraph H , $\gamma_w(H)$ lies between $\gamma_i(H)$ and $\beta_0(H)$. In fact, we have a *conjecture* that for any hypergraph H with *cyclomatic number* (see [3], [4], [6], [9], [10], [11], [17]) $\mu(H)$ equal to zero

$$(2) \quad \gamma_i(H) \leq \gamma_w(H) \leq \beta_0(H).$$

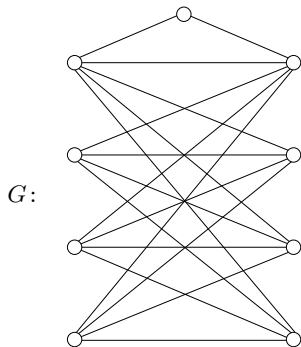
The notion of a WEDD-set in a graph was first introduced by E. Sampathkumar and L. Pushpa Latha [16] who conjectured that (2) must hold for any tree H and this was almost instantly proved to be true by J. H. Hattingh and R. C. Laskar [12] and the present authors [7] (see [13], [15]). In substantiation of our more general conjecture as mentioned above, we shall show in this paper that for any hypertree (i.e., a connected acyclic hypergraph) H the inequalities in (2) hold.

It is not hard to find hypergraphs with nonzero cyclomatic number which still satisfy (2) (e.g., see Fig. 2(a)), as also such hypergraphs that do not satisfy (2) (e.g., see Fig. 1(b), (c)), thus pointing at the interesting *open problem of characterizing hypergraphs H that satisfy (2)* as well as the importance of settling our above conjecture.

1. SOME GENERAL RESULTS

Given a hypergraph $H = (X, E)$, by the *edge-degree* $\text{ed}(x)$ of a vertex $x \in X(H)$ we mean the cardinality $|E_x|$ of the *edge-neighbourhood* E_x of x . Hence, by a *pendant vertex* we mean a vertex x in H for which $\text{ed}(x) = 1$; that is, x is contained in exactly one edge of H (see [1], [2]). Clearly, every vertex of an edge e of H is pendant if and only if e is a component of H . The set of pendant vertices in H is denoted by $\mathfrak{P}(H)$. We shall call an edge of H a *pendant edge* if it contains exactly one “nonpendant” vertex (i.e., a vertex x for which $\text{ed}(x) \geq 2$). A nonpendant vertex of H is called a *support* if it is contained in a pendant edge. By the *removal* of an edge e from H we shall mean the operation of deleting e from E and deletion of the pendant vertices contained in e from X so that the hypergraph obtained by *removing* e from H is given by $H - e = (X - e^0, E - \{e\})$ where e^0 denotes the set of pendant vertices in the edge e .

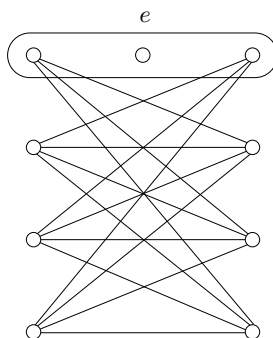
Next, for any $u \in X(H)$, let $N_w(u) = \{v \in X(H) : u, v \in e \text{ for some } e \in E(H) \text{ and } \text{ed}(u) \geq \text{ed}(v)\}$ denote the *weak edge-degree vertex neighbourhood* of u ; any particular element of $N_w(u)$ is called a *weak neighbour* of u . Clearly, $D \subseteq X(H)$ is



G :

A graph G with
 $\gamma(G) = \gamma_w(G) = 3 < 4 = \gamma_i(G) = \beta_0(G)$

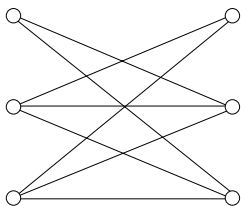
(a)



H :

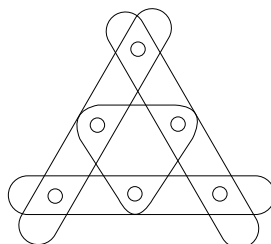
A hypergraph H with
 $\gamma_w(H) = 2 < 3 = \gamma_i(H)$

(b)



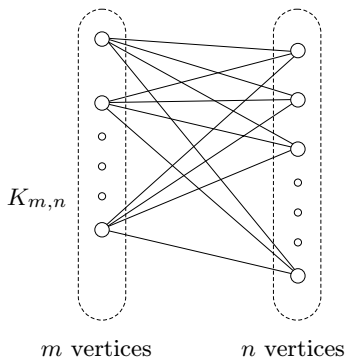
A graph G with
 $\gamma(G) = \gamma_w(G) = \gamma_i(G) = 2 < 3 = \beta_0(G)$

(c)



A hypergraph H with $\gamma_w(H) = \gamma_i(H) = 2$

(d)



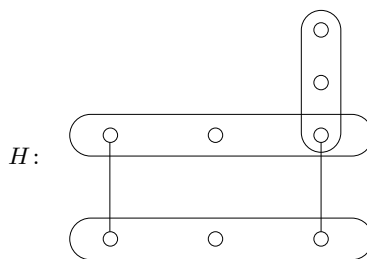
$K_{m,n}$

m vertices

n vertices

A graph $G = K_{m,n}$ with
 $\gamma_i(G) = m < n = \gamma_w(G)$ for $m < n$

(e)

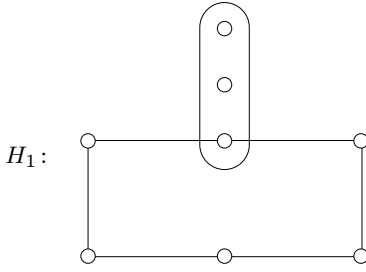


H :

A hypergraph H with
 $\gamma_i(H) = 2 < 3 = \gamma_w(H)$

(f)

Figure 1

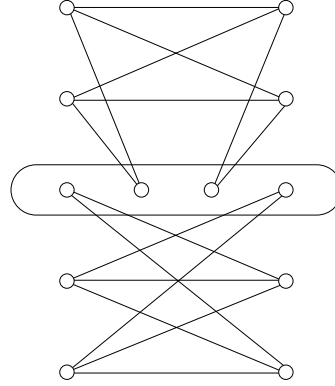


H_1 :

A hypergraph $H_1 = (X, \mathcal{E})$ with cyclomatic number

$$\mu(H_1) = \sum_{E \in \mathcal{E}} |E| - |X| - m(H_1) = 15 - 8 - 6 = 1 > 0 \text{ and } 2 = \gamma_i(H_1) < 3 = \gamma_w(H_1) < \beta_0(H_1) = 4$$

(a)



H_2 :

A non-edge-degree regular hypergraph H_2 with $\gamma_w(H_2) = 4 < 5 = \gamma_i(H_2)$

(b)

Figure 2

a WEDD-set of H if and only if

$$(3) \quad |D \cap N_w(u)| \geq 1 \quad \text{for every } u \in X - D.$$

The set of all WEDD-sets of H will be denoted by $\mathbf{D}_w(H)$. The following results can be easily established in the same way as their analogues in graph theory (see [7]).

Theorem 1. *Let $H = (X, E)$ be any hypergraph and $D \in \mathbf{D}_w(H)$. Then D is a minimal WEDD-set of H if and only if for each $d \in D$ one of the following conditions is satisfied:*

- (i) $D \cap N_w(d) = \emptyset$,
- (ii) $\exists x \in X - D$ such that $D \cap N_w(x) = \{d\}$.

Theorem 2. *In any hypergraph $H = (X, E)$, $N_w(u) = \emptyset \Leftrightarrow u \in \bigcap_{S \in \mathbf{D}_w(H)} S$; that is, u has no weak neighbour if and only if u lies in every WEDD-set of H .*

Theorem 3. *In any hypergraph $H = (X, E)$, every WEDD-set D contains a vertex of minimum edge-degree $\delta_e(H)$.*

Theorem 4. *In any hypergraph $H = (X, E)$, the set $W^0 = \{x \in X : N_w(x) = \emptyset\}$ is an independent subset of every WEDD-set of H .*

2. MAIN RESULTS

A connected acyclic hypergraph is called a *hypertree*. Clearly, any hypertree $H = (X, E)$ satisfies the “Linearity Property”, viz.,

$$(4) \quad |e_1 \cap e_2| \leq 1 \quad \text{for any two } e_1, e_2 \in E$$

(e.g., see C. Berge [9], p. 8). Next, let $(u_0, e_1, u_1, e_2, u_2, e_3, u_3, \dots, u_{k-1}, e_k, u_k)$ be a diametrical path in H . Then the following observations are straightforward:

- O1. e_1 is a pendant edge and every vertex of e_1 except u_1 is a pendant vertex of H .
- O2. Every edge which has a nonempty intersection with e_2 , except possibly e_3 , is a pendant edge. If $k = 3$, then e_3 also becomes a pendant edge.
- O3. $k = 2$ if and only if $a_1 \cap a_2 = \{u_1\}$ for all edges $a_1, a_2 \in E$.
- O4. $e_i \cap e_j = \emptyset$ for all distinct $i, j \in \{1, 2, \dots, k\}$ with $|i - j| > 1$.

The following general observations are also useful in our investigation.

Lemma 1. *If S is any WEDD-set of a hypergraph H and if e is any edge of H with $e^0 \neq \emptyset$ then $S \cap e^0 \neq \emptyset$; further, if S is a minimal WEDD-set of H then $|S \cap e^0| = 1$.*

A hypergraph is *edge-degree regular* if all its vertices have the same edge-degree and hence the following result is easy to see.

Theorem 5. *For any edge-degree regular hypergraph $H = (X, E)$, $\gamma_w(H) = \gamma(H) \leq \gamma_i(H)$.*

Fig. 2 (b) exhibits an edge-degree regular hypergraph H such that $\gamma_w(H) < \gamma_i(H)$ as well as a hypergraph which is not edge-degree regular but still satisfies the inequality of Theorem 5; the latter example illustrates that edge-degree regularity is a sufficient condition for H to satisfy the conclusion of Theorem 5 but it is not necessary. Thus, it is important to characterize hypergraphs H for which $\gamma_w(H) = \gamma_i(H)$.

We shall now proceed to establish the following first main result of the paper.

Theorem 6. *For any hypertree $H = (X, E)$, $\gamma_i(H) \leq \gamma_w(H)$.*

Proof. We shall prove the result by induction on the *size* (i.e., the number of edges) of hypertrees.

If H is any hypertree of size $q = 1$, then H consists of just one edge whence every vertex in it is pendant so that each vertex constitutes a minimal independent domset as well as a minimum WEDD-set of H ; thus, we have $\gamma_i(H) = 1 = \gamma_w(H)$ and the result follows.

Next, let H be any hypertree of size $q = 2$ with $E = \{e_1, e_2\}$. Then $e_1 \cap e_2 = \{u_1\}$ for some vertex u_1 whence, again, $\gamma_i(H) = 1 < 2 = \gamma_w(H)$ and the result follows.

If H is any hypertree of size $q = 3$, let $E = \{e_1, e_2, e_3\}$. Then either $e_1 \cap e_2 \cap e_3 = \{u_1\}$ for some vertex u_1 or there exist vertices u_1 and u_2 such that $e_1 \cap e_2 = \{u_1\}$, $e_2 \cap e_3 = \{u_2\}$ and $e_1 \cap e_3 = \emptyset$. In the first case, $\{u_1\}$ is a minimal independent domset whence, again, $\gamma_i(H) = 1$. Further, any set consisting of exactly one pendant vertex from each of the edges e_1 , e_2 and e_3 is a minimum WEDD-set of H so that $\gamma_w(H) = 3$, implying the result. Consider the latter case. If e_2 has a pendant vertex then $\gamma_i(H) = 2 < 3 = \gamma_w(H)$ and if e_2 has no pendant vertex then any set consisting of one pendant vertex v_1 chosen from e_1 and one pendant vertex v_3 chosen from e_3 turns out to be both a minimal independent domset and a minimum WEDD-set of H , whence we get $\gamma_i(H) = 2 = \gamma_w(H)$. Thus, the result is seen to hold in this case as well.

Hence, suppose the result holds for all hypertrees of size less than an arbitrarily given positive integer q . Let H be any hypertree of size $q > 3$. Let $(u_0, e_1, u_1, e_2, u_2, e_3, u_3, \dots, u_{k-1}, e_k, u_k)$ be a diametrical path in H . If $k \leq 3$ then the above arguments and observations O1–O4 yield the desired result. Hence, we let $k \geq 4$.

Case 1: Suppose H contains a WEDD-set S of cardinality $\gamma_w(H)$ (or, henceforth, a “ $\gamma_w(H)$ -set” for brevity), which contains a support vertex x of H . Then we let $H' = (X', E')$ denote the subhypergraph of H obtained after removing all the pendant edges in E_x . Since H' is a hypertree with $|E'| < |E|$, by the induction hypothesis we get

$$(5) \quad \gamma_i(H') \leq \gamma_w(H').$$

Let $P(x)$ denote the set of pendant edges in H with x as their support. Then

$$(6) \quad |(e - \{x\}) \cap S| = 1 \quad \text{for every } e \in P(x).$$

Hence, if $S' = \bigcup_{e \in P(x)} ((e - \{x\}) \cap S)$ then it is easy to see that the set $S - S'$ is a WEDD-set of H' , whence we get

$$(7) \quad \gamma_w(H') \leq \gamma_w(H) - |P(x)|.$$

Next, let T be a $\gamma_i(H')$ -set. If $x \in T$, then T is a maximal independent set of H . Also, if $x \notin T$ then $T \cup S'$ is a maximal independent set of H . Thus, in either case, we have

$$(8) \quad \gamma_i(H) \leq \gamma_i(H') + |P(x)|.$$

The relations (5), (7) and (8) yield the desired result in this case.

Case 2: Next, consider the case when H does not contain any $\gamma_w(H)$ -set that contains the support of H .

Subcase 1: e_2 contains a pendant vertex (i.e., $e_2^0 \neq \emptyset$).

Let S be any $\gamma_w(H)$ -set and let H' be the subhypergraph obtained after removing all edges with their supports in e_2 , one at a time successively. Let the set of edges so removed be denoted by $P(e_2)$. Clearly, for each $e \in P(e_2)$, Lemma 1 implies that $|S \cap e^0| = 1$. Hence we observe that $S - \bigcup_{e \in P(e_2)} (S \cap e)$ is a WEDD-set of H' , irrespective of whether or not u_2 belongs to S . Thus, it follows that

$$(9) \quad \gamma_w(H') \leq \gamma_w(H) - |P(e_2)|.$$

Next, let T be a $\gamma_i(H')$ -set. Since $T \cap e_2 \neq \emptyset$ and $|T \cap e_2| = 1$, let $\{w_0\} = T \cap e_2$. If $w_0 \in e_2^0$ in H then $T \cup T''$ where T'' is a set of pendant vertices, one chosen arbitrarily from each member of $P(e_2)$, is a maximal independent set of H and hence

$$(10) \quad \gamma_i(H) \leq \gamma_i(H') + |P(e_2)|.$$

If $w_0 \in e_2 - e_2^0$ then $T \cup T'$ where T' is a set of pendant vertices, one chosen arbitrarily from each member of $P(e_2 - \{w_0\})$, is a maximal independent set of H and hence

$$(11) \quad \gamma_i(H) \leq \gamma_i(H') + |P(e_2 - \{w_0\})|.$$

Thus, in every case we have

$$(12) \quad \gamma_i(H) \leq \gamma_i(H') + |P(e_2)|.$$

Since H' is a hypertree of size less than q the induction hypothesis implies that $\gamma_i(H') \leq \gamma_w(H')$, whence the inequality (12) yields the desired result in this subcase.

Subcase 2: e_2 does not contain a pendant vertex.

If u_2 is a support, then let H' be the subhypergraph obtained by removing e_2 and all pendant edges with their supports in $e_2 - \{u_2\}$, one by one in succession. By the assumption of the case, $u_2 \notin S$. Also, for every $e \in P(e_2 - \{u_2\})$, $|S \cap e| = 1$. Then $S - \bigcup_{e \in P(e_2 - \{u_2\})} (S \cap e)$ is a WEDD-set of H' and so

$$(13) \quad \gamma_w(H') \leq \gamma_w(H) - |P(e_2 - \{u_2\})|.$$

Also, if T is a $\gamma_i(H')$ -set then $T \cup T'$ where T' is a set of pendant vertices, one chosen arbitrarily from each member of $P(e_2 - \{u_2\})$, is a maximal independent set of H and hence $\gamma_i(H) \leq \gamma_i(H') + |P(e_2 - \{u_2\})| \leq \gamma_w(H') + |P(e_2 - \{u_2\})| \leq \gamma_w(H)$, as desired.

Next, suppose u_2 is not a support. Let H' be the subhypergraph obtained after removing all members of $P(e_2)$ along with e_2 . Then, whether $u_2 \in S$ or $u_2 \notin S$, it is not hard to see that $\gamma_i(H) \leq \gamma_i(H') + |P(e_2)| \leq \gamma_w(H') + |P(e_2)| \leq \gamma_w(H)$, as desired. This completes the proof. \square

The second main result is the following one.

Theorem 7. *For any hypertree $H = (X, E)$, $\gamma_w(H) \leq \beta_0(H)$.*

Proof. We shall prove the result by induction on the size of hypertrees. If H is any hypertree of size $q = 1$, then H consists of just one edge, whence $\gamma_w(H) = 1 = \beta_0(H)$ and the result follows. Next, let H be any hypertree of size $q = 2$ with $E = \{e_1, e_2\}$. Then, since H is connected, there must exist a vertex u_1 such that $e_1 \cap e_2 = \{u_1\}$. Then any set consisting of exactly one pendant vertex from each of the two edges forms a maximum independent set which is also a minimum WEDD-set of H , so that $\gamma_w(H) = 2 = \beta_0(H)$ implying the result. Further, let H be any hypertree of size $q = 3$ and let $E = \{e_1, e_2, e_3\}$. Then either $e_1 \cap e_2 \cap e_3 = \{u_1\}$ for some vertex u_1 or there exist vertices u_1 and u_2 such that $e_1 \cap e_2 = \{u_1\}$, $e_2 \cap e_3 = \{u_2\}$ and $e_1 \cap e_3 = \emptyset$. In the former case, $\text{diam}(H)$ (i.e., the largest length of a shortest path between any two vertices in H) = 2 and in the latter case $\text{diam}(H) = 3$. It is easily verified that in the former case $\gamma_w(H) = 3 = \beta_0(H)$ and in the latter case $\gamma_w(H) = 3 = \beta_0(H)$ or $\gamma_w(H) = 2 = \beta_0(H)$ according to whether e_2 is a pendant edge of H or not. Thus, the result is seen to hold in this case as well.

Hence, suppose the result holds for all hypertrees of size less than an arbitrarily given size $q \geq 4$. Let $(u_0, e_1, u_1, e_2, u_2, e_3, u_3, \dots, u_{k-1}, e_k, u_k)$ be a diametrical path in H (i.e., $k = \text{diam}(H)$). Trivially, $k > 1$ since $q \geq 4$. If $k = 2$ then $\gamma_w(H) = |P(u_1)| = |E| = \beta_0(H)$. If $k = 3$, then $\gamma_w(H) = |P(e_2)| = \beta_0(H)$ when e_2 has no pendant vertices and $\gamma_w(H) = |P(e_2)| + 1 = \beta_0(H)$ when e_2 has a pendant vertex, again implying the desired result in either case.

Hence, let H be any hypertree of size $q \geq 4$ and $k \geq 4$.

Case 1: e_2 does not contain a pendant vertex.

Let H' be the subhypergraph of H obtained after removing e_2 and all pendant edges having their supports in $e_2 - \{u_2\}$ one by one in succession. Let S be any $\gamma_w(H')$ -set.

Subcase 1 (a): $u_2 \notin S$

Let S' be formed by choosing one vertex each arbitrarily from each member of $P(e_2 - \{u_2\})$. Then $S \cup S'$ is a WEDD-set of H , whence we see that $\gamma_w(H) \leq \gamma_w(H') + |S'|$. Further, let T be a $\beta_0(H')$ -set. Then $T \cup S'$ is a maximal independent set of H and, therefore, $\beta_0(H) \geq \beta_0(H') - |S'|$. Also, by the induction hypothesis, we have $\gamma_w(H') \leq \beta_0(H')$. The above inequalities yield $\gamma_w(H) \leq \beta_0(H)$.

Subcase 1 (b): $u_2 \in S$

Since S is a minimal WEDD-set of H , there exists a subset A of $X' - S$ such that $S \cap N_w(u) = \{u_2\}$ for every $u \in A$. Further, since in H' not every vertex u in any edge other than e_3 can satisfy $S \cap N_w(u) = \{u_2\}$, it follows that $A \subseteq e_3$. Now, $d_{H'}(u) \geq d_{H'}(u_2)$ for every $u \in A$. However, $d_{H'}(u) = d_H(u)$ for every $u \in A$ because $A \subseteq e_3$. Let $A_1 \subseteq A$ be such that $d_{H'}(u) = d_{H'}(u_2)$ for every $u \in A_1$ and $d_{H'}(v) > d_{H'}(u_2)$ for every $v \in A - A_1$. Since $d_{H'}(u_2) + 1 = d_H(u_2)$ we see that $d_H(u_2) \leq d_H(v)$ for every $v \in A - A_1$. Also, for every $u \in A_1$, $d_{H'}(u) = d_H(u) = d_H(u_2) - 1$, which yields $d_H(u) < d_H(u_2)$ for every $u \in A_1$. Now, choose a vertex $w_0 \in A_1$. Then for every $u \in A_1 - \{w_0\}$, $d_H(u) = d_H(w_0)$ and for every $v \in A - A_1$ we have $d_H(v) \geq d_H(\{u_2\}) > d_H(w_0)$. Thus, $d_H(w_0) < d_H(v)$ for every $v \in (A - \{w_0\}) \cup \{u_2\}$ and so $(S - \{u_2\}) \cup \{w_0\} \cup S'$ is a WEDD-set of H and hence $\gamma_w(H) \leq \gamma_w(H') + |P(e_2 - \{u_2\})|$. The other part, viz., $\beta_0(H') + |P(e_2 - \{u_2\})| \leq \beta_0(H)$, follows as in the Subcase 1 (a), whence we get $\gamma_w(H) \leq \beta_0(H)$ as desired.

Case 2: e_2 has a pendant vertex.

Let H'' be the subhypergraph of H obtained after removing all pendant edges having their supports in $e_2 - \{u_2\}$ one by one in succession. In H'' , e_2 is a pendant edge. Let S be any $\gamma_w(H'')$ -set. Then $S \cap (e_2 - \{u_2\}) = \emptyset$ and, in fact, this set consists of a single vertex, say x_0 . Without loss of generality, we may assume that x_0 is a pendant vertex also in H then $x_0 \in e_2^0$, since otherwise $S_y = (S - \{x_0\}) \cup \{y\}$ for some $y \in e_2^0$ would also be a $\gamma_w(H')$ -set. Let S' be a set of pendant vertices, one chosen from each member of $P(e_2 - \{u_2\})$. Then $S \cup S'$ is a WEDD-set of H , whence we get $\gamma_w(H) \leq \gamma_w(H') + |P(e_2 - \{u_2\})|$. Now, let T be a $\beta_0(H')$ -set. Then $|T \cap e_2| = 1$. Let $T \cap e = \{w\}$. If $w = u_2$, then T along with S' is a maximal independent set of H . On the other hand, if $w \neq u_2$, then $w \in e - \{u_2\}$ and w is a pendant vertex in H'' . Without loss of generality, we may assume that w is a pendant vertex also in H ; that is, $w \in e_2^0$. Then $T \cup S'$ is a maximal independent set of H , whence we see that $\beta_0(H) \geq \beta_0(H'') + |P(e_2 - \{u_2\})|$. Now, by the induction hypothesis, recall that $\gamma_w(H'') \leq \beta_0(H'')$; this inequality, together with the foregoing ones, yields the result that $\gamma_w(H) \leq \beta_0(H)$, as desired. \square

3. CONCLUDING REMARKS

Thus, we have shown the following result:

Theorem 8. *For any hypertree H , $\gamma_i(H) \leq \gamma_w(H) \leq \beta_0(H)$.*

Fig. 2(a) exhibits a hypergraph which is not a hypertree but still satisfies the inequalities of Theorem 8.

It is strongly believed that the conclusion of Theorem 8 must also hold for any hypergraph whose cyclomatic number is zero. Nevertheless, as illustrated already in Fig. 2 (a), there do exist hypergraphs with nonzero cyclomatic number that satisfy relation (2). This raises a natural problem of characterizing in general the hypergraphs that satisfy (2). However, a solution of this problem appears to be complex and hence may force a step-by-step approach to solve it finally. A natural next step would be to attempt settling the problem for hypergraphs without significant cycles (cf. [14]), which is being tried presently.

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