Czechoslovak Mathematical Journal

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Czechoslovak Mathematical Journal, Vol. 43 (1993), No. 4, 713–722

Persistent URL: http://dml.cz/dmlcz/128429

Czechoslovak Mathematical Journal, 43 (118) 1993, Praha

DECOMPOSITION OF THE WEIGHTED SOBOLEV SPACE $W^{1,p}(\Omega, d_M^\varepsilon)$ AND ITS TRACES

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(Received March 8, 1992)

1. INTRODUCTION

This paper continues [1] and we shall keep the corresponding notation. Let $N > 0$, $k \geqslant 0$ be integers, let ε, p be real numbers, $1 < p < \infty$. Denote by p' the conjugate Lebesgue exponent, i.e. $p' = \frac{p}{p-1}$. Let Ω be a non-empty, open, bounded subset of $\mathbf{R}^N$. Let M be a closed subset of $\partial\Omega$ and let $d_M(x)$ be the distance function, $d_M(x) = \operatorname{dist}(x, M)$. Given an integer m, $1 \leqslant m \leqslant N$, the symbol Q_m stands for the cube $(0,1)^m$.

Definition 1.1. We shall write $(\Omega, M) \in B(k, N)$ for $1 \leqslant k \leqslant N-1$, $N \geqslant 2$ if and only if there exists a bilipschitz mapping

$$B\colon Q_N \to \Omega$$

such that $B(\overline{Q}_k) = M$.

By $C^\infty(\overline{\Omega})$ we denote the set of real functions u defined on $\overline{\Omega}$ such that the derivatives $D^\alpha u$ can be continuously extended to $\overline{\Omega}$ for all multiindices α. Set $C_M^\infty(\overline{\Omega}) = \{u \in C^\infty(\overline{\Omega})\colon \operatorname{supp} u \cap M = \emptyset\}$. Define the weighted Sobolev space $W^{1,p}(\Omega, d_M^\varepsilon)$ as the closure of $C^\infty(\overline{\Omega})$ with respect to the norm

$$\|u|W^{1,p}(\Omega, d_M^\varepsilon)\| = \left(\int_\Omega |u(x)|^p d_M^\varepsilon(x)\,\mathrm{d}x + \int_\Omega \sum_{i=1}^N |D_i u(x)|^p d_M^\varepsilon(x)\,\mathrm{d}x\right)^{1/p}$$

where $D_i u = \frac{\partial u}{\partial x_i}$ stands for the generalized derivative of the function u, $W_M^{1,p}(\Omega, d_M^\varepsilon)$ as the closure of $C_M^\infty(\overline{\Omega})$ in the space $W^{1,p}(\Omega, d_M^\varepsilon)$ and $H^{1,p}(\Omega, d_M^\varepsilon)$ as the class of all functions u with a finite norm

$$\|u|H^{1,p}(\Omega, d_M^\varepsilon)\| = \Big(\int_\Omega |u(x)|^p d_M^{\varepsilon-p}(x)\,\mathrm{d}x + \int_\Omega \sum_{i=1}^N |D_i u(x)|^p d_M^\varepsilon(x)\,\mathrm{d}x\Big)^{1/p}.$$

Now, let $(\Omega, M) \in B(k, N)$. Define $X_{\varepsilon,M}^p(\partial\Omega)$ as the class of all real functions u on $\partial\Omega$ vanishing on M with a finite norm

$$\|u|X_{\varepsilon,M}^p(\partial\Omega)\| = \Big(\int_{\partial\Omega - M} |u(x)|^p d_M^{\varepsilon-p+1}(x)\,\mathrm{d}x + \iint_{(\partial\Omega-M)^2} \frac{|u(x)d_M^{\varepsilon/p}(x) - u(y)d_M^{\varepsilon/p}(y)|^p}{|x-y|^{N+p-2}}\,\mathrm{d}x\,\mathrm{d}y\Big)^{1/p}.$$

For $0 < s < 1$ we recall the definition of the Slobodeckij space $W^{s,p}(M)$ as the set of all functions u defined on M with a finite norm

$$\|u|W^{s,p}(M)\| = \Big(\int_M |u(x)|^p\,\mathrm{d}x + \int_M\int_M \frac{|u(x)-u(y)|^p}{|x-y|^{k+sp}}\,\mathrm{d}x\,\mathrm{d}y\Big)^{1/p}.$$

Maz'ja and Plamenevskij [5] proved the following decomposition lemma:

Lemma 1.1. *Let Ω have a Lipschitz boundary, i.e. $\Omega \in C^{0,1}$ in the sense of Definition 5.5.6 in* [6]. *Let $x_0 \in \partial\Omega$, $M = \{x_0\}$ and $-N < \varepsilon < p - N$. Then*

$$W^{1,p}(\Omega, d_M^\varepsilon) = H^{1,p}(\Omega, d_M^\varepsilon) \oplus \mathbf{R}^1$$

and the norms in the spaces $W^{1,p}(\Omega, d_M^\varepsilon)$ and $H^{1,p}(\Omega, d_M^\varepsilon) \oplus \mathbf{R}^1$ are equivalent.

The paper extends this result to the case $(\Omega, M) \in B(k, N)$.

2. Decomposition of $W^{1,p}(\Omega, d_M^\varepsilon)$

Let us recall four assertions we shall need in this paper.

Theorem 2.1 (see [2]). *Let Ω have a Lipschitz boundary and let M be a non-empty closed subset of $\partial\Omega$. Then $C_M^\infty(\overline{\Omega})$ is dense in $H^{1,p}(\Omega, d_M^\varepsilon)$.*

Theorem 2.2 (see [3]). *Let Ω have a Lipschitz boundary and let M be a non-empty closed subset of $\partial\Omega$. Then*

(i) *there exists a unique bounded linear operator*

$$T\colon H^{1,p}(\Omega, d_M^\varepsilon) \to X_{\varepsilon,M}^p(\partial\Omega)$$

such that

$$Tu = u\big|_{\partial\Omega\setminus M}$$

for all functions $u \in C_M^\infty(\overline{\Omega})$,

(ii) *there exists a bounded linear operator*

$$R\colon X^p_{\varepsilon,M}(\partial\Omega) \to H^{1,p}(\Omega, d_M^\varepsilon)$$

such that

$$TRu = u$$

for all functions $u \in X^p_{\varepsilon,M}(\partial\Omega)$.

Theorem 2.3 (see [1]). *Let* $N \geqslant 2$, $1 \leqslant k \leqslant N-1$, $k - N < \varepsilon < p + k - N$ *and let* $(\Omega, M) \in B(k, N)$. *Then*

(i) *there exists a unique bounded linear operator*

$$T\colon W^{1,p}(\Omega, d_M^\varepsilon) \to W^{1-\frac{N-k+\varepsilon}{p},p}(M)$$

such that

$$Tu = u\Big|_M$$

for all $u \in C^\infty(\overline{\Omega})$,

(ii) *there exists a bounded linear operator*

$$R\colon W^{1-\frac{N-k+\varepsilon}{p},p}(M) \to W^{1,p}(\Omega, d_M^\varepsilon)$$

such that

$$TRu = u$$

for all functions $u \in W^{1-\frac{N-k+\varepsilon}{p},p}(M)$.

Theorem 2.4 (see [4]). *Let* $N \geqslant 2$, $0 \leqslant k \leqslant N-1$, $\varepsilon \leqslant k - N$ *or* $\varepsilon > p + k - N$ *and* $(\Omega, M) \in B(k, N)$. *Then*

$$H^{1,p}(\Omega, d_M^\varepsilon) = W^{1,p}(\Omega, d_M^\varepsilon)$$

and the norms in the two spaces are equivalent.

According to Lemma 1.1 we can restrict ourselves to the case $N \geqslant 2$ and $1 \leqslant k \leqslant N-1$.

Lemma 2.5. *Let $N \geqslant 2$, $1 \leqslant k \leqslant N-1$ and $\varepsilon < p+k-N$. Let $(\Omega, M) \in B(k,N)$. Then the bounded imbedding*

$$W_M^{1,p}(\Omega, d_M^\varepsilon) \hookrightarrow L^p(\Omega, d_M^{\varepsilon-p})$$

holds.

Proof. Without loss of generality we can assume $\Omega = Q_N$ and $M = Q_k$. Let $u \in C_M^\infty(\overline{Q}_N)$. We shall write $x = (x', x'')$, where $x' = (x_1, \ldots, x_k)$, $x'' = (x_{k+1}, \ldots, x_N)$. Obviously, $d(x) = |x''|$ on Q_N. Hence, using the general cylindrical coordinates (x', r, φ) (see the proof of Lemma 2.10 in [1]) we have

$$\int_{Q_N} |u(x)|^p d_M^{\varepsilon-p}(x)\,\mathrm{d}x$$

$$= \int_M \int_{(0,\frac{\pi}{2})^{N-k-1}} \left[\int_0^{a(\varphi)} |u(x', r, \varphi)|^p r^{\varepsilon-p+N-k-1} \mathrm{d}r\right] J(\varphi)\,\mathrm{d}\varphi\,\mathrm{d}x' = I,$$

where $a(\varphi)$ is the function corresponding to the set $\{(x', x'') \colon x' \in M,\ 0 \leqslant x_j \leqslant 1$ for $j = k+1, \ldots, N\}$ and $J(\varphi) r^{-N+k+1}$ is the Jacobian. Note that $J(\varphi) \geqslant 0$. Obviously, from the Hardy inequality (note that $u = 0$ on M) we obtain

$$I \leqslant c \int_M \int_{(0,\frac{\pi}{2})^{N-k-1}} \left[\int_0^{a(\varphi)} \left|\frac{\partial u}{\partial r}(x', r, \varphi)\right|^p r^{\varepsilon+N-k-1}\,\mathrm{d}r\right] J(\varphi)\,\mathrm{d}\varphi\,\mathrm{d}x'$$
$$\leqslant c_1 \|u | W^{1,p}(Q_N, d_M^\varepsilon)\|^p.$$

This completes the proof. □

Lemma 2.6. *Let $N \geqslant 2$, $1 \leqslant k \leqslant N-1$, $\varepsilon < p+k-N$ and let $(\Omega, M) \in B(k,N)$. Then*

$$W_M^{1,p}(\Omega, d_M^\varepsilon) = H^{1,p}(\Omega, d_M^\varepsilon).$$

Moreover, the norms in the two spaces are equivalent.

Proof. Again, we can assume $\Omega = Q_N$, $M = Q_k$. The imbedding

$$W_M^{1,p}(\Omega, d_M^\varepsilon) \hookrightarrow H^{1,p}(\Omega, d_M^\varepsilon)$$

follows from Lemma 2.5. Due to the imbedding $H^{1,p}(Q_N, d_M^\varepsilon) \hookrightarrow W^{1,p}(Q_N, d_M^\varepsilon)$ it suffices to prove that any function $u \in H^{1,p}(Q_N, d_M^\varepsilon)$ can be approximated in the space $W^{1,p}(\Omega, d_M^\varepsilon)$ by functions from the set $C_M^\infty(\overline{\Omega})$. This will prove the inverse imbedding. Let $\{\Phi_h : h > 0\}$ be a family of real functions defined on $[0, \infty)$ and satisfying the following conditions:

$$\Phi_h(t) = 0 \quad \text{for} \quad t \in [0, h), \tag{2.1}$$

$$\Phi_h(t) = 1 \quad \text{for} \quad t \in (2h, \infty), \tag{2.2}$$

$$\Phi_h \in C^\infty(0, \infty), \quad 0 \leqslant \Phi_h \leqslant 1, \tag{2.3}$$

$$|\Phi_h'(t)| \leqslant \frac{c}{h}, \quad h > 0, \quad t > 0, \tag{2.4}$$

where c is a positive constant independent of h and t. Let $u \in H^{1,p}(\Omega, d_M^\varepsilon)$. For every $h > 0$ define a function u_h by

$$u_h(x', x'') = u(x', x'')\Phi_h(|x''|).$$

Then $u_h \in W^{1,p}(Q_N, d_M^\varepsilon)$ for every $h > 0$. Put

$$J_h = \|u_h - u | W^{1,p}(Q_N, d_M^\varepsilon)\|^p.$$

The properties of $\Phi_h(t)$ yield

$$\begin{aligned} J_h \leqslant c\Big(& \int_{Q_N} \Big| u(x', x'')\big(1 - \Phi_h(|x''|)\big)\Big|^p |x''|^\varepsilon \, dx'' \, dx' \\ & + \int_{Q_N} \Big|\sum_{i=1}^N D_i u(x', x'')\big(1 - \Phi_h(|x''|)\big)\Big|^p |x''|^\varepsilon dx'' \, dx' \\ & + \int_{Q_N} |u(x', x'')|^p \sum_{i=k+1}^N |\Phi_h'(|x''|)|^p |x''|^\varepsilon \, dx'' \, dx' \Big) \\ = {} & c(J_{1h} + J_{2h} + J_{3h}). \end{aligned} \tag{2.5}$$

Set $Q(2h) = \{(x', x'') : x' \in M, |x''| < 2h\}$ and $Q(h, 2h) = \{(x', x'') : x \in M, h < |x''| < 2h\}$. Using (2.1)–(2.4) we obtain the estimates

$$J_{1h} \leqslant \int_{Q_{2h}} |u(x', x'')|^p |x''|^\varepsilon \, dx'' \, dx',$$

$$J_{2h} \leqslant \int_{Q_{2h}} \Big|\sum_{i=1}^N D_i u(x', x'')\Big|^p |x''|^\varepsilon \, dx'' \, dx',$$

$$J_{3h} \leqslant Nc \int\limits_{Q(h,2h)} |u(x',x'')|^p |x''|^{\varepsilon - p}.$$

Since $H^{1,p}(Q_N, d_M^\varepsilon) \hookrightarrow W^{1,p}(Q_N, d_M^\varepsilon)$ and $u \in H^{1,p}(Q_N, d_M^\varepsilon)$, the absolute continuity of the Lebesgue integral yields

(2.6) $$\lim_{h \to 0} J_{ih} = 0.$$

Now, (2.5) and (2.6) imply

$$\lim_{h \to 0} J_h = 0 \quad \text{and} \quad u \in W_M^{1,p}(Q_N, d_M^\varepsilon),$$

which completes the proof. □

As a consequence of Lemma 2.6 we have

Theorem 2.7. *Let $N \geqslant 2$, $1 \leqslant k \leqslant N-1$, $\varepsilon < p+k-N$. Let $(\Omega, M) \in B(k,N)$. Then $H^{1,p}(\Omega, d_M^\varepsilon)$ is a closed subspace of $W^{1,p}(\Omega, d_M^\varepsilon)$.*

Note that $H^{1,p}(\Omega, d_M^\varepsilon) \neq W^{1,p}(\Omega, d_M^\varepsilon)$ for $k - N < \varepsilon < p + k - N$. We can take $u(x) \equiv 1$ on Ω to prove it.

Definition 2.1. Let $N \geqslant 2$, $1 \leqslant k \leqslant N-1$, $k - N < \varepsilon < p + k - N$. Let $(\Omega, M) \in B(k,N)$. Let

$$R\colon W^{1-\frac{N-k+\varepsilon}{p},p}(M) \to W^{1,p}(\Omega, d_M^\varepsilon)$$

be the linear bounded extension operator from Theorem 3.4 in [1]. We denote the range of the operator R by $D^p_{\varepsilon,M}(\Omega)$. On $D^p_{\varepsilon,M}(\Omega)$ we define the norm by

$$\|u|D^p_{\varepsilon,M}(\Omega)\| = \left\|Tu|W^{1-\frac{N-k+\varepsilon}{p},p}(M)\right\|,$$

where T is the trace operator from Theorem 2.11 in [1].

The space $D^p_{\varepsilon,M}(\Omega)$ is isometrically isomorphic to the space $W^{1-\frac{N-k+\varepsilon}{p},p}(M)$.

Lemma 2.8. *Let $N \geqslant 2$, $1 \leqslant k \leqslant N-1$, $k - N < \varepsilon < p+k-N$. Then the linear operator A defined by*

(2.7) $$Au = u - RTu$$

is a bounded linear mapping of $W^{1,p}(Q_N, d_M^\varepsilon)$ to $H^{1,p}(Q_N, d_M^\varepsilon)$.

Proof. Obviously, it suffices to prove only that

$$A\colon W^{1,p}(Q_N, d_M^\varepsilon) \to L^p(Q_N, d_M^{\varepsilon-p})$$

is bounded. Let $u \in C^\infty(\overline{Q}_N)$. Let S be the bounded linear operator from Lemma 3.2 in [1]. We have

$$\begin{aligned}
(2.8)\qquad & \|Au|L^p(Q_N, d_M^{\varepsilon-p})\|^p \\
&= \int\limits_{Q_N} |u(x',x'') - (RSTu)(x',x'')|^p |x''|^{\varepsilon-p} \,\mathrm{d}x'' \,\mathrm{d}x' \\
&= \int\limits_{Q_N} \Bigl| u(x',x'') - \frac{1}{|x''|^k} \int\limits_{|x'-y'|\leqslant |x''|} \Phi\Bigl(\frac{x'-y'}{|x''|}\Bigr) Su(y',0)\,\mathrm{d}y' \Bigr|^p |x''|^{\varepsilon-p} \,\mathrm{d}x'' \,\mathrm{d}x' \\
&\leqslant 2^{p-1} \Bigl[\int\limits_M \int\limits_{(0,1)^{N-k}} |u(x',x'') - u(x',0)|^p |x''|^{\varepsilon-p} \,\mathrm{d}x'' \,\mathrm{d}x' \\
&\quad + \int\limits_M \int\limits_{(0,1)^{N-k}} \Bigl| \int\limits_{|s'|<1} \Phi(s')\bigl(u(x',0) - Su(x'-s'|x''|,0)\bigr) \,\mathrm{d}s' \Bigr|^p |x''|^{\varepsilon-p} dx'' \,\mathrm{d}x' \Bigr] \\
&= 2^{p-1}(J_1 + J_2).
\end{aligned}$$

As in the proof of Lemma 2.5, we obtain

$$\begin{aligned}
(2.9)\quad J_1 &= \int\limits_M \int\limits_{(0,\frac{\pi}{2})^{N-k-1}} \Bigl[\int\limits_0^{a(\varphi)} |u(x',r,\varphi) - u(x',0,\varphi)|^p r^{\varepsilon-p+N-k-1} \mathrm{d}r \Bigr] J(\varphi) \,\mathrm{d}\varphi \,\mathrm{d}x' \\
&\leqslant c_1 \int\limits_M \int\limits_{(0,\frac{\pi}{2})^{N-k-1}} \Bigl[\int\limits_0^{a(\varphi)} \Bigl| \frac{\partial u}{\partial r}(x',r,\varphi) \Bigr|^p r^{\varepsilon+N-k-1} \mathrm{d}r \Bigr] J(\varphi) \,\mathrm{d}\varphi \,\mathrm{d}x' \\
&\leqslant c_2 \|u|W^{1,p}(Q_N, d_M^\varepsilon)\|^p.
\end{aligned}$$

Obviously, using the general cylindrical coordinates we have

$$J_2 \leqslant c_3 \int\limits_{(-K,K)^k} \int\limits_{0<r<b(x')} \int\limits_{|s'|<1} \frac{|Su(x',0) - Su(x'-s'r,0)|^p}{r^p} r^{N-k-1+\varepsilon} \mathrm{d}s' \,\mathrm{d}r \,\mathrm{d}x',$$

where $b(x') = K - \max\limits_{i=1,2,\dots,k} |x_i|$ and K is the real number from the proof of Lemma 3.3 in [1]. This integral can be estimated in a similar way as the integral I_i in the proof of Lemma 3.1 from [1] to obtain

$$(2.10)\qquad J_2 \leqslant c_4 \|u|W^{1,p}(Q_N, d_M^\varepsilon)\|^p.$$

The imbedding (2.7) now follows from (2.8), (2.9) and (2.10). □

Lemma 2.9. *Let $N \geqslant 2$, $1 \leqslant k \leqslant N-1$, $k-N < \varepsilon < p+k-N$, $M = [0,1]^k$. Then*

$$W^{1,p}(Q_N, d_M^\varepsilon) = H^{1,p}(Q_N, d_M^\varepsilon) \oplus D^p_{\varepsilon,M}(Q_N).$$

Moreover, the norms in the spaces $W^{1,p}(Q_N, d_M^\varepsilon)$ and $H^{1,p}(Q_N, d_M^\varepsilon) \oplus D^p_{\varepsilon,M}(Q_N)$ are equivalent.

Proof. Let $u \in W^{1,p}(Q_N, d_M^\varepsilon)$. We can write

$$u = (u - RTu) + RTu = u_1 + u_2.$$

From Lemma 2.8 we obtain $u_1 \in H^{1,p}(Q_N, d_M^\varepsilon)$ and according to Definition 2.1 we have $u_2 \in D^p_{\varepsilon,M}(Q_N)$. In [2] and [4] it is proved that $H^{1,p}(Q_N, d_M^\varepsilon)$ is the closure of the set $C_M^\infty(\overline{Q}_N)$ in the norm of the space $W^{1,p}(Q_N, d_M^\varepsilon)$. It immediately implies that the functions from $H^{1,p}(Q_N, d_M^\varepsilon)$ have zero traces on M. From the linearity of the opearator R we get $R(0) = 0$. This yields

$$H^{1,p}(Q_N, d_M^\varepsilon) \cap D^p_{\varepsilon,M}(Q_N) = \{0\}.$$

Now, let $u_1 \in H^{1,p}(Q_N, d_M^\varepsilon)$, $u_2 \in D^p_{\varepsilon,M}(Q_N)$. Taking into account the trivial imbedding $H^{1,p}(Q_N, d_M^\varepsilon) \hookrightarrow W^{1,p}(Q_N, d_M^\varepsilon)$ and Theorem 3.4 in [1] we get

$$\begin{aligned}
&\|u_1 + u_2 | W^{1,p}(Q_N, d_M^\varepsilon)\| \\
&\leqslant \|u_1 | W^{1,p}(Q_N, d_M^\varepsilon)\| + \|RTu | W^{1,p}(Q_N, d_M^\varepsilon)\| \\
&\leqslant c_1\big(\|u_1 | H^{1,p}(Q_N, d_M^\varepsilon)\| + \|Tu | W^{1-\frac{N-k+\varepsilon}{p},p}(M)\|\big) \\
&= c_1\big(\|u_1 | H^{1,p}(Q_N, d_M^\varepsilon)\| + \|u_2 | D^p_{\varepsilon,M}(Q_N)\|\big),
\end{aligned}$$

which proves

$$H^{1,p}(Q_N, d_M^\varepsilon) \oplus D^p_{\varepsilon,M}(Q_N) \hookrightarrow W^{1,p}(Q_N, d_M^\varepsilon).$$

On the other hand, let $u \in W^{1,p}(Q_N, d_M^\varepsilon)$. We can write

$$u = (u - RTu) + RTu.$$

Lemma 2.8 yields

$$\|u - RTu | H^{1,p}(Q_N, d_M^\varepsilon)\| \leqslant c_2 \|u | W^{1,p}(Q_N, d_M^\varepsilon)\|,$$

and by Theorems 3.4 and 2.11 in [1] we have

$$\|RTu | D^p_{\varepsilon,M}(Q_N)\| \leqslant c_3 \|u | W^{1,p}(Q_N, d_M^\varepsilon)\|.$$

Thus,

$$W^{1,p}(Q_N, d_M^\varepsilon) \hookrightarrow H^{1,p}(Q_N, d_M^\varepsilon) \oplus D^p_{\varepsilon,M}(Q_N).$$

□

It is not difficult to extend Lemma 2.9 in the following way.

Theorem 2.10. *Let $N \geqslant 2$, $1 \leqslant k \leqslant N-1$, $k-N < \varepsilon < p+k-N$ and let $(\Omega, M) \in B(k,N)$. Then*

$$W^{1,p}(\Omega, d_M^\varepsilon) = H^{1,p}(\Omega, d_M^\varepsilon) \oplus D^p_{\varepsilon,M}(\Omega)$$

and the norms in the spaces $W^{1,p}(\Omega, d_M^\varepsilon)$ and $H^{1,p}(\Omega, d_M^\varepsilon) \oplus D^p_{\varepsilon,M}(\Omega)$ are equivalent.

Definition 2.2. Let the assumptions of Theorem 2.10 be satisfied. Since the trivial imbedding

$$W^{1,p}(\Omega, d_M^\varepsilon) \hookrightarrow W^{1,1}(\Omega)$$

holds, there exists a trace operator $\tilde{T}$ such that

$$\tilde{T}\colon W^{1,p}(\Omega, d_M^\varepsilon) \hookrightarrow L^1(\partial\Omega).$$

Define the space $Y^p_{\varepsilon,M}(\partial\Omega)$ as the range of the operator

$$\tilde{T}R\colon W^{1-\frac{N-k+\varepsilon}{p},p}(M) \to L^1(\partial\Omega),$$

endowed with the norm

$$\|v|Y^p_{\varepsilon,M}(\partial\Omega)\| = \|(\tilde{T}R)^{-1}u|W^{1-\frac{N-k+\varepsilon}{p},p}(M)\|.$$

Theorem 2.11. *Let $N \geqslant 2$, $1 \leqslant k \leqslant N-1$, $k-N < \varepsilon < p+k-N$, $(\Omega, M) \in B(k,N)$. Then*

(i) there exists a unique bounded linear operator

$$T\colon W^{1,p}(\Omega, d_M^\varepsilon) \hookrightarrow X^p_{\varepsilon,M}(\partial\Omega) \oplus Y^p_{\varepsilon,M}(\partial\Omega)$$

such that

$$Tu = u\Big|_{\partial\Omega}$$

for every $u \in C^\infty(\overline{\Omega})$,

(ii) there exists a bounded linear operator

$$R\colon X^p_{\varepsilon,M}(\partial\Omega) \oplus Y^p_{\varepsilon,M}(\partial\Omega) \to W^{1,p}(\Omega, d_M^\varepsilon)$$

such that

$$TRu = u \quad \text{on} \quad \partial\Omega.$$

Proof. The theorem follows easily from Theorems 2.2 and 2.10. □

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