Ján Jakubík; Milan Kolibiar
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EIGHTY YEARS OF PROFESSOR ŠTEFAN SCHWARZ

JÁN JAKUBÍK, Košice, and MILAN KOLIŠIAR, Bratislava

A prominent Slovak mathematician, Professor Štefan Schwarz, Doctor of Science, an outstanding specialist in the theory of semigroups and in number theory, has reached eighty years of age on May 18, 1994 continuing his creative work in mathematics.

He was born at Nové Mesto nad Váhom. After having completed his studies at Charles University in Prague in 1936, he became Assistant Professor at the Mathematical Institute of the Faculty of Science, Charles University, and later, in 1939, he became member of staff of the newly established Slovak Technical University.

In 1944, he was deported and imprisoned in a German concentration camp, from where he was liberated in 1945. Two his sisters died in such camps.

In 1946 he was appointed Associate Professor at the Faculty of Science in Bratislava and since 1947 he has been Full Professor at the Slovak Technical University. He was elected corresponding member and ordinary member (Academician) of the Czechoslovak Academy of Sciences in 1952 and 1960, respectively. In 1953 he also became member of the then established Slovak Academy of Sciences. Since 1966 until 1988 he has been Head of the Mathematical Institute of this Academy. In the years 1965–1970 he was President of the Slovak Academy of Sciences and Vice-President of the Czechoslovak Academy of Sciences.

The scientific papers of Štefan Schwarz concern the following regions: a) theory of semigroups and its applications in other fields of mathematics; b) theory of finite fields; c) number theory; d) non-negative matrices and Boolean matrices.

The results of Štefan Schwarz have been incorporated and reproduced in detail in several monographs; in particular, cf. [1a] and [2a] for the case of algebraic semigroups, [3a] for topological semigroups, [4a] for Boolean matrices and [5a] for the theory of finite fields.

His work was appraised twenty years ago and ten years ago in Czechoslovak Mathematical Journal, Časopis pro pěstování matematiky, Mathematica Slovaca; cf., e.g., [1c], [2c], [3c].
The purpose of the present article is to characterize the papers of Štefan Schwarz which have been published after 1981.

For a semigroup $S$ having a minimal left ideal let $K$ be the union of all minimal left ideals of $S$. Schmutterer proposed the problem of characterizing those semigroups for which $Ka$ is a minimal left ideal of $S$ for each $a \in S$. In [94] Š. Schwarz finds such a characterization; a semigroup has the mentioned property if and only if it can be obtained by a right composition of a special type of semigroups (called $U_l$-semigroups). He also obtains several results concerning the converse problem: to decide whether a given family of $U_l$-semigroups admits at least one right composition.

The results of the paper [90] concern the structure of semigroups $S$ such that (i) $S$ contains a universally minimal left ideal $L$ (i.e., the last left ideal $L$) which is a left group, and (ii) $S$ has an $L$-homomorphism.

In the paper [92], the author continues in studying the semigroup of binary relations on a finite set which was begun in [72] and [73]. At the same time he shows that there exists a close connection of the question under consideration with the theory of Boolean matrices and the theory of directed graphs. Let $B_n(V)$ be the semigroup of all binary relations on a finite $V$ with $n$ elements, where $n \geq 2$. Further let $M_n$ be the set of all $n \times n$ matrices over the Boolean algebra $\{0, 1\}$; $M_n$ is a semigroup under the Boolean matrix multiplication. Finally, let $G_n(V)$ be the set of all directed graphs having the set $V$ as the set of all vertices with allowable loops and simple directed edges. There exists a natural one-to-one correspondence between the sets $B_n(V), M_n$ and $G_n(V)$. By applying this correspondence and by a masterful synthesis of methods belonging to semigroup theory, theory of Boolean matrices and graph theory Š. Schwarz reused to solve a combinatorial problem which was proposed by A. Paz in his monograph [6a] on probabilistic automata. A series of further results in the field under consideration is deduced in [92].

Under the usual notation, let $GF(q)$ be a finite field with $q$ elements and let $S_n$ be the multiplicative semigroup of all $n \times n$ matrices over $GF(q)$. I. B. Marshall [2b], 1911, L. Niven [3b], 1948 and V. Klein [1b], 1980, dealt with the identity $A^\kappa = A^{\kappa + \delta}$, where $A$ runs over the set of all rectangular matrices belonging to $S_n$, and the integers $\kappa$ and $\delta$ are as small as possible. In [93] Š. Schwarz proved the following theorem concerning singular matrices (by using semigroup-theoretical methods):

For each $n \times n$ matrix $A$ over $GF(q)$ with $1 \leq \text{rank}(A) \leq h \leq n - 1$ the relation

$$A^{h+1} = A^{h+1+\lambda(h, q)}$$

is valid, and this result is the best possible. Here, $\lambda(h, q)$ is the least common multiple of $h$ and $q$.

The powers of non-negative matrices have been investigated by several authors (cf., e.g., the monographs [4a] and [7a]). In particular, the following question has
been considered: Suppose that a non-negative \( n \times n \) matrix \( P \) satisfies the condition
\[
(\ast) \text{ some power of } P \text{ has a positive column.}
\]

What is the least integer \( k \) such that \( P^k \) has a positive column? D. Isaacson and P. Madsen [4b], 1974, proved that \( k \leq n^2 - 3n + 3 \); the same result is contained in a paper of L. T. Kou [5b], 1984. In his article [95] Š. Schwarz proved a stronger result, namely:

Let \( P \) be a non-negative \( n \times n \) matrix satisfying the condition \((\ast)\). Put \( L = n^2 - 3n + 3 \) and \( K = n^2 - 5n + 8 \).

(i) If \( P \) is primitive, then \( P^L \) contains a positive column. For each \( n \geq 2 \) there are primitive \( n \times n \) matrices for which the number \( L \) cannot be replaced by a smaller one.

(ii) If \( P \) is not primitive, then \( P^K \) has a positive column. For each \( n \geq 3 \) there are \( n \times n \) matrices for which the number \( K \) cannot be replaced by a smaller one.

In the paper [97] the author gives a new proof of the existence of a normal basis for cyclic extensions over any field. This proof is shorter and more transparent than those given until now in the literature. Next, an effective method to find all normal bases is described.

Pei, Wang and Oniura [6b] studied normal bases of finite fields having the form \( GF(2^m) \) in connection with the coding theory. In [98] Š. Schwarz considers the following question. Let \( f(x) \) be an irreducible polynomial of degree \( n \) over \( F_q = GF(q) \) and let \( \alpha \) be a fixed root of \( f(x) \); the problem is to find an effective method for deciding whether the roots of \( f(x) = 0 \) represent a normal basis for the cyclic extension \( F_q(\alpha) \) over \( F_q \). A criterion giving such an effective method is described in Schwarz's theorem which is the main result of [98]. This theorem widely generalizes the result of Pei, Wang and Oniura; also, the method of proof is different.

As a former student of Charles University Š. Schwarz often remembers on and speaks about his teachers, especially Professors Karel Petr (1868-1950), Bohumil Bydžovský (1880-1969), Vojtěch Jarník (1897-1970), Vladimír Kořínek (1899-1981), and his elder colleagues František Vyčichlo (1905-1958) and Vladimír Knichal (1908-1974).

In June 1993, on the occasion of the celebration (organized in Prague) of the 100th anniversary of birth of Eduard Čech, Professor Štefan Schwarz presented a lecture on this greatest personality of Czech mathematics.

In the name of Czech and Slovak mathematical communities we cordially congratulate Professor Štefan Schwarz to his anniversary, and we do it also in the name of a large number of his thankful former students. We wish him all the best for the next years, good health, further deep and nice mathematical results. We thank him for his scientific, pedagogical and organizing work.
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B. Scientific articles:

C. Other articles:

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