Anton Dekrét
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Mathematica Slovaca, Vol. 27 (1977), No. 3, 257–265

Persistent URL: http://dml.cz/dmlcz/128859

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HORIZONTAL STRUCTURES ON FIBRE MANIFOLDS

ANTON DEKRÉT

Libermann, [3], has defined a connection of the first order on a fibre space \( E(B, F, \pi) \) as a global cross-section \( \Gamma: E \rightarrow J^1E \). In this paper we find some properties of this structure. Our consideration are in the category \( C^\infty \). The standard terminology and notations of the theory of jets are used throughout the paper, see [2].

1. Let \( VTE \) denote the fibre bundle of vertical vectors on \( E(B, F, \pi) \). A tensor field \( \sigma: E \rightarrow VTE \otimes T^*E \) will be said to be a v-field. Let \( X \) be a vector field on \( E \). Denote by \( L_x(\sigma) \) the Lie derivative of \( \sigma \) by \( X \). Locally, let \((x', y^a), i=1, \ldots, n = \dim B, a=1, \ldots, \dim F, \) be local coordinates on \( E \). Direct evaluation yields for the v-field \( \sigma: (x, y) \mapsto (a_k(x, y)dx^k + b^a(x, y)dy^a) \otimes \partial y_a \) and the vector field \( X = a'(x, y)dx^1 + b''(x, y)dy^a \):

\[
L_x(\sigma) = - (a^a dx^i + b^a dy^a) \frac{\partial a^i}{\partial y^a} \otimes \partial x_i + \left\{ \left( \frac{\partial a^a}{\partial x^i} a^i + \frac{\partial a^a}{\partial y^a} \right) dx^i + \right. \\
+ \frac{\partial a^a}{\partial y^a} b^a + a^a \frac{\partial a^i}{\partial x^k} + b^a \frac{\partial b^a}{\partial x^i} \frac{\partial b^a}{\partial y^a} a^a \right\} dx^k + \\
\left. + \left( \frac{\partial a^a}{\partial y^a} + \frac{\partial b^a}{\partial x^i} a^i + \frac{\partial b^a}{\partial y^a} b^a + b^a \right) dy^a - \frac{\partial b^a}{\partial y^a} b^a \right\} \otimes \partial y_a.
\]

This immediately gives

**Lemma 1.** Let \( X \) be a vector field on \( E \). Then the Lie derivative of every v-field on \( E \) by \( X \) is a v-field on \( E \) if and only if \( X \) is projectable.

Let \( \sigma \) be a v-field, hence \( \sigma(u) \in \text{Hom}(T_uE, T_uE) \), \( \pi u = x \). If \( \sigma(u)|T_uE \) is regular for any \( u \in E \), then \( \sigma \) determines a horizontal distribution of the kernels of \( \sigma(u) \), i.e. a global cross-section \( E \rightarrow J^1E \). Denote by \( \kappa(E) \) the set of all such v-fields on \( E \) that \( \sigma(u)|T_uE = \text{id}|T_uE \) for any \( u \in E \). Let \( \Gamma_E \) be the set of all cross-sections \( E \rightarrow J^1E \). There is a one to one correspondence \( \delta: \kappa(E) \rightarrow \Gamma_E \), where
\(\delta(\sigma)\) is a cross-section \(E \to J^1E\) determined by the horizontal distribution of the kernels of \(\sigma(u), \ u \in E\).

2. **Definition 1.** Let \(\Gamma: E \to J^1E\) be a cross-section. The pair \((E, \Gamma)\) or the \(v\)-field \(\delta^{-1}(\Gamma) = \tau \sigma\) will be called an \(H\)-structure or a tensor of the \(H\)-structure, respectively.

Every 1-jet \(\Gamma(u)\) determines an element of \(\text{Hom}(T_xB, T_xE)\), \(\pi u = x\). Thus we get a cross-section \(\tilde{\Gamma}: E \to TE \otimes T^*B\). Locally, let \((x', y'^n, y'^i)\) be local coordinates on \(J^1E\). If \(\Gamma: (x', y'^n) \mapsto (x', y'^n, \ y'^i = - a'^i(x', y'^n))\), then

\[
\tau \sigma(x, y) \mapsto (a'^n(x, y)dx^n + dy^n) \otimes \partial y_n,
\]

\[
\tilde{\Gamma}: (x, y) \mapsto dx' \otimes \partial x' - a'^n(x, y)dx^n \otimes \partial y_n.
\]

By direct evaluation we get

**Lemma 2.** Let \(X\) be a projectable vector field on \(E\). Then \(L_X(\tilde{\Gamma})\) is a global cross-section \(E \to VTE \otimes T^*M\) and

\[
(L_X \tau \sigma)(u) = -(L_X \tilde{\Gamma})(u)\pi_*.
\]

Let \(X\) be a projectable vector field on \(E\) and \(^1X\) be the first prolongation of \(X\) on \(J^1E\). Let \(\Gamma(E)\) be the set of all values of the cross-section \(\Gamma: E \to J^1E\). By [1] a projectable field \(X\) on \(E\) is conjugate with \(\Gamma\) if \(\Gamma^*(X)(h) = ^1X(h)\). It is easy to prove

**Proposition 1.** Let \((E, \Gamma)\) be an \(H\)-structure. Let \(X\) be a projectable vector field on \(E\). Then \(X\) is conjugate with \(\Gamma\) if and only if \(L_X (\tau \sigma) = 0\).

Denote by \(\tilde{Y}\) the \(\Gamma\)-lift of a vector field \(Y\) on \(B\). Let \(Z_1, Z_2 \in T_xB\). Let \(Y_1\) or \(Y_2\) be such a vector field on \(B\) that \(Y_1(x_0) = Z_1\), or \(Y_2(x_0) = Z_2\), respectively. Put

\[
\Theta(u)(Z_1, Z_2) = \tau \sigma([\tilde{Y}_1, \tilde{Y}_2])(u))
\]

It is easy to prove that \(\Theta(u)(Z_1, Z_2)\) does not depend on the choice of the vector fields \(Y_1, Y_2\) and that the mapping \(u \mapsto \Theta(u)\) determines a global cross-section

\[
\Theta: E \to VTE \otimes \Lambda^2T^*B,
\]

which will be said to be the curvature field of the \(H\)-structure.

Let \(\Gamma_1: E \to J^1E\) denote the first prolongation of \(\Gamma: E \to J^1E\), see [4]. In local coordinates, if

\[
\Gamma: (x', y'^n) \mapsto (x', y'^n, y'^i = - a'^i(x', y'^n)),
\]

then

\[
(2) \quad \Gamma': (x', y'^n) \mapsto \left(x', y'^n, y'^i = - a'^i, y'^i = \frac{\partial a'^n}{\partial y'^p} a'^i - \frac{\partial a'^i}{\partial x'}\right).
\]

Kolář, [4], introduced the difference tensor \(X(\sigma)\) of an arbitrary semi-holonomic
jet $X$. We recall that if $h \in \mathcal{J}_x E$, $\beta h = u \in E$, then $\Delta(h) \in T_x E \otimes T^* B$. Locally, if $h = (x', y'', y''', y''''', y''')$, then $\Delta(h) = y''''', a_i \ dx^i \wedge dx^k \otimes \partial y_a$.

In the case of the $H$-structure $(B, \Gamma)$ we obtain a global cross-section $\Delta(\Gamma'') : E \to VTE \otimes T^* B$. By the direct evaluation in local coordinates we get

**Proposition 2.** Let $(E, \Gamma)$ be an $H$-structure. Then

$$\Theta(u) = -\Delta(\Gamma''(u))$$

for any $u \in E$.

By the relation (3) the curvature field $\Theta$ of the $H$-structure $(E, \Gamma)$ is the curvature of the connection $\Gamma$ by Libermann [3]. Relation (3) also gives in the comparison the curvature of the differential system $\Gamma$ by Prad nes [6].

Let $\tilde{X} = a^i \partial x_i - a_k^a \partial y_a$ be the $\Gamma$-lift of a vector field $X$ on $M$. Using (1) we have

$$L_x(\Gamma') = \left[ \frac{\partial a_k^a}{\partial x_i} \frac{\partial a_k^a}{\partial y_a}, a_i \frac{\partial a_k^a}{\partial y_a} a_k^a - \frac{\partial a_k^a}{\partial x_i} \right] a^i dx^k \otimes \partial y_a$$

It immediately yields that the mapping

$$X \mapsto L_x(\Gamma')$$

is a linear mapping of the modul $D(M)$ of all vector fields on $M$ to the modul of all tensor fields $E \to VTE \otimes T^* M$. Moreover if the curvature field of $(B, \Gamma)$ vanishes, then the $\Gamma$-lift $X$ of $X$ is conjugate with $\Gamma$.

Let $w \in \mathcal{J}_x E$, $\beta w = u$, $\pi u = x$. Denote by $L(w)$ the element of $T_x E \otimes T^*_x M$ determined by $w$. Then $L(w) - L(\Gamma(u)) \in T_x E \otimes T^*_x M$ and determines a 1-jet of $\mathcal{J}_x(B, E_x)$, which we will denote by $w - \Gamma(u)$ and call the developement of $w$ into $E_x$ by means of $\Gamma$.

Let $v \in \mathcal{J}_x E$, $\beta v = u$. Then the tensor $\tilde{\tau}(v) = \Delta(v) - \Delta(\Gamma'(u))$ will be said to be the torsion of the 2-jet $v$. Let $\mathcal{J} : B \to \mathcal{J} E$ be a global section of $\mathcal{J} E$ over $B$. Let $(E, \Gamma)$ be an $H$-structure. Then the threetuple $(E, \Gamma, \mathcal{J})$ will be called the $SH$-space. The tensor

$$\tilde{\tau}(x) = \Delta(\mathcal{J}(x)) - \Delta(\Gamma'(\mathcal{J}(x)))$$

will be said to be the torsion of the $SH$-space at $x \in B$.

**Remark.** The second prolongation of the section $S : B \to E$ gives a holonomic section $S^{(2)} : B \to \mathcal{J}^2 E$ and determines the $SH$-space $(E, \Gamma, S^{(2)})$, the torsion of which has the property

$$\tilde{\tau}(x) = \Theta(S(x)).$$

3. Let us compare our consideration with the theory of connections. Let $\Phi$ be a Lie grupoid of the operators on a fibre bundle $E(B, F, \pi)$. Let $a, b$ be the projections of $\Phi$ and let $1, \epsilon \in \Phi$ denote the unit over $x \in B$. Let us recall (see [5])
that the connection (of the first order) on $\Phi$ is a global cross-section $C: B \to \bigcup_{x \in B} Q_x$, where $Q_x$ denotes the set of all such elements $h \in J^1_x(a^{-1}(x), b, B)$ that $\beta h = 1_x$.

Let $C$ be a connection on $\Phi$, $C(x) = j_x^i \eta$. Let $v \in J^1 E$, $v = j_x^i \xi$. We recall that

$$(6) \quad C^{-1}(x)(v) = j_x^i [\eta^{-1}(z)[\xi(z)]] \in J^1(B, E_x)$$

is the development of $v$ into $E_x$ by means of $C$ and analogously if $w \in J^1 E$, $w = j_x^i \xi$, then

$$(7) \quad C'^{-1}(x)(w) = C^{-1}(x)[j_x^i c^{-1}(z)(\xi(z))] \in \hat{J}^1(B, E_x)$$

is the development of $w$ into $E_x$ by means of $C$.

Let $u \in E$, $\pi u = x$, $C(x) = j_x^i \eta$. Using the diffeomorphism $\eta(z): E_x \to E_x$ put

$$(8) \quad C^\tau(u) = j_x^i[z \mapsto \eta(z)(u)] \in J^1 E.$$ 

It is easy to see that the mapping $u \mapsto C^\tau(u)$ determines a global cross-section $C^\tau: E \to J^1 E$. The $H$-structure $(E, C^\tau)$ will be said to be the representative of the connection $C$ on $E$.

Denote by $U$ the domain of the local cross-section $\eta$. We have a mapping $f: \pi^{-1}(U) \to E_x$ determined by $h \mapsto \eta^{-1}(z)(h)$, $\pi h = z$. Let $dC_u$ be the differential of $f$ at $u \in E$, $\pi u = x$.

**Proposition 3.** Let $C$ be a connection on $\Phi$. Then

$$dC_u = c^\sigma(u), \quad u \in E,$$

where $c^\sigma$ denotes the tensor of the $H$-structure $(E, C^\tau)$.

**Proof.** Since $\beta C(x) = 1_x$, $dC_u|T_u(E_x) = \text{id}|T_u(E_x)$. Let $Y \in H_u \subset T_u(E)$, where $H_u$ is the subspace determined by $C^\tau(u)$. Then $dC_u(Y) = O$. It proves our assertion.

**Lemma 3.** Let $v \in J^1 E$, $\beta v = u$. Then

$$L(C^{-1}(x)(w)) = L(v) - L(C^\tau(u)).$$

**Proof.** It is easy to see that $L(v) - L(C^\tau(u)) = c^\sigma(u)L(v)$ and that $dC_u$ $L(v) = L(C^{-1}(x)(v))$. Then the relation (9) completes our proof.

Using Lemma 3 the following assertion can be proved by direct evaluation in local coordinates.

**Proposition 4.** Let $w \in \hat{J}^1 E$, $\beta w = u$, $\pi u = x$. Then

$$\hat{\tau}(w) = \Delta C'^{-1}(x)(w).$$

Let $P(B, G, p)$ be a principal fibre bundle and let $E(B, F, \pi)$ be a fibre bundle associated with $P$. Let $\Phi = PP^{-1}$ be the grupoid associated with $P$. Let us recall that
\Phi = (P \times P)\mid G, (h_1, h_2g) \sim (h_1, h_2); if \vartheta = (h_1, h_2) \in \Phi, then a\vartheta = ph_2, b\vartheta = ph_1; if \vartheta_1 = (h_1, h_2) and \vartheta_2 = (h_3, h_4), then the composition \vartheta_1 \vartheta_2 is defined if and only if h_1 = h_2 and \vartheta_1 \vartheta_2 = (h_1, h_4). Let us also recall that \Phi = P P^{-1} is a grupoid of operators on \(E(B, F, \pi)\). Let \(C\) be a connection on \(\Phi\) and let \(\Gamma: P \to J^1P\) be the representative of \(C\) on \(P\). It is known that \(\Gamma(g) = \Gamma h g\) (i.e. \(\Gamma\) is a connection on \(P\)). Hence the tensor \(s\) of the \(H\)-structure \((P, \Gamma)\) is equivariant, i.e. if \(Y \in T_hP\) is generated by \(Y \in \mathcal{G}\) (\(\mathcal{G}\) denotes the Lie algebra of \(G\)) and \(s(X) = \hat{Y}\), then
\[s(\rho W(X)) = \Ad g^{-1}(Y)\]. Let \(h \in P, \rho(h) = x\). Denote by \(h\) the map \(P \to G, \quad h(q) = h(hg) = g\). Let \(\varphi\) be the canonical form of the connection \(\Gamma\). Then \(\varphi(h) \in \mathcal{G}\otimes T^*_hP\) and
\[\varphi(h) = \hat{h}s(h)\).

Let \(\Omega\) be the curvature form of the connection \(\Gamma\) on \(P\), denoted by \(\Omega(h)\) the element of \(\mathcal{G} \otimes \Lambda^2 T^*_hM\) determined by \(\Omega\) at \(h \in P, \rho h = x\).

**Proposition 5.** Let \(\Theta\) be the curvature field of the \(H\)-structure \((P, \Gamma)\) determined by the connection \(\Gamma\) on \(P\). Let \(\Omega\) be the curvature form of \(\Gamma\). Then
\[\hat{h}\Theta(h) = -\Omega(h)\).

**Proof.** Let \(\hat{X}, \hat{Y}\) be the \(\Gamma\)-lifts of vector fields \(X, Y\) on \(B\). Using (12), the definitions of \(\Omega\) and \(\Theta\) yield
\[\Omega(h)(X, Y) = \varphi(\hat{X}, \hat{Y})(\hat{h}(h)) = -\hat{h}s(h)[\hat{X}, \hat{Y}] = -\hat{h}\Theta(h)(X, Y) \cdot QED\).

Denote by \((E, \tilde{\Gamma})\) the \(H\)-structure, which is the representative of the connection \(C\) on \(E\). Every \(h \in P, \rho h = x\), determines a mapping \(\hat{h}: P \to a^{-1}(x) \subset \Phi, \quad \hat{h}(q) = (q, h)\). Analogously denote by \(\tilde{u}: a^{-1}(x) \to E\) the map \(\vartheta \to \vartheta(u), u \in E_x\). Therefore \(\tilde{u}h: P \to E\) is a fibre morphism from \(P\) to \(E\). Let \((\tilde{u}h)': J^1P \to J^1E\) denote the prolongation of the map \(\tilde{u}h\). It is easy to see that the diagram

\[
P \xrightarrow{\tilde{u}h} E \\
\Gamma/ \xrightarrow{\tilde{\Gamma}} \tilde{E} \\
J^1P \xrightarrow{(\tilde{u}h)^*} J^1E
\]

is commutative. Let \((\tilde{u}h)^*\) denote the differential of \(\tilde{u}h\) at \(h \in P\). Using (14) we obtain

**Proposition 6.** Let \(\tilde{\sigma}\) or \(\tilde{s}\) be the tensor field of the \((E, \tilde{\Gamma})\), or \((P, \Gamma)\), respectively. Then
\[(\tilde{u}h)^*\tilde{s}(h)(X) = \tilde{\sigma}(\tilde{u}h)^*(X), \quad X \in T_h(P)\).

261
Proposition 7. Let \( h \in P_{x} \), \( u \in E_{x} \). Let \( \hat{\Theta} \) be the curvature field of the \( H \)-structure \((E, \tilde{\Gamma})\). Then

\[
(16) \quad \hat{\Theta}(u) = (uh)^{\ast} \Theta(h).
\]

Remark. Let \( G_{x} \) be the isotropy group of \( \Phi \) over \( x \in B \) and let \( \mathcal{G}_{x} \) be its Lie algebra. Let \( h \in P_{x} \). Denote by \( \hat{h} \) the differential of the mapping \( \hat{h} \colon G \to G_{x} \), \( \hat{h}(g) = [hg, h] = \theta \in \Phi \), at \( e \in G \), where \( e \) denotes the unit of \( G \). Let \( \Omega \) be the curvature form \( \tilde{\Gamma} \) the connection \( \Gamma \) on \( P \) which is the representative of the connection \( C \). In [5] Kolář has introduced the curvature form of the connection \( C \) at \( x \in B \) by

\[
\Omega(x) = \hat{h} \ast \cdot \Omega(h),
\]

where the dot denotes the composition of mappings, and also introduced a generalized space with connection as a quadruple \( \mathcal{F} = S(P(B, G), F, C, \eta) \), where \( \eta : B \to E \) is a global cross-section. Let \( u \in E_{x} \). Let \( \hat{u} \) denote the differential of mapping \( \hat{u} : G_{x} \to E_{x} \), \( u(\hat{1}) = \hat{1}(u) \), at \( 1, e \in G \). Then the form

\[
\tau(x) = (\eta(x))^{\ast} \cdot \Omega(x)
\]

is called by Kolář the torsion form of the generalized space \( \mathcal{F} \) with connection at \( x \in B \). The relations (13) and (16) give

\[
(17) \quad \hat{\Theta}(\eta(x)) = - \tau(x).
\]

Moreover the generalized space \( \mathcal{F}(P(B, G), F, C, \eta) \) with connection determines the \( SH \)-space \((E, \tilde{\Gamma}, \eta^{\odot})\). Let \( \tilde{\tau}(x) \) be the torsion of this \( SH \)-space. Then comparing (5) with (17) we get

\[
\tilde{\tau}(x) = - \tau(x).
\]

4. Let us consider the special case of a vector bundle \( E(B, x) \). Denote by \( V \) the Liouville field on \( E \) determined by the 1-parametric group of all homothetics on \( E \). Locally, \( V = y^{\alpha} \partial y_{\alpha} \). A \( v \)-field \( \sigma \) on \( E \) will be said to be \( k \)-homogeneous, if \( L_{v} \sigma = k \sigma \).

Lemma 4. Locally let \( \sigma = (a_{i}(x^{i}, y^{\alpha})dx^{i} + b_{\alpha}(c^{i}, y^{\alpha})dy^{\gamma}) \otimes \partial y_{\alpha} \). Then \( \sigma \) is \( k \)-homogeneous if and only if \( a_{i} \) or \( b_{\alpha}^{\gamma} \) are homogeneous functions of the degree \( k+1 \) or \( k \) with respect to variables \( y^{\alpha} \).
Proof. Relation (1) gives

\[ L_v \sigma = \left[ \frac{\partial a^a_k}{\partial y^i} y^i - a^a_k \right] dx^k + \frac{\partial b^a_{\nu}}{\partial y^\nu} y^\nu dy^a \bigotimes \partial y_a. \]

This proves our assertion.

**Proposition 8.** Let \((E, \Gamma)\) be an \(H\)-structure. Then \(\sigma\) is \(O\)-homogeneous if and only if the Liouville field \(V\) is conjugate with \(\Gamma\).

**Proof.** In the case of the tensor field \(\sigma\) of the \(H\)-structure we have

\[ L_{\nu} \sigma = \left[ \frac{\partial a^a_k}{\partial y^i} y^i - a^a_k \right] dx^k + \frac{\partial b^a_{\nu}}{\partial y^\nu} y^\nu dy^a \bigotimes \partial y_a. \]

Using proposition 1, relation (19) and Lemma 4 complete our assertion.

Let \(\dot{X}\) be the \(\Gamma\)-lift of a field \(X\) on \(B\). Then

\[ (L_{\nu} \sigma)(\dot{X}) = [V, \dot{X}]. \]

This gives

**Proposition 9.** The tensor field \(\sigma\) of the \(H\)-structure \((E, \Gamma)\) is \(O\)-homogeneous if and only if \([V, \dot{X}] = 0\) for every vector field \(X\) on \(B\).

Let \((E, \Gamma)\) be an \(H\)-structure, \(Z\) be a vertical field on \(E\). Then \(\Gamma^*(Z)\) is a vector field on the submanifold \(\Gamma(E)\). The values of \(\Gamma^*(Z)\) are vertical tangent vectors on the vector bundle \(J^1E\) over \(B\). Let \(i: T^*_{\nu}(\nu)E \rightarrow J^1E\) be the canonical identification. Then \(u \mapsto i \cdot \Gamma^*(Z(u))\) determines a mapping \(\zeta: E \rightarrow J^1E\). Locally, \(Z = b^a(x', y^\alpha)\partial y_a\) and

\[(x', y^\alpha) \mapsto (x', b^a(x', y^\alpha), \frac{\partial a^a_i}{\partial y^\alpha} b^\alpha).\]

therefore \(\zeta\) is a global cross-section of \(J^1E\) over \(E\) if and only if \(Z = V\). In this case denote by \((E, \nu(\Gamma))\) the \(H\)-structure determined by \(\zeta\). Locally

\[ \nu(\Gamma) \sigma = (dy^a + \frac{\partial a^a_i}{\partial y^\alpha} y^\alpha dx^i) \bigotimes \partial y_a. \]

**Proposition 10.** Let \((E, \Gamma)\) be an \(H\)-structure. Then

\[ (L_\nu \sigma)(u) = (\bar{\nu}(u) - V(\Gamma)(u))\pi\nu. \]

**Proof.** \(\bar{\nu}(x', y^a) \mapsto \bar{dx}' \bigotimes \partial x_i - a^a_i(x, y)dx' \bigotimes \partial y_a,\)

\[ \bar{V}(\Gamma): (x', y^a) \mapsto dx' \bigotimes \partial x_i - \frac{\partial a^a_i}{\partial y^\alpha} y^\alpha dx' \bigotimes \partial y_a. \]

Using (19) we get (21).
**Corollary.** An H-structure \((E, \Gamma)\) is O-homogeneous if and only if \(\Gamma = V(\Gamma)\).

**Remark.** As it is well known, the H-structure \((E, \Gamma)\) is a connection on \(E\) if and only if the cross-section \(\gamma: E \rightarrow J^1E\) is a vector bundle morphism over \(B\). Locally, \(\gamma\) is a connection on \(E\) if and only if \(a_i^\nu = \Gamma^\nu_{\mu}(x)y^\mu\). Hence the Liouville field \(V\) is conjugate with every connection on \(E\).

Further, if \((E, \Gamma)\) is an H-structure and \(\varepsilon: B \rightarrow E\) is a global cross-section, then, using the identifications \(j: E \rightarrow T_{\varepsilon(x)}E, \ i: T_{\varepsilon(x)}J^1E \rightarrow J^1E\), we get the mapping

\[ \Gamma^r(x) = i \cdot \Gamma \cdot j \]

from \(E\) to \(J^1E\). It is easy to see that \(\Gamma^r\) is a connection on \(E\). Locally, if the functions \(a_i^\nu(x, y)\) determine the H-structure \((E, \Gamma)\), then the functions

\[ \frac{\partial a_i^\nu(x^k, \varepsilon^r(x^k))}{\partial y^\mu} \]

determine the connection \(\Gamma^r\).

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Received December 8, 1975
Пусть $E$ расслоенное пространство. Горизонтальная структура или обобщенная связность это сечение $\gamma: E \rightarrow J'E$ расслоения $J'E$. В статье определено поле и форма кривизны горизонтальной структуры. Пользуясь теорией струй найден джет-вид формы кривизны. Обоснованы некоторые свойства производной Ли поля горизонтальной структуры. Специально исследованы горизонтальные структуры на векторных расслоенных пространствах. Результаты соединены с полем и формой кривизны горизонтальной структуры сравны с теорией связности на главном расслоенном пространстве и пространствах ассоциированных с этим пространством.