Ivan Dobrakov

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ON EXTENSION OF BAIRE SUBMEASURES

IVAN DOBRAKOV

Let T be a locally compact Hausdorff topological space. It is well known and important, for example in harmonic analysis, that each Baire measure on T can be uniquely extended to a regular Borel measure on T, see § 54 in [6], or § 65 in [1]. The extension procedure described therein consists of the generation of a regular Borel content and its extension to a regular Borel measure. This procedure was modified in [3, § 3] to obtain the analogous result for the so-called submeasures (non additive set functions having many common properties with measures, see later). For measures, and especially for submeasures, this method is rather technical. The purpose of this note is to propose a more simple and transparent method of extension. It was inspired by the following simple observation. If μ is a regular Borel (sub)measure, then to each Borel set A there is a Baire set E such that $A = E \triangle N$, where N is a Borel μ-null set, see § 68 in [1] and Theorem 17 in [3]. Hence to obtain the required extension we have to add to Baire sets by symmetric difference a suitable class of null sets, and make the obvious extension which neglects in value these null sets. In this note this method will be described in details. Since we consider finite, hence bounded (sub)measures, we in fact obtain the extension to the so called weakly Borel sets (= the σ-algebra generated by all open subsets of T), see [2]. (Recently for most authors these are the Borel sets.)

For convenience let us remind the notions of submeasures. Let $\mathcal{R}$ be a ring of subsets of a non empty set T. According to Definition 1 in [3] we say that a set function $\mu: \mathcal{R} \to [0, +\infty)$ is a submeasure if it is 1) monotone, 2) continuous: $A_n \in \mathcal{R}$, $n = 1, 2, \ldots$, and $A_n \searrow \emptyset$ implies $\mu(A_n) \to 0$, and 3) subadditively continuous: For every $A \in \mathcal{R}$ and $\varepsilon > 0$ there is a $\delta > 0$ such that $B \in \mathcal{R}$ and $\mu(B) < \delta$ implies $\mu(A) - \varepsilon \leqq \mu(A - B) \leqq \mu(A) \leqq \mu(A \cup B) \leqq \mu(A) + \varepsilon$. If the δ in condition 3) is uniform with respect to $A \in \mathcal{R}$, then we say that μ is a uniform submeasure. It is easy to verify, see page 68 in [4], that subadditive continuity is equivalent to the following property 3)*: If $A, A_n \in \mathcal{R}$, $n = 1, 2, \ldots$ and $\mu(A_n \triangle A) \to 0$, then $\mu(A_n) \to \mu(A)$. Similarly, the uniform subadditive continuity is equivalent to the following one: 3u)*: for each $\varepsilon > 0$ there is a $\delta > 0$ such that $A, B \in \mathcal{R}$ and $\mu(A \triangle B) < \delta \Rightarrow |\mu(A) - \mu(B)| < \varepsilon$. If instead of 3) we have $\mu(A \cup B) \leqq \mu(A) + \mu(B)$ for every $A, B \in \mathcal{R}$, or $\mu(A \cup B) = \mu(A) + \mu(B)$ for every

$A, B \in \mathscr{R}$ such that $A \cap B = \emptyset$, then we say that μ is a subadditive or an additive submeasure, respectively. Obviously subadditive, and particularly additive submeasures (i.e., countably additive measures) are uniform.

For our next general result we need Theorem 12 from [5] According to this theorem, if $\mathscr{D} \subset 2^T$ is a δ-ring (a ring closed with respect to the formation of countable intersections) and $\mathscr{N} \subset 2^T$ is a hereditary σ-ring, then the smallest δ-ring containing both $\mathscr{D}$ and $\mathscr{N}$ is the class $\mathscr{D} \triangle \mathscr{N} = \{A\colon A = D \triangle N,\ D \in \mathscr{D},\ N \in \mathscr{N}\}$.

Theorem 1. *Let $\mathscr{D} \subset 2^T$ be a δ-ring, let $\mathscr{N} \subset 2^T$ be a hereditary σ-ring and let $\mu\colon \mathscr{D} \to [0, +\infty)$ be a submeasure. Suppose $\mu(E) = 0$ for $E \in \mathscr{D} \cap \mathscr{N}$, and for $A = D \triangle N$, where $D \in \mathscr{D}$ and $N \in \mathscr{N}$ put $\mu_{\mathscr{N}}(A) = \mu(D)$. Then $\mu_{\mathscr{N}}\colon \mathscr{D} \triangle \mathscr{N} \to [0, +\infty)$ is an unambiguously defined submeasure of the same type as μ, which extends μ to the smallest δ-ring $\mathscr{D} \triangle \mathscr{N}$ containing both $\mathscr{D}$ and $\mathscr{N}$.*

Proof. Throughout this proof all D-s belong to $\mathscr{D}$ and all N-s to $\mathscr{N}$. The unambiguity and monotonicity of $\mu_{\mathscr{N}}$ follows by standard methods, see the proof of Theorem 13 B in [6] on the completion of a measure.

Let $A_n = D_n \triangle N_n \searrow \emptyset$, $n = 1, 2, \ldots$. Then $\bigcap_{n=1}^{\infty} D_n - \bigcup_{n=1}^{\infty} N_n = \bigcap_{n-1}^{\infty} (D_n - N_n) \subset \bigcap_{n=1}^{\infty} A_n = \emptyset$, hence $\mu\left(\bigcap_{n=1}^{\infty} D_n\right) = 0$. However, $\mu\left(\bigcap_{i=1}^{n} D_i - \bigcap_{i-1}^{\infty} D_i\right) \to 0$ by continuity of μ, hence $\mu\left(\bigcap_{i=1}^{n} D_i\right) \to 0$ by subadditive continuity of μ. Clearly $D_n - \bigcap_{i=1}^{n} D_i \subset \bigcup_{i=1}^{n} (D_n - D_i)$. Since $A_n \subset A_i$ for $i \leqq n$, $\mu(D_n - D_i) = 0$ for $i \leqq n$. Thus $\mu(D_n) = \mu\left(D_n - \bigcap_{i=1}^{n} D_i\right) \cup \bigcap_{i=1}^{n} D_i\Big) = \mu\left(\bigcap_{i=1}^{n} D_i\right) \to 0$. Hence $\mu_{\mathscr{N}}\colon \mathscr{D} \triangle \mathscr{N} \to [0, +\infty)$ is continuous.

Let $A = D \triangle N$, let $A_n = D_n \triangle N_n$, $n = 1, 2, \ldots$ and let $\mu_{\mathscr{N}}(A_n \triangle A) \to 0$. Then $\mu_{\mathscr{N}}(A_n \triangle A) = \mu_{\mathscr{N}}(D_n \triangle D \triangle N_n \triangle N) = \mu_{\mathscr{N}}(D_n \triangle D)$, hence $\mu(D_n \triangle D) \to 0$. But then $\mu_{\mathscr{N}}(A_n) - \mu_{\mathscr{N}}(A) = \mu(D_n) - \mu(D) \to 0$ by subadditive continuity of μ. Thus $\mu_{\mathscr{N}}$ is subadditively continuous on $\mathscr{D} \triangle \mathscr{N}$.

Clearly $\mu_{\mathscr{N}}$ is uniform or subadditive, respectively if μ is such. Suppose finally that μ is additive. Let $A = D_1 \triangle N_1$, $B = D_2 \triangle N_2$, and let $A \cap B = \emptyset$. Then $(D_1 \cap D_2) \triangle N_3 = \emptyset$ for some $N_3 \in \mathscr{N}$. Hence $\mu(D_1 \cap D_2) = 0$. Thus $\mu_{\mathscr{N}}(A \cup B) = \mu(D_1 \cup D_2) = \mu((D_1 - D_2) \cup (D_2 - D_1) \cup (D_1 \cap D_2)) = \mu(D_1 - D_2) + \mu(D_2 - D_1) = \mu(D_1 - D_1 \cap D_2) + \mu(D_2 - D_1 \cap D_2) = \mu(D_1) + \mu(D_2) = \mu_{\mathscr{N}}(A) + \mu_{\mathscr{N}}(B)$ by the additivity of μ. Hence $\mu_{\mathscr{N}}$ is additive on $\mathscr{D} \triangle \mathscr{N}$. The theorem is proved.

In the following T will be a locally compact Hausdorff topological space. By $\mathscr{C}_0$ ($\mathscr{C}$) we denote the lattice of all compact G_δ (compact) subsets of T. $\sigma(\mathscr{C}_0)$ ($\sigma(\mathscr{C})$) denotes the smallest σ-ring over $\mathscr{C}_0$ ($\mathscr{C}$), and its elements are called Baire (Borel) subsets of T. By $\mathscr{U}$ ($\mathscr{U}_0$) we denote the lattice of all open (open

Baire) subsets of T. $\sigma(\mathscr{U})$ denotes the σ-algebra of the so-called weakly Borel subsets of T, see [2].

Lemma 1. *Let U, $V \in \mathscr{U}$, let $C \in \mathscr{C}_0$ and let $C \subset U \cup V$. Then there are C_1, $C_2 \in \mathscr{C}_0$ such that $C_1 \subset U$, $C_2 \subset V$ and $C = C_1 \cup C_2$.*

Proof. By Urysohn's lemma there is a continuous function $f: C \to [0, 1]$ such that $f(t)=0$ for $t \in C - V$, and $f(t)=1$ for $t \in C - U$. Now it is enough to put $C_1 = \{t \in C, f(t) \leqq 2^{-1}\}$ and $C_2 = \{t \in C, f(t) \geqq 2^{-1}\}$.

Let now $\mu_0: \sigma(\mathscr{C}_0) \to [0, +\infty)$ be a Baire submeasure. According to Theorem 11 in [3] μ_0 is regular, i.e., for each $A \in \sigma(\mathscr{C}_0)$ and each $\varepsilon > 0$ there are $U \in \mathscr{U}_0$ and $C \in \mathscr{C}_0$ such that $C \subset A \subset U$ and $\mu_0(U - C) < \varepsilon$.

For $U \in \mathscr{U}$ put

$$\mu_*(U) = \sup \{\mu_0(C), C \in \mathscr{C}_0, C \subset U\},$$

for $A \subset T$ put

$$\mu^*(A) = \inf \{\mu_*(U), U \in \mathscr{U}, A \subset U\},$$

and define

$$\mathscr{N} = \{N: N \subset T, \mu^*(N) = 0\}.$$

Clearly $\mu^*(U) = \mu_*(U)$ for any $U \in \mathscr{U}$, and μ^* is monotone on 2^T and $\mu^*(T) < +\infty$ by Theorem 4 in [3]. According to Theorem 3-b) in [3] the submeasure μ_0 (as well as any submeasure on a σ-ring) has the following property: for each $\varepsilon > 0$ there is a $\delta > 0$ such that A, $B \in \sigma(\mathscr{C}_0)$ and $\mu_0(A)$, $\mu_0(B) < \delta$ implies $\mu_0(A \cup B) < \varepsilon$ (the so-called pseudometric generating property, see [4]). From this fact we immediately obtain.

Lemma 2. *For each $\varepsilon > 0$ there is a $\delta > 0$ such that U, $V \in \mathscr{U}$ and $\mu_*(U)$, $\mu_*(V) < \delta$ implies $\mu_*(U \cup V) < \varepsilon$.*

Using this we easily have

Lemma 3. *There is a sequence of numbers δ_k, $k = 1, 2, \ldots$ such that $0 < \delta_k \leqq 2^{-k}$ for each k, $\delta_k \searrow 0$, and $U_k \in \mathscr{U}$ and $\mu_*(U_k) < \delta_k$ for each $k = 1, 2, \ldots$ imply $\mu_*\left(\bigcup_{i=k+1}^{\infty} U_i\right) \leqq \delta_k$ for each k.*

Proof. Let $0 < \delta_1 \leqq 2^{-1}$. By Lemma 2 there is a $\delta_1 \leqq 2^{-1}\delta_1$ such that U, $V \in \mathscr{U}$ and $\mu_*(U)$, $\mu_*(V) < \delta_2$ implies $\mu_*(U \cup V) < \delta_1$. Again by Lemma 2 there is a $\delta_3 \leqq 2^{-1}\delta_2$ such that U, $V \in \mathscr{U}$ and $\mu_*(U)$, $\mu_*(V) < \delta_3$ implies $\mu_*(U \cup V) < \delta_2$. Continuing in this way we obtain a sequence δ_k, $0 < \delta_k \leqq 2^{-k}$ for each $k = 1, 2, \ldots$, $\delta_k \searrow 0$, and such that U, $V \in \mathscr{U}$ and $\mu_*(U)$, $\mu_*(V) < \delta_{k+1}$ implies $\mu_*(U \cup V) < \delta_k$ for each $k = 1, 2, \ldots$. Take a sequence $U_k \in \mathscr{U}$, $k = 1, 2, \ldots$ so that $\mu_*(U_k) < \delta_k$ for each k, and let $C \in \mathscr{C}_0$ and $C \subset \bigcup_{i=k+1}^{\infty} U_i$. Then by compactness of C there is a positive integer p such that $C \subset \bigcup_{i=k+1}^{k+p} U_i$. But then we easily compute that $\mu_0(C) \leqq \mu_*\left(\bigcup_{i=k+1}^{k+p} U_i\right) < \delta_k$. Since $C \in \mathscr{C}_0$, $C \subset \bigcup_{i=k+1}^{\infty} U_i$ was arbitrary, we have the required inequality $\mu_*\left(\bigcup_{i=k+1}^{\infty} U_i\right) \leqq \delta_k$.

Lemma 4. *$\mathscr{N}$ is a hereditary σ-ring and $\mathscr{N}_0 = \{E: E \in \sigma(\mathscr{C}_0),\ \mu_0(E) = 0\}$ $= \mathscr{N} \cap \sigma(\mathscr{C}_0)$.*

Proof. The first assertion of the lemma immediately follows from Lemma 3 and the definition of $\mathscr{N}$.

Let $E \in \sigma(\mathscr{C}_0)$. Then $\mu(E) = \inf\{\mu_0(U): U \in \mathscr{U}_0,\ E \subset U\} \geqq \inf\{\mu_*(U): U \in \mathscr{U}, E \subset U\} = \mu^*(E)$ by regularity of the Baire submeasure μ_0, see Theorem 11 in [3]. Hence $\mathscr{N}_0 \subset \sigma(\mathscr{C}_0) \cap \mathscr{N}$.

Let $E \in \sigma(\mathscr{C}_0) \cap \mathscr{N}$. Since $\mu_0(E) = \sup\{\mu_0(C): C \in \mathscr{C}_0, C \subset E\}$ by regularity of μ_0, it is enough to show that $\mu_0(C) = 0$ for any $C \in \mathscr{C}_0$, $C \subset E$. Let C be such a set. Then owing to Theorem D in §50 in [6] and the regularity of μ_0 we obtain the required equality $0 = \mu^*(C) = \inf\{\mu_*(U): U \in \mathscr{U}, C \subset U\} = \inf\{\mu_*(U): U \in \mathscr{U}_0, C \subset U\}$ $= \inf\{\mu_0(U): U \in \mathscr{U}_0,\ C \subset U\} = \mu_0(C)$.

Before the next lemma and theorem let us recall that $\sigma(\mathscr{C}_0) \triangle \mathscr{N}$ is the smallest σ-ring containing both $\sigma(\mathscr{C}_0)$ and $\mathscr{N}$, see Theorem 12 in [5].

Lemma 5. *$\sigma(\mathscr{U}) \subset \sigma(\mathscr{C}_0) \triangle \mathscr{N}$ and $\mu_{\mathscr{N}}(U) \leqq \mu_*(U)$ for each $U \in \mathscr{U}$, where $\mu_{\mathscr{N}}$ is defined on $\sigma(\mathscr{C}_0) \triangle \mathscr{N}$ as in Theorem 1.*

Proof. Let $U \in \mathscr{U}$. If $\mu_*(U) = \mu^*(U) = 0$, then $U \in \mathscr{N}$. If $\mu_*(U) > 0$, then take $C_1 \in \mathscr{C}_0$, $C_1 \subset U$ so that $\mu_0(C_1) > 2^{-1}\mu_*(U)$. If $\mu_*(U - C_1) = 0$, then $U - C_1 \in \mathscr{N}$, hence $U \in \sigma(\mathscr{C}) \triangle \mathscr{N}$, and $\mu_{\mathscr{N}}(U) = \mu_0(C_1) \leqq \mu_*(U)$. If $\mu_*(U - C_1) > 0$, then take $C_2 \in \mathscr{C}_0$, $C_2 \subset U - C_1$ so that $\mu_0(C_2) > 2^{-1}\mu_*(U - C_1)$. Continuing in this way we either arrive at a k such that $\mu_*\left(U - \bigcup_{i=1}^{k} C_i\right) = 0$, or $\mu_*\left(U - \bigcup_{i=1}^{k} C_i\right) > 0$ for each $k = 1, 2, \dots$. In the first case $U - \bigcup_{i=1}^{k} C_i \in \mathscr{N}$, hence $U \in \sigma(\mathscr{C}_0) \triangle \mathscr{N}$, and $\mu_{\mathscr{N}}(U)$ $= \mu_0\left(\bigcup_{i=1}^{k} C_i\right) \leqq \mu_*(U)$. In the second case $\mu_*\left(U - \bigcup_{i=1}^{\infty} C_i\right) \leqq \mu_*\left(U - \bigcup_{i=1}^{k} C_i\right) <$ $2\mu_0(C_{k+1})$ for each $k = 1, 2, \dots$. Since C_k, $k = 1, 2, \dots$ are pairwise disjoint sets, $\mu_0(C_k) \to 0$ by exhaustivity of μ_0, see Theorem 1-c) in [3]. Thus $U - \bigcup_{i=1}^{\infty} C_i \in \mathscr{N}$, hence $U \in \sigma(\mathscr{C}_0) \triangle \mathscr{N}$. Further, $\mu_{\mathscr{N}}(U) = \mu_0\left(\bigcup_{i=1}^{\infty} C_i\right) = \lim_{k \to \infty} \mu_0\left(\bigcup_{i=1}^{k} C_i\right) \leqq \mu_*(U)$ by Theorem 1 in [3]. Since $\sigma(\mathscr{C}_0) \triangle \mathscr{N}$ is a σ-ring $\sigma(\mathscr{U}) \subset \sigma(\mathscr{C}_0) \triangle \mathscr{N}$, and the lemma is proved.

Now we are prepared to prove our main

Theorem 2. *Let $\mu_{\mathcal{N}}: \sigma(\mathcal{C}_0)\triangle\mathcal{N}\to[0, +\infty)$ be defined as in Theorem 1. Then $\mathcal{M}=\{E: E\in\sigma(\mathcal{C}_0)\triangle\mathcal{N}, \mu_{\mathcal{N}}(E)=0\}=\mathcal{N}$, and $\mu_{\mathcal{N}}: \sigma(\mathcal{C}_0)\triangle\mathcal{N}\to[0, +\infty)$ is a $(\mathcal{C}, \mathcal{U})$-regular and complete submeasure of the same type as $\mu_0: \sigma(\mathcal{C}_0)\to[0, +\infty)$. Further, its restriction $\mu_{\mathcal{N}}: \sigma(\mathcal{U}) \to [0, +\infty)$ ($\sigma(\mathcal{U})\subset\sigma(\mathcal{C}_0)\triangle\mathcal{N}$ by Lemma 5) is the unique $(\mathcal{C}, \mathcal{U})$-regular submeasure which extends $\mu_0: \sigma(\mathcal{C}_0) \to [0, +\infty)$, and $\sigma(\mathcal{C}_0)\triangle\mathcal{N}$ is the completion of $\mu_{\mathcal{N}}: \sigma(\mathcal{U}) \to [0, +\infty)$.*

Proof. $\mathcal{N}\subset\mathcal{M}$ by definition of $\mu_{\mathcal{N}}$. Let $E\in\mathcal{M}$. Then $E=A\triangle N$ with $A\in\sigma(\mathcal{C}_0)$, $\mu_0(A)=0$ and $N\in\mathcal{N}$. But then $A\in\mathcal{N}$ by Lemma 4, hence $E\in\mathcal{N}$. Thus $\mathcal{M}=\mathcal{N}$. Since $\mathcal{N}$ is a hereditary class, $\mu_{\mathcal{N}}: \sigma(\mathcal{C}_0)\triangle\mathcal{N} \to [0, +\infty)$ is complete. According to Theorem 1 $\mu_{\mathcal{N}}: \sigma(\mathcal{C}_0)\triangle\mathcal{N} \to [0, +\infty)$ is a submeasure of the same type as $\mu_0: \sigma(\mathcal{C}_0) \to [0, +\infty)$.

To prove the $(\mathcal{C}, \mathcal{U})$-regularity of $\mu_{\mathcal{N}}$ on $\sigma(\mathcal{C}_0)\triangle\mathcal{N}$, let $A\in\sigma(\mathcal{C}_0)\triangle\mathcal{N}$ and let $\varepsilon>0$. According to Lemma 2 take $\delta>0$ so that $U, V\in\mathcal{U}$ and $\mu_*(U), \mu_*(V)<\delta$ implies $\mu_*(U\cup V)<\varepsilon$. Suppose $A=E\triangle N$ with $E\in\sigma(\mathcal{C}_0)$ and $N\in\mathcal{N}$. Then $E-N\subset A\subset E\cup N$. Since $\mu_0: \sigma(\mathcal{C}_0) \to [0, +\infty)$ is regular, see Theorem 11 in [3], there are $U_1\in U_0$ and $C_1\in\mathcal{C}_0$ such that $C_1\subset E\subset U_1$ and $\mu_*(U_1-C_1) = \mu_0(U_1-C_1)<\delta$. Since $0=\mu^*(N)=\inf\{\mu_*(U): U\in\mathcal{U}, N\subset U\}$, there is a $U_2\in\mathcal{U}$ such that $N\subset U_2$ and $\mu_*(U_2)<\delta$. Clearly $C_1-U_2\in\mathcal{C}$, $C_1-U_2\subset A\subset U_1\cup U_2\in\mathcal{U}$, and $(U_1\cup U_2)-(C_1-U_2) \subset (U_1-C_1)\cup U_2$. Thus using Lemma 5 we have the inequalities $\mu_{\mathcal{N}}((U_1\cup U_2) - (C_1-U_2)) \leqq \mu_*((U_1\cup U_2) - (C_1-U_2)) \leqq \mu_*((U_1-C_1)\cup U_2)<\varepsilon$. Hence $\mu_{\mathcal{N}}: \sigma(\mathcal{C}_0)\triangle\mathcal{N} \to [0, +\infty)$ is $(\mathcal{C}, \mathcal{U})$-regular.

If $\mu_1, \mu_2: \sigma(\mathcal{C}_0)\triangle\mathcal{N} \to [0, +\infty)$ are two $(\mathcal{C}, \mathcal{U})$-regular submeasures both extending $\mu_0: \sigma(\mathcal{C}_0)\to[0, +\infty)$, and if $A\in\sigma(\mathcal{C}_0)\triangle\mathcal{N}$, then their $(\mathcal{C}, \mathcal{U})$-regularity and Theorem D in § 50 in [6] imply the existence of a sequence $C_n\in\mathcal{C}_0$, $n=1, 2, \dots$ such that $\mu_1(A) = \lim_{n\to\infty}\mu_0(C_n) = \mu_2(A)$. Hence $\mu_{\mathcal{N}}: \sigma(\mathcal{C}_0)\triangle\mathcal{N} \to [0, +\infty)$ is the unique $(\mathcal{C}, \mathcal{U})$-regular extension of μ_0.

Denote by $\mathcal{S}$ the completion of $\mu_{\mathcal{N}}: \sigma(\mathcal{U})\to[0, +\infty)$. Since $\mu_{\mathcal{N}}: \sigma(\mathcal{C}_0)\triangle\mathcal{N} \to [0, +\infty)$ is complete, $\mathcal{S}\subset\sigma(\mathcal{C}_0)\triangle\mathcal{N}$. To prove the converse inclusion it is enough to prove that $\mathcal{N}\subset\mathcal{S}$. Let $N\in\mathcal{N}$. Then by Lemma 5 $0=\mu^*(N) = \inf\{\mu_*(U), U\in\mathcal{U}, N\subset U\} \geqq \inf\{\mu_{\mathcal{N}}(U), U\in\mathcal{U}, N\subset U\}$. Since $\mathcal{U}$ is a lattice and $\mu_{\mathcal{N}}: \sigma(\mathcal{U}) \to [0, +\infty)$ is monotone, there is a non-increasing sequence $U_n\in\mathcal{U}$, $n=1, 2, \dots$ such that $N\subset U_n$ for each $n=1, 2, \dots$ and $\mu_{\mathcal{N}}(U_n)\to 0$. Thus $N\subset\bigcap_{n=1}^{\infty} U_n\in\sigma(\mathcal{U})$ and $\mu_{\mathcal{N}}\left(\bigcap_{n=1}^{\infty} U_n\right)=0$. Hence $\mathcal{N}\subset\mathcal{S}$. The theorem is proved.

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Matematický ústav SAV
Obrancov mieru 49
814 73 Bratislava

О РАСШИРЕНИИ БЭРОВСКИХ СУБМЕР

Ivan Dobrakov

Резюме

Пусть μ_0–беровская субмера (неаддитивное обобщение бэровской меры, см. [3]) на локально компактном топологическом пространстве T. Присоединяя к бэровским множествам надлежащий класс нулевых множеств, мы получим единственное регулярное борелевское расширение μ_0. Этот метод нагляднее обычного метода, использующего расширение субобъемов.