Jerzy Niewiarowski
Convergence of sequences of real functions with respect to small systems


Persistent URL: [http://dml.cz/dmlcz/130685](http://dml.cz/dmlcz/130685)

**Terms of use:**

© Mathematical Institute of the Slovak Academy of Sciences, 1988

Institute of Mathematics of the Academy of Sciences of the Czech Republic provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use.*

This paper has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library* [http://project.dml.cz](http://project.dml.cz)
CONVERGENCE OF SEQUENCES OF REAL FUNCTIONS
WITH RESPECT TO SMALL SYSTEMS

JERZY NIEWIAROWSKI

Small systems were introduced by Riečan [3] and studied by many authors (see [2 p. 498], for references).

In this paper we define the convergence with respect to a small system. In [4] Wagner defined the convergence with respect to an \( \sigma \)-ideal. We shall study the instances when both the convergence with respect to a small system and that with respect to a suitable \( \sigma \)-ideal are equivalent.

**Definition 1.** Let \( (X, S) \) be a measurable space and \( \{N_r\}_{r=1}^{\infty} \) a sequence of subfamilies of \( S \) such that

1. \( \emptyset \in N_r \) for \( r = 1, 2, \ldots \)
2. For each positive integer \( r \), there exists a sequence \( \{k_i\}_{i=1}^{\infty} \) of positive integers such that \( E_i \in N_{k_i} \) (i.e., \( i = 1, 2, \ldots \)) implies \( \bigcup_{i=1}^{\infty} E_i \in N_r \).
3. For each positive integer \( r \), if \( E \in N_r \), \( F \subseteq E \), \( F \in S \), then \( F \in N_r \).
4. \( N_r \supset N_r+1 \) for \( r = 1, 2, \ldots \).
5. For each positive integer \( r \), if \( E \in N_r \), \( F \subseteq \bigcap_{r=1}^{\infty} N_r \), then \( E \cup F \in N_r \).

The sequence \( \{N_r\}_{r=1}^{\infty} \) satisfying all the above properties will be called a small system on \( S \) and will be denoted by \( \mathcal{N} \).

It is not difficult to verify (see [2 p. 491]) that \( \mathcal{N} = \bigcap_{r=1}^{\infty} N_r \) is a \( \sigma \)-ideal on \( S \).

**Definition 2.** A small system \( \mathcal{N} \) will be called upper semicontinuous if and only if, for every nonincreasing sequence of the sets \( \{E_i\}_{i=1}^{\infty} \) the following is true: if there exists a positive integer \( r_0 \) such that \( E_i \notin N_{r_0} \) (for \( i = 1, 2, \ldots \)), then \( \bigcap_{i=1}^{\infty} E_i \notin \mathcal{N} \).

Let \( I \) be an \( \sigma \)-ideal in an \( \sigma \)-field \( S \). We say that \( I \)-almost every point of \( E \subseteq X \) has some property (or that this property holds \( I \)-almost everywhere, in abbreviation \( I \)-a.e., on \( E \)) if and only if the set of points in \( E \) at which this property does not hold belongs to the \( \sigma \)-ideal \( I \).

**Definition 3.** We say that a sequence \( \{f_n\}_{n=1}^{\infty} \) of \( S \)-measurable functions defined on \( X \) converges with respect to \( I \) to an \( S \)-measurable function \( f \) defined on \( X \) if and
only if each subsequence \( \{f_{n_r}\}_{n_r = 1}^{\infty} \) of \( \{f_n\}_{n = 1}^{\infty} \) contains a subsequence \( \{f_{n_m}\}_{m = 1}^{\infty} \) converging to \( f \) \( I \)-a.e. on \( X \). We shall use the notation \( f_n \xrightarrow{n \to \infty} f \).

**Definition 4.** Let \( \mathcal{N} \) be a small system on \( S \) and \( \{f_n\}_{n = 1}^{\infty} \) a sequence of \( S \)-measurable functions defined on \( X \). We assume that the functions \( f \) and \( f_n \) \( (n = 1, 2, \ldots) \) are \( N \)-a.e. finite \( \left( \text{where } N = \bigcap_{r=1}^{\infty} N_r \right) \). We say that \( \{f_n\}_{n = 1}^{\infty} \) converges with respect to the small system \( \mathcal{N} \) to an \( S \) measurable function \( f \) defined on \( X \) if and only if, for each \( \varepsilon > 0 \) and for each positive integer \( m \), there exists a positive integer \( n \geq n_{r,m} \), such that, for each positive integer \( n \geq n_{r,m} \), the set

\[
\{x: |f_n(x) - f(x)| \geq \varepsilon \} \in N_m.
\]

We shall use the notation \( f_n \xrightarrow{n \to \infty} f \).

We say that two \( S \)-measurable functions \( f \) and \( g \) are equivalent if and only if \( f - g \) vanishes \( N \)-a.e. on \( X \).

It is not difficult to observe that both the limit with respect to the small system \( \mathcal{N} \) and that with respect to the \( \sigma \)-ideal \( N \) are determined up to equivalent functions. In the above definitions we can also suppose that all functions \( f_n \) and \( f \) are defined only \( N \)-a.e. on \( X \) (see definition 1(5)).

From now on, we shall use only the properties (1)—(4) from definition 1. We shall assume that all functions \( f_n, f \) under consideration are \( S \)-measurable and \( N \)-a.e. finite \( \left( N = \bigcap_{r=1}^{\infty} N_r \right) \).

**Remark 1.** If \( f_n = f \) for every \( n \), then \( f_n \xrightarrow{n \to \infty} f \). If \( f_n \xrightarrow{n \to \infty} f \), then, for each subsequence \( \{f_{n_m}\}_{m = 1}^{\infty} \) of \( \{f_n\}_{n = 1}^{\infty} \), we have \( f_{n_m} \xrightarrow{m \to \infty} f \).

**Lemma 1.** Let \( \mathcal{N} \) be a small system; then there exists a sequence \( \{k_i\}_{i = 1}^{\infty} \) of positive integers such that, if \( E_i \subseteq N_{k_i} \), then \( \bigcup_{i=1}^{\infty} E_i \subseteq N_n \) for each positive integer \( n \).

**Proof.** For each positive integer \( j \), there exists a sequence \( \{k_i^{(j)}\}_{i = 1}^{\infty} \) of positive integers such that, if \( E_i \subseteq N_{k_i^{(j)}} \), then \( \bigcup_{i=1}^{\infty} E_i \subseteq N_j \). Let \( k_i = \max_{j \leq i} k_i^{(j)}, \ E_i \subseteq N_{k_i} \), \( i = 1, 2, \ldots \). If \( i \geq n \), then \( k_i \geq k_i^{(n)} \), hence \( E_i \subseteq N_{k_i^{(n)}} \) and \( \bigcup_{i=n}^{\infty} E_i \subseteq N_n \).

**Theorem 1.** If a sequence \( \{f_n\}_{n = 1}^{\infty} \) of functions converges on \( X \) with respect to a small system \( \mathcal{N} \) to a function \( f \), then there exists a subsequence \( \{f_{n_j}\}_{j = 1}^{\infty} \) of \( \{f_n\}_{n = 1}^{\infty} \) converging to \( f \) \( N \)-a.e. on \( X \).

**Proof.** Let \( A = \{x; |f(x)| = +\infty\} \), \( A_n = \{x; |f_n(x)| = +\infty\} \), \( C = A \cup \bigcup_{n=1}^{\infty} A_n \),
$E_n(\varepsilon_i) = \{x \in X - C; |f_n(x) - f(x)| \geq \varepsilon_i\}$, where $\{\varepsilon_i\}_{i=1}^{\infty}$ is a sequence of positive numbers converging on 0. There exists a sequence $\{n_i\}_{i=1}^{\infty}$ such that $E_{n_i}(\varepsilon_i) \in N_{k_i}$, where $\{k_i\}_{i=1}^{\infty}$ is a sequence from Lemma 1, and $n_{i+1} > n_i$ for $i = 1, 2, \ldots$. Let us put $E_i = E_n(\varepsilon_i)$, $R_n = \bigcup_{i=n}^{\infty} E_i$, $Q = \bigcap_{n=1}^{\infty} R_n$. By Lemma 1, the sets $R_n$ belong to $N_n$ for $n = 1, 2, \ldots$. We assume that $Q \notin N$. There exists an $n_0$ such that $Q \notin N_{n_0}$, but $R_{n_0} \in N_{n_0}$ and $Q \in S$, so $Q \in N_{n_0} - a$ contradiction. Hence $Q \in N$. If $x_0 \in X - (Q \cup C)$, then there exists an $i_0$ such that $|f_n(x_0) - f(x_0)| < \varepsilon$, for each positive integer $i \geq i_0$. Since $Q \cup C \in N$, we obtain that $f_n \rightarrow f \text{ N-a.e. on } X$.

**Theorem 2.** If a sequence $\{f_n\}_{n=1}^{\infty}$ of functions defined on $X$ converges to $f$ with respect to a small system $N$, then this sequence converges to $f$ with respect to the $\sigma$-ideal $N = \bigcap_{r=1}^{\infty} N_r$.

**Proof.** By Remark 1, each subsequence $\{f_{n_m}\}_{m=1}^{\infty}$ of $\{f_n\}_{n=1}^{\infty}$ converges with respect to the small system $N$ to $f$ and, by Theorem 1, there exists a subsequence $\{f_{n_{m_p}}\}_{p=1}^{\infty}$ of $\{f_{n_m}\}_{m=1}^{\infty}$ converging to $f$ $N$-a.e. on $X$, so $f_n \rightarrow f$ $\text{ N-a.e. on } X$.

**Theorem 3.** Let $N$ be an upper semicontinuous small system. If a sequence $\{f_n\}_{n=1}^{\infty}$ of functions defined on $X$ converges to $f$ $N$-a.e. on $X$, then $\{f_n\}_{n=1}^{\infty}$ converges on $f$ with respect to the small system $N$.

**Proof.** Let $C$ be defined as in the proof of Theorem 1, $B = \{x; f_n(x) \rightarrow f(x)\}$, $Q = B \cup C$. Of course, $Q \in N$.

We have defined the sets $E_k(\varepsilon) = \{x; |f_k(x) - f(x)| \geq \varepsilon\}$ for $k = 1, 2, \ldots$ and each positive $\varepsilon$. Let $R_n(\varepsilon) = \bigcup_{k=n}^{\infty} E_k(\varepsilon)$, $M(\varepsilon) = \bigcap_{n=1}^{\infty} R_n(\varepsilon)$. If $x_0 \notin Q$, then lim $f_k(x_0) = f(x_0)$ and there exists a positive integer $n$ such that, for each positive $k \geq n$, $|f_k(x_0) - f(x_0)| < \varepsilon$. Hence $x_0 \in M(\varepsilon)$ and $x_0 \in M$; so $M \in Q$. We obtain that $M \in N$ because $M \in S$ and $Q \in N$.

Now suppose that the sequence $\{f_n\}_{n=1}^{\infty}$ does not converge to $f$ with respect to $N$. There exist a positive $\varepsilon_0$ and a positive integer $m_0$ such that, for every positive integer $i$, there exists a positive integer $k_i \geq i$ such that $E_{k_i}(\varepsilon_0) \notin N_{m_0}$. Hence $R_{k_i}(\varepsilon_0) \notin N_{m_0}$ for $i = 1, 2, \ldots$. The sequence $(R_{k_i}(\varepsilon_0))_{i=1}^{\infty}$ is a nonincreasing sequence of sets and the small system $N$ is upper semicontinuous, therefore $M \notin N$, which is a contradiction. We obtain that $f_n \rightarrow f.$

**Lemma 2.** If, for each subsequence $\{f_{n_m}\}_{m=1}^{\infty}$ of $\{f_n\}_{n=1}^{\infty}$, there exists a subsequence $\{f_{n_{m_p}}\}_{p=1}^{\infty}$ of $\{f_{n_m}\}_{m=1}^{\infty}$ such that $f_{n_{m_p}} \rightarrow f$, then $f_n \rightarrow f.$
Proof. Suppose that the sequence \( \{f_n\}_{n=1}^\infty \) does not converge to \( f \) with respect to \( \mathcal{N} \). Then there exist \( \varepsilon_0 > 0 \) and a positive integer \( m_0 \) such that, for each positive integer \( i \), there exists a positive integer \( n_i > i \) such that
\[
\{x : |f_{n_i}(x) - f(x)| \geq \varepsilon_0\} \notin N_{m_0}.
\]
By our assumption, there exists a subsequence \( \{f_{n_p}\}_{p=1}^\infty \) of \( \{f_n\}_{n=1}^\infty \) convergent to \( f \) with respect to \( \mathcal{N} \). Then there exists some \( p_{r_0} \) such that \( \{x : |f_{n_p}(x) - f(x)| \geq \varepsilon_0\} \notin N_{m_0} \) for \( p \geq p_{r_0} \), a contradiction.

**Theorem 4.** Let \( \mathcal{N} \) be an upper semicontinuous small system If a sequence \( \{f_n\}_{n=1}^\infty \) converges to \( f \) with respect to an \( \sigma \)-ideal \( N = \bigcap_{r=1}^\infty N_r \), then \( \{f_n\}_{n=1}^\infty \) converges to \( f \) with respect to the small system \( \mathcal{N} \).

**Proof.** Suppose that \( f_n \xrightarrow{n \to \infty} f \); then there exists a subsequence \( \{f_{n_p}\}_{p=1}^\infty \) with no subsequence converging to \( f \) with respect to \( \mathcal{N} \). By Theorem 3, no subsequence \( \{f_{n_p}\}_{p=1}^\infty \) converges to \( f \) \( N \)-a.e. — a contradiction.

**Corollary 1.** If \( \mathcal{N} \) is an upper semicontinuous small system, then both the convergence with respect to a small system \( \mathcal{N} \) and that with respect to a \( \sigma \)-ideal \( N = \bigcap_{r=1}^\infty N_r \), are equivalent.

**Remark 2.** If \( \mathcal{N} \) is not upper semicontinuous, then the above-mentioned convergences are not equivalent. The convergence with respect to an \( \sigma \)-ideal \( N = \bigcap_{r=1}^\infty N_r \), does not imply that with respect to \( \mathcal{N} \).

**Proof.** By the assumption there exists a nonincreasing sequence \( \{E_i\}_{i=1}^\infty \) of sets belonging to \( S \) and, for this sequence, there exists a positive integer \( r_0 \) such that, for each positive integer \( i \), \( E_i \notin N_{r_0} \) and \( \bigcap_{i=1}^\infty E_i \in N \). Let us put \( f_i(x) = X_{E_i}(x) \) and \( f(x) = 0 \) for \( x \in X \). If \( x \notin \bigcap_{i=1}^\infty E_i \), then \( f_i(x) \xrightarrow{i \to \infty} f(x) \). Hence \( f_i(x) \xrightarrow{i \to \infty} f(x) \) \( N \)-a.e. on \( X \), so \( f_i \xrightarrow{N} f \). Since, for each \( i \), \( \{x : |f_i(x) - f(x)| \geq \varepsilon\} = E_i \notin N_{r_1} \), where \( \varepsilon > 0 \), therefore \( f_i \xrightarrow{N} f \).

**Remark 3.** Let \( \mathcal{N} \), \( \mathcal{N}' \) be small systems such that \( N = \bigcap_{r=1}^\infty N_r = \bigcap_{r=1}^\infty N'_r \). The convergence with respect to \( \mathcal{N} \) need not be equivalent to the convergence with respect to \( \mathcal{N}' \).

**Proof.** Let \( (X, S, m) \) be a measurable space with the finite measure \( m \). We put \( N_r = \left\{ E \in S : m(E) < \frac{1}{r} \right\} \) for \( r = 1, 2, \ldots \). The convergence with respect to \( \mathcal{N} \)
is the convergence in measure $m$. The set $N = \bigcap_{r=1}^{\infty} N_r$ is the $\sigma$-ideal of sets of null measure. Let $N'_r = N$ for $r = 1, 2, \ldots$. The convergence with respect to $N'$ is the uniform convergence almost everywhere on $X$ (i.e. there exists a set $A \in N$ such that $f_n \Rightarrow f$ on $X - A$). It is easy to show when the convergences with respect to the different small systems are equivalent. For this reason, we recall the following definitions and theorems (see [2, p. 492—493]).

**Definition** [2]. Let $\mathcal{N}$, $\mathcal{M}$ be two small systems on $S$ and $N = \bigcap_{r=1}^{\infty} N_r$, $M = \bigcap_{r=1}^{\infty} M_r$. We say that $\mathcal{N}$ is the absolutely continuous system with respect to $\mathcal{M}$ if and only if, for each positive integer $r$, there exists a positive integer $n_r$ such that if $E \in M_{n_r}$, then $E \in N_r$. We use the notation $\mathcal{N} \ll \mathcal{M}$. We say that $\mathcal{N}$ is predominated by a system $\mathcal{M}$ if and only if $M \subseteq N$. We use the notation $\mathcal{N} \ll \mathcal{M}$.

**Theorem** [2]. If $\mathcal{N} \ll \mathcal{M}$, then $\mathcal{N} \ll \mathcal{M}$.

**Theorem** [2]. Let $\mathcal{N}$, $\mathcal{M}$ be two small systems on $S$. If $\mathcal{N}$ is the upper semicontinuous system, then the following conditions are equivalent

1. $\mathcal{N} \ll \mathcal{M}$
2. $\mathcal{N} \ll \mathcal{M}$.

**Example** 1. Let $N_r$ be the family of all measurable subsets of the set $\bigcup_{n=1}^{\infty} \left( n, n + \frac{1}{r + 3} \right)$ for $r = 1, 2, \ldots$. Let $M_r$ be the family of all measurable subsets of the set $\bigcup_{n=1}^{\infty} \left( n - \frac{1}{r + 3}, n \right)$ for $r = 1, 2, \ldots$.

The sequences $\{N_r\}$ and $\{M_r\}$ are small systems such that $\bigcap_{r=1}^{\infty} N_r = \bigcap_{r=1}^{\infty} M_r$, but $\mathcal{N}$ is not absolutely continuous with respect to $\mathcal{M}$ and $\mathcal{M}$ is not absolutely continuous with respect to $\mathcal{N}$.

**Theorem 5.** Let $\mathcal{N}$ and $\mathcal{M}$ be two small systems. The convergences with respect to these systems are equivalent if and only if $\mathcal{N} \ll \mathcal{M}$ and $\mathcal{M} \ll \mathcal{N}$.

**Proof.** Let there exist for each positive integer $m$ a positive integer $n_m$ such that if $E \in M_{n_m}$, then $E \in N_m$. Let $f_n \xrightarrow{\mathcal{M}} f$, then, for each $\varepsilon > 0$ and for each positive integer $m$, there exists a positive integer $n_{\varepsilon, m}$ such that

$$\{x: |f_n(x) - f(x)| \geq \varepsilon\} \subseteq M_{n_m}, \text{ for } n \geq n_{\varepsilon, m},$$

so $\{x: |f_n(x) - f(x)| \geq \varepsilon\} \subseteq N_m$ and $f_n \xrightarrow{\mathcal{N}} f$.

Now we suppose that $\mathcal{N}$ is not absolutely continuous with respect to $\mathcal{M}$. Then there exists a positive integer $n_0$ such that for each positive integer $n$ there exists a set $E_n$ such that $E_n \in M_n$ and $E_n \notin N_{n_0}$. Let $f_n(x) = X_{E_n}(x)$, $f(x) \equiv 0$ and
0 < \varepsilon < 1. We observe that for each \( n \) \( \{x : |f_n(x) - f(x)| \geq \varepsilon \} = E_n \notin N_{n_0} \) and \( f_n \xrightarrow{n \to \infty} f \) but for each positive integer \( m \) there exists \( n_m = m \) such that for \( n \geq n_m \) \( \{x : |f_n(x) - f(x)| \geq \varepsilon \} \in M_m \) and \( f_n \xrightarrow{n \to \infty} f \).

**Corollary 2.** If \( N \) is not predominated by \( \mathcal{M} \), then the convergence with respect to \( \mathcal{M} \) does not imply the convergence with respect to \( N \).

**Corollary 3.** If \( N, \mathcal{M} \) are upper semicontinuous small systems such that \( \bigcap_{r=1}^{\infty} N_r = \bigcap_{r=1}^{\infty} M_r \), then the convergences with respect to these systems are equivalent.

**Corollary 4.** If \( N \) is an \( \sigma \)-ideal, \( N_r = N \) for \( r = 1, 2, \ldots \) and \( \mathcal{N} \) is upper semicontinuous, and \( \mathcal{M} \) is a small system such that \( \bigcap_{r=1}^{\infty} M_r = N \), then \( \mathcal{M} \) is an upper semicontinuous system and the convergences with respect to those systems are equivalent.

Let \([f]\) denote the class of all \( S \)-measurable real functions equivalent to \( f \) and \( \mathfrak{M} \) denotes the family of these classes of equivalence.

**Definition 5.** We say that a sequence \( \{(f_n)_{n=1}^{\infty}\} \) of elements \( \mathfrak{M} \) converges to \( [f] \in \mathfrak{M} \) with respect to the small system \( \mathcal{N} \) (abbr. \( [f_n] \xrightarrow{\mathcal{N}} [f] \)) if and only if \( f_n \xrightarrow{n \to \infty} f \).

**Theorem 6.** The space \( \mathfrak{M} \) is equipped with the Fréchet topology generated by the convergence with respect to a small system \( \mathcal{N} \).

**Proof.** By Remark 1, Lemma 2 and Definition 5 the space \( \mathfrak{M} \) equipped with the convergence with respect to \( \mathcal{N} \) is an \( L^* \)-space ([1, p. 90]). We shall show that the following condition is fulfilled. By Definition 5 we can formulate this condition for representatives of suitable classes of equivalence.

If \( f_j \xrightarrow{j \to \infty} f \) and \( f_{j,n} \xrightarrow{n \to \infty} f_j \) for \( j = 1, 2, \ldots \), then there exist two sequences of positive integers \( \{j_p\}_{p=1}^{\infty}, \{n_p\}_{p=1}^{\infty} \) such that \( f_{j_p,n_p} \xrightarrow{p \to \infty} f \).

If the above-mentioned \( L^* \)-space fulfills this condition, then it is equipped with the Fréchet topology generated by the convergence with respect to \( \mathcal{N} \).

For each positive integer \( p \), there exists positive integers \( k_1^{(p)} \) and \( k_2^{(p)} \) such that if \( E_1 \in N_{k_1^{(p)}}, E_2 \in N_{k_2^{(p)}} \), then \( E_1 \cup E_2 \in N_p \).

For each positive integer \( p \), there exists a positive integer \( j_p \) such that, for each positive integer \( j \geq j_p \), \( \{x : |f_j(x) - f(x)| \geq \frac{1}{2p}\} \in N_{k_1^{(j)}} \). We may assume that \( j_{p+j} > j_p \) for \( p = 1, 2, \ldots \).
For each positive integer $p$, there exists a positive integer $n_p$ such that, for each positive integer $n \geq n_p$, \( \left\{ x : |f_{j_p,n}(x) - f_j(x)| \geq \frac{1}{2p} \right\} \in N_k(p) \). We may assume that $n_{p+1} > n_p$ for $p = 1, 2, \ldots$. Let $\epsilon > 0$ and $m$ a positive integer. Let us put $p_{\epsilon,m} = \max \left( \left\lfloor \frac{1}{\epsilon} \right\rfloor + 1, m \right)$; then, for each $p \geq p_{\epsilon,m}$, we have the following inclusions:

\[
\left\{ x : |f_{j_p,n}(x) - f_j(x)| \geq \epsilon \right\} \subseteq \left\{ x : |f_{j_p,n}(x) - f_j(x)| \geq \frac{1}{p} \right\} \subseteq \\
\left\{ x : |f_{j_p}(x) - f_j(x)| \geq \frac{1}{2p} \right\} \cup \left\{ x : |f_{j_p,n}(x) - f_j(x)| \geq \frac{1}{2p} \right\}.
\]

The last set belongs to $N_p$, thus it belongs to $N_m$.

The sets \( \left\{ x : |f_{j_p,n}(x) - f_j(x)| \geq \epsilon \right\} \) belong to $N_m$ for $p \geq p_{\epsilon,m}$ because they are subsets of sets belonging to $N_m$.

Since the above-mentioned condition is fulfilled, the proof is completed.

**Corollary 5.** Let $S$ be the class of sets having the Baire property and $N$ the class of sets of the first category; then there is no upper semicontinuous small system such that $N = \bigcap_{r=1}^{\infty} N_r$.

**Proof.** Suppose that there exists such a small system. Then the convergence with respect to it is equivalent to the convergence with respect to an $\sigma$-ideal $N$ and generates the Fréchet topology, a contradiction (see [4, p. 93]).

**REFERENCES**


Received August 9, 1985

Institute of Mathematics
University of Łódź
S. Banacha 22, POLAND.
В этой работе введено понятие сходимости по малых системах. Доказано, что сходимость по малых системах и сходимость по соответствующем $c$-идеале эквивалентны в случае полунепрерывной сверху малой системы.

Дальнее рассматривается пространство $\mathcal{M}$ всех классов эквивалентности $S$-измеримых действительных функций. Доказывается, что сходимость по малых системах преображает это пространство в топологическое пространство.