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ON TOTAL MATCHING NUMBERS AND TOTAL COVERING NUMBERS FOR k-UNIFORM HYPERGRAPHS

FRANTIŠEK OLEJNIK

In [3] P. Erdős and A. Meir investigate upper and lower bounds for $\alpha_2(G) + \alpha_2(\bar{G})$ and $\beta_2(G) + \beta_2(\bar{G})$, where $G$ is an undirected graph without loops and multiple edges and $\bar{G}$ is the complement of $G$. $\alpha_2(G)$ or $\alpha_2(\bar{G})$ is the total covering number of $G$ or $\bar{G}$ respectively and $\beta_2(G)$ or $\beta_2(\bar{G})$ is the total matching number of $G$ or $\bar{G}$ respectively. In this paper these results are generalized for $k$-uniform hypergraphs. First let us introduce the necessary notions.

(Cf. Berge [1].) By a hypergraph $H$ we mean a couple $(X, \mathcal{E})$, where $X$ is a finite set of elements called vertices and $\mathcal{E} = \{E_1, \ldots, E_m\}$ is a finite system of non-empty subsets of $X$ called edges, where $E_i \neq E_j$ for $i, j \in \{1, \ldots, m\}, i \neq j$.

A hypergraph is said to be $k$-uniform, $k > 1$, if all its edges have cardinality $k$. A $k$-uniform hypergraph with $n \geq k$ vertices is called complete if its set of edges has the cardinality $\binom{n}{k}$.

The complement of a $k$-uniform hypergraph $H = (X, \mathcal{E})$ is the hypergraph $\bar{H} = (X, \bar{\mathcal{E}})$ if $|\mathcal{E} \cup \bar{\mathcal{E}}| = \binom{n}{k}$ and $\mathcal{E} \cap \bar{\mathcal{E}} = \emptyset$. ($|\mathcal{E} \cup \bar{\mathcal{E}}|$ denotes the cardinality of the set $\mathcal{E} \cup \bar{\mathcal{E}}$.)

A hypergraph $H(N) = (X, \mathcal{E}_N)$ is said to be a $k$-uniform subhypergraph of a $k$-uniform hypergraph $H = (X, \mathcal{E})$ induced by a set $N$ if $N \subseteq X$ and $\mathcal{E}_N$ is the system of all edges $E_i \in \mathcal{E}$ such that $E_i \subseteq N$.

A vertex $x$ of a $k$-uniform hypergraph $H$ is said to cover itself, all edges incident with $x$ and all vertices adjacent to $x$. An edge $E_i$ of a $k$-uniform hypergraph $H$ covers itself, the vertices incident with $E_i$ and all edges adjacent to $E_i$.

A subset $P$ of elements of $X \cup \mathcal{E}$ is called a total covering of $H = (X, \mathcal{E})$ if the elements of $P$ cover $H$ and $P$ is a minimal set with this property.

Two elements of the set $X \cup \mathcal{E}$ are called strongly independent if they do not cover each other. A subset $F$ of $X \cup \mathcal{E}$ is called a strong total matching if elements of $F$ are pairwise strongly independent and $F$ is maximal.
A subset $N$ of $X$ is called stable if for each edge $E_i \in \mathcal{E}$, $|E_i \cap N| \leq k - 1$. A subset $S$ of $X$ is called strongly stable if for each edge $E_i$, $|E_i \cap S| \leq 1$.

A subset $T$ of $X \cup \mathcal{E}$ is said to be a weak total matching if $T$ is maximal and has the following properties:

1° The elements of $T \cap \mathcal{E}$ are pairwise independent (disjoint)
2° No element of $T \cap \mathcal{E}$ covers an element of $T \cap X$
3° The elements of $T \cap X$ form a stable set of $H$.

The cardinality of a minimum set which is a total covering of $H$ is called the total covering number $\alpha_2(H)$ of $H$.

The cardinality of a maximum strong total matching of $H$ is called the strong total matching number $\beta_2(H)$ of $H$.

The cardinality of a maximum weak total matching of $H$ is called the weak total matching number $\gamma_2(H)$ of $H$.

The cardinality of a maximum stable set of $H$ is called the stability number $\alpha(H)$ of $H$.

The cardinality of a maximum strong stable set of $H$ is called the strong stability number $\alpha_0(H)$ of $H$.

In the sequel we suppose that $n \geq k \geq 3$.

**Theorem 1.** For a $k$-uniform hypergraph $H = \langle X, \mathcal{E} \rangle$ with $n$ vertices and its complement $\bar{H}$

\[
\left\lfloor \frac{n}{k} \right\rfloor + 2 \leq \beta_2(H) + \beta_2(\bar{H}) \leq \left\lfloor \frac{(k+1)n}{k} \right\rfloor
\]

holds.

**Proof.** Let $F$ or $\bar{F}$ be a strong total matching of $H$ or $\bar{H}$ with cardinality $\beta_2(H)$ or $\beta_2(\bar{H})$ respectively. Let $F = F_x \cup F_y$ and $\bar{F} = \bar{F}_x \cup \bar{F}_y$, where $F_x$ or $\bar{F}_x$ is a set of vertices of $F$ or $\bar{F}$ respectively and $F_y$ or $\bar{F}_y$ is a set edges of $F$ or $\bar{F}$ respectively. Thus $\beta_2(H) = |F_x| + |F_y|$ and $\beta_2(\bar{H}) = |\bar{F}_x| + |\bar{F}_y|$ holds.

Let $V(F_x)$ or $V(\bar{F}_y)$ be the set of vertices incident with edges of $F_x$ or $\bar{F}_y$ respectively. Without loss of generality we can suppose that the sets $F_x$ or $\bar{F}_y$ are maximal independent sets of $H$ or $\bar{H}$ respectively, so that the subhypergraphs $H \langle X - V(F_x) \rangle$ and $\bar{H} \langle X - V(\bar{F}_y) \rangle$ have no edges.

A. We prove the upper bound from Theorem 1.

\[
\beta_2(H) = |F_x| + |F_y|
\]

holds, thus

\[
\beta_2(\bar{H}) \leq \left\lfloor \frac{|X - V(F_x)|}{k} \right\rfloor + k|F_y|.
\]

Then

\[
\beta_2(H) + \beta_2(\bar{H}) \leq |F_x| + |F_y| + \left\lfloor \frac{n - k|F_y|}{k} \right\rfloor + k|F_y|.
\]

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Since

\[ |F_x| + k|F_y| \leq n, \]

\[ \beta_2(H) + \beta_2(\tilde{H}) \leq n + \left\lfloor \frac{n}{k} \right\rfloor = \frac{(k+1)n}{k} \]

holds.

B. We prove the lower bound from Theorem 1.

For \( k \leq n \leq 2k \) the theorem holds.

Let \( n > 2k \).

Since \( H(X - V(F_y)) \) or \( \tilde{H}(X - V(\tilde{F}_y)) \) are empty subhypergraphs of \( H \) or \( \tilde{H} \) respectively,

\[ |F_x| + |\tilde{F}_x| \geq \left\lceil \frac{n}{k} \right\rceil \]  \hspace{1cm} (2)

holds.

Let us analyse five possibilities:

I. If \(|F_y| = \left\lfloor \frac{n}{k} \right\rfloor \) and \( n \equiv 0 \pmod{k} \), then \( \beta_2(\tilde{H}) \geq 2 \), thus the assertion of the theorem holds.

II. If \(|F_y| = \left\lfloor \frac{n}{k} \right\rfloor \) and \( n \not\equiv 0 \pmod{k} \), then \( |F_x| \geq 1 \) and \( \beta_2(\tilde{H}) \geq 2 \), thus the assertion of the theorem holds.

III. If \( 0 < |F_y| < \left\lfloor \frac{n}{k} \right\rfloor \) and \( n \equiv 0 \pmod{k} \), then \( |F_x| \geq 1 \) and \( |\tilde{F}_x| + |\tilde{F}_y| \geq \left\lceil \frac{n}{k} \right\rceil - |F_y| + 1 \), thus the assertion of the theorem holds.

IV. If \( 0 < |F_y| < \left\lfloor \frac{n}{k} \right\rfloor \) and \( n \not\equiv 0 \pmod{k} \) and \( |F_x| + |\tilde{F}_y| \geq \left\lceil \frac{n}{k} \right\rceil \), then \( |F_x| \geq 1 \), \( |\tilde{F}_x| \geq 1 \), thus the assertion of the theorem holds.

V. Let \( 0 < |F_y| < \left\lfloor \frac{n}{k} \right\rfloor \) and \( n \not\equiv 0 \pmod{k} \) and

\[ |F_y| + |\tilde{F}_y| = \left\lceil \frac{n}{k} \right\rceil, \]  \hspace{1cm} (3)

then

\[ |F_x| + |\tilde{F}_x| > 2. \]

Suppose in fact the assertion does not hold.

Then

\[ |F_x| + |\tilde{F}_x| = 2, \quad \text{(i.e. } |F_x| = 1, \ |\tilde{F}_x| = 1). \]  \hspace{1cm} (4)
We will show that the hypergraph satisfying both the hypotheses of V and (4) does not exist. We can suppose that the sets $V(F_y)$ and $V(\tilde{F}_y)$ are disjoint, because $H(X - V(F_y))$ has no edges, hence as a maximal set of disjoint edges of $\tilde{H}(X - V(F_y))$ we can consider $\tilde{F}_y$.

Let $N = X - V(F_y) - V(\tilde{F}_y)$.

The hypergraph satisfying both the hypotheses of V and (4) must have the following properties:

(a) $0 < |N| \leq k - 1$, because $|V(F_y)| + |V(\tilde{F}_y)| = k \left\lceil \frac{n}{k} \right\rceil$ and $|N| = |X - V(F_y) - V(\tilde{F}_y)| = n - k \left\lceil \frac{n}{k} \right\rceil$.

(b) $\tilde{H}(V(F_y) \cup N)$ or $\tilde{H}(V(\tilde{F}_y) \cup N)$ is a complete subhypergraph of $H$ or $\tilde{H}$ respectively.

If $\tilde{H}(V(F_y) \cup N)$ is not complete, then in $\tilde{H}(V(F_y) \cup N)$ there exists at least one edge, which is a contradiction to (3).

(c) Each vertex of $X$ covers all vertices of both $H$ and $\tilde{H}$. Let there exist vertices $x_1$, $x_2$, which are not incident in $H$. From (b) it follows that in the set $N$ all vertices are incident, i.e.

(i) $x_1 \in V(F_y)$ and $x_2 \in V(\tilde{F}_y)$, or

(ii) $x_1, x_2 \in V(\tilde{F}_y)$

(in the case $x_1 \in N$ and $x_2 \in V(\tilde{F}_y)$ there would be a contradiction to $|F_y| = 1$).

In case (i) all edges containing the vertices $x_1, x_2$ are in $\tilde{H}$. Let us take such an edge $E$ from $\tilde{H}$, which has $(k - 1)$ vertices in the set $V(F_y)$. From (b) it follows that in $\tilde{H}(V(\tilde{F}_y) \cup N - \{x_2\})$ there exists an independent set of edges $\tilde{F}_y$, for which $|\tilde{F}_y| = |\tilde{F}_y|$. But in $\tilde{H}$ we can add an edge $E$ to $\tilde{F}_y$, and obtain an independent set $\tilde{F}_y$, whose cardinality is $|\tilde{F}_y| = |\tilde{F}_y| + 1$. Then $|F_y| + |\tilde{F}_y| > \left\lceil \frac{n}{k} \right\rceil$, which is a contradiction to (3). In case (ii) we take $F_z = \{x_1, x_2\}$, which is a contradiction to $|F_y| = 1$.

(d) Each vertex of $V(F_y) \cup N$ forms an edge with arbitrary $(k - 1)$ vertices of $V(\tilde{F}_y)$ in $\tilde{H}$. Otherwise there exist $(k - 1)$ vertices $x_2, \ldots, x_k$ in $V(\tilde{F}_y)$ and $x_0 \in V(F_y) \cup N$, that $\{x_0, x_2, \ldots, x_k\}$ forms an edge in $H$. But in $H(V(F_y) \cup N - \{x_0\})$ there exists an independent set of edges of cardinality $|F_y|$ and thus in $H$ there exist a set of disjoint edges of cardinality $|F_y| + 1$, which is a contradiction to (3).

(e) Each vertex of $V(\tilde{F}_y) \cup N$ forms an edge with arbitrary $(k - 1)$ vertices of $V(F_y)$ in the hypergraph $H$, which follows from an analogous consideration to that in (d).

For $k = 3$, (c), (d), (e) and (3) can not hold the same time, thus for a 3-uniform hypergraph satisfying the condition from V the Theorem 1 holds.

Let $k \geq 4$. By induction we will prove an assertion (A):
(A) In a hypergraph \( H \) which satisfies (3) and (4), there does not exist an edge which has exactly \( i \) vertices in \( V(F_i) \), for \( i = 2, 3, \ldots, k - 1 \). This will be a contradiction to (e), because according to (e) each edge exactly \( (k - 1) \) of whose vertices are in \( V(F_i) \) must belong to \( H \).

Proof of (A):

1. Let \( i = 2 \). Let there exist an edge \( E_1 \) in \( H \) such that \( |E_1 \cap V(F_2)| = 2 \). According to (d), in \( \tilde{H} \) there exists an edge \( E_2 \) such that \( |E_1 \cap E_2 \cap V(F_2)| = 1 \) and \( |E_1 \cap E_2| = k - 1 \). Let us consider a set of vertices \( R \subseteq V(F_2) \cup V(\tilde{F}_2) \) such that \( |R \cap V(F_2)| = k - 2 \), \( |R \cap V(\tilde{F}_2)| = 2 \), \( R \cap E_1 = \emptyset \) and \( |R \cap E_2| = 1 \). Subhypergraph \( \tilde{H} \langle R \cup N \rangle \) does not contain any edge (otherwise in \( \tilde{H} \langle R \cup V(F_2) \cup E_1 \rangle \) there exists an independent set of edges of cardinality \( |F_2| + 1 \) which is a contradiction with (3)), and so \( \tilde{H} \langle R \cup N \rangle \) is a complete subhypergraph of \( \tilde{H} \). But in this case \( \tilde{H} \langle R \cup N \cup V(F_j) \cup E_2 \rangle \) contains an independent set of edges of cardinality at least \( |F_j| + 1 \), which is a contradiction to (3). Let \( v \in N \). Then the set of vertices \( E_3 = (R - E_2) \cup \{ v \} \) forms an edge in \( \tilde{H} \) and \( E_3 \cap E_2 = \emptyset \), which is a contradiction to (3), thus for \( i = 2 \) the assertion (A) holds.

2. Suppose that for \( i = r \), \( 2 < r \leq k - 2 \), the assertion (A) holds and for \( i = r + 1 \) it does not hold, then in \( H \) there exists an edge \( E_1 \) such that \( |E_1 \cap V(F_r)| = r + 1 \). According to the induction assumption there exists in \( \tilde{H} \) an edge \( E_2 \) such that \( |E_1 \cap E_2| = k - 1 \) and \( |E_1 \cap E_2 \cap V(F_r)| = r \). Let us consider a set of vertices \( R \subseteq V(F_r) \cup V(\tilde{F}_r) \) for which \( |R \cap V(F_r)| = k - r \), \( |R \cap V(\tilde{F}_r)| = r \), \( R \cap E_1 = \emptyset \) and \( |R \cap E_2| = 1 \). Then \( \tilde{H} \langle R \cup N \rangle \) is a complete subhypergraph of \( \tilde{H} \), otherwise we have a contradiction to (3). But in this case \( \tilde{H} \langle R \cup N \cup V(F_r) \cup E_2 \rangle \) contains an independent set of edges of cardinality at least \( |F_r| + 1 \), which is a contradiction to (3). Thus the auxiliary assertion is proved.

From (A) it follows for \( i = k - 1 \) that in \( H \) there does not exist any edge \( E \) for which \( |E \cap V(F_k)| = k - 1 \), which is a contradiction to (e). This completes the proof of the assertion for case V and therefore also of Theorem 1.

Remark. The equality in the upper bound (1) holds for an arbitrary complete \( k \)-uniform hypergraph.

The equality in the lower bound (1) holds, e.g., for \( H = \langle X, \mathcal{E} \rangle \) with the following structure:

1° There exists a vertex \( x \in X \) such that \( H \langle X - \{ x \} \rangle \) is a complete subhypergraph of \( H \).

2° In \( H \) there exist exactly \( \left[ \frac{n - 1}{k - 1} \right] \) edges containing a vertex \( x \), among which there exist \( \left[ \frac{n - 1}{k - 1} \right] \) edges such that any two edges have in common exactly the vertex \( x \).

3° The vertex \( x \) is adjacent to all vertices of \( H \).
For such a hypergraph $H$

$$\beta_2(H) = \left\lfloor \frac{n}{k} \right\rfloor$$

and $\beta_2(\bar{H}) = 2$ holds.

This means that the upper and lower bounds (1) are the best possible.

**Theorem 2.** For a $k$-uniform hypergraph $H = \langle X, \mathcal{E} \rangle$ and its complement $\bar{H} = \langle X, \bar{\mathcal{E}} \rangle$

$$\left\lfloor \frac{n-1}{k} \right\rfloor + 1 \leq \alpha_2(H) + \alpha_2(\bar{H}) \leq \left\lfloor \frac{(k+1)n}{k} \right\rfloor$$

(5)

holds.

**Proof.** The upper bound in (5) follows from the inequality

$$\alpha_2(H) \leq \beta_2(H), \quad \alpha_2(\bar{H}) \leq \beta_2(\bar{H})$$

and from Theorem 1.

Let $P = P_x \cup P_y$ be a total covering of $H$, where $P_x$ is a set of vertices and $P_y$ is a set of edges.

If $|P_x| = 0$, then $n \leq k|P_y|$, thus

$$\alpha_2(H) = |P_y| \geq \left\lfloor \frac{n}{k} \right\rfloor.$$

As $\alpha_2(\bar{H}) \geq 1$, the lower bound in (5) is satisfied.

Let $|P_x| \geq 1$. Let us denote $N = X - P_x - V(P_y)$. If $|N| \leq k - 1$, then

$$\alpha_2(H) = |P_x| + |P_y| \geq |P_x| + \frac{|V(P_y)|}{k} \geq \frac{k|P_x|}{k} + \frac{|V(P_y)|}{k} =$$

$$= \left\lfloor \frac{k|P_x| + |V(P_y)|}{k} \right\rfloor \geq \left\lfloor \frac{n}{k} \right\rfloor.$$

holds and $\alpha_2(\bar{H}) \geq 1$, thus the lower bound in (5) is satisfied.

If $|N| \geq k$, then $\bar{H}(N)$ is a complete $k$-uniform subhypergraph of $\bar{H}$, thus

$$\alpha_2(\bar{H}) \geq \left\lfloor \frac{|N|}{k} \right\rfloor.$$

It follows that

$$\alpha_2(H) + \alpha_2(\bar{H}) \geq |P_x| + |P_y| + \left\lfloor \frac{|N|}{k} \right\rfloor = \frac{1}{k} \left( |P_x| + k|P_y| + |N| + (k - 1)|P_x| \right).$$

$$|P_x| + k|P_y| + |N| \geq n,$$
\[ \alpha_2(H) + \alpha_2(\bar{H}) \geq \frac{1}{k} (n + (k - 1)|P_1|) \geq \frac{n - |P_1|}{k} |P_1| + |P_1| \geq \frac{n - 1}{k} + 1. \]

The proof of Theorem 2 is now complete.

Remark. The equality in the upper bound (5) holds for an arbitrary complete \( k \)-uniform hypergraph.

The equality in the lower bound (5) holds, e.g., for \( H = \langle X, \emptyset \rangle \) with the following structure:

1° There exists a vertex \( x \in X \) such that the subhypergraph \( H \langle X - \{x\} \rangle \) is complete.

2° The vertex \( x \) is incident with exactly one edge of \( H \).

For such a hypergraph \( H \)

\[ \alpha_2(H) = \frac{n - 1}{k} \quad \text{and } \quad \alpha_2(\bar{H}) = 1 \]

holds. This shows that the upper and lower bounds in (5) are the best possible ones.

**Lemma 1.** For a \( k \)-uniform hypergraph \( H = \langle X, \emptyset \rangle \) and its complement \( \bar{H} = \langle X, \emptyset \rangle \)

\[ \alpha(H) + \alpha(\bar{H}) \leq n + k - 1 \]

holds. \( \text{Proof.} \) Let \( \alpha(H) = r \). Then in \( \bar{H} \) there exists a complete subhypergraph with \( r \) vertices, thus \( \alpha(\bar{H}) \leq n - r + k - 1 \). From this, the assertion of the lemma follows.

**Theorem 3.** For a \( k \)-uniform hypergraph \( H = \langle X, \emptyset \rangle \) and its complement \( \bar{H} = \langle X, \emptyset \rangle \)

\[ \gamma_2(H) + \gamma_2(\bar{H}) \leq \left[ \frac{(k + 1)n + 1}{k} \right] + k - 2 \]

holds. \( \text{Proof.} \) Let \( \alpha(H) \) be the cardinality of the greatest stable set of vertices in \( H \). Then

\[ \gamma_2(H) \leq \alpha(H) + \left[ \frac{n - \alpha(H)}{k} \right] \]

holds. Also

\[ \gamma_2(\bar{H}) \leq \alpha(\bar{H}) + \left[ \frac{n - \alpha(\bar{H})}{k} \right] \]

holds. After the addition of these inequalities we get

\[ \gamma_2(H) + \gamma_2(\bar{H}) \leq \alpha(H) + \alpha(\bar{H}) + \left[ \frac{2n - (\alpha(H) + \alpha(\bar{H}))}{k} \right]. \]
By using Lemma 1 we get
\[ \gamma_2(H) + \gamma_2(\bar{H}) \leq n + k - 1 + \left\lfloor \frac{n - k + 1}{k} \right\rfloor, \]
after appropriate modifications we get the assertion of Theorem 3.

Remark. The equality in (7) holds for an arbitrary complete \( k \)-uniform hypergraph \( H \).

A \( k \)-uniform hypergraph \( H = \langle X, \mathcal{E} \rangle \) is connected if for each non-empty set of vertices \( S \subseteq X \) the following holds: \( \mathcal{E}_1 \cup \mathcal{E}_2 \neq \mathcal{E} \), where \( \mathcal{E}_1 \) or \( \mathcal{E}_2 \) is a set of edges of the subhypergraph \( H(S) \) or \( H(X - S) \), respectively.

**Lemma 2.** For a connected \( k \)-uniform hypergraph \( H = \langle X, \mathcal{E} \rangle \)
\[ \alpha_2(H) \leq \left\lfloor \frac{n}{2} \right\rfloor \]  
holds.

Proof. From a hypergraph \( H = \langle X, \mathcal{E} \rangle \) we construct an undirected graph \( G = \langle X, E \rangle \) without loops or multiple edges, by which the vertices \( x_i, x_j \in X \) form the edge in \( G \), if in \( H \) there exists at least one edge which contains them. \( G \) is connected and \( \alpha_2(H) \leq \alpha_2(G) \). For a connected graph with \( n \) vertices, the inequality
\[ \alpha_2(G) \leq \left\lfloor \frac{n}{2} \right\rfloor \]  
holds [2]. From this, the assertion of Lemma 2 follows.

**Lemma 3.** For a connected \( k \)-uniform hypergraph \( H = \langle X, \mathcal{E} \rangle \)
\[ \alpha_2(H) \leq n - \alpha_0(H) + 2 - k \]  
\[ \beta_2(H) \leq \alpha_0(H) + \frac{n - \alpha_0(H)}{k} \]  
\[ \alpha_2(H) \leq n - \alpha(H) \]  
\[ \gamma_2(H) \leq \alpha(H) + \frac{n - \alpha(H)}{k} \]  
holds.

Proof. The above follows directly from the definition of the characteristic numbers treated and from the connectivity of \( H \).

**Theorem 4.** For a connected \( k \)-uniform hypergraph \( H = \langle X, \mathcal{E} \rangle \)
\[ \alpha_2(H) + \beta_2(H) \leq n + \left\lfloor \frac{1}{k} \left( \left\lfloor \frac{n}{2} \right\rfloor - 2 \right) \right\rfloor + 3 - k \]  
(13)

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\[
\alpha_2(H) + \gamma_2(H) \leq n + \left[ \frac{1}{k} \right] \frac{n}{2} \tag{14}
\]
\[
\beta_2(H) + \gamma_2(H) \leq 2n - k \tag{15}
\]
holds.

Proof. From (10) it follows that
\[
\alpha_0(H) \geq \frac{k\beta_2(H) - n}{k - 1}
\]
and after substitution into (9) we get
\[
\alpha_2(H) \leq n - \frac{k\beta_2(H) - n}{k - 1} + 2 - k,
\]
and further
\[
\alpha_2(H) + \beta_2(H) \leq n - \frac{\alpha_2(H)}{k} + 3 - k + \frac{2}{k}.
\]

After substitution for \( \alpha_2(H) \) from (8) we get the assertion (13). From (11) and (12) it follows that
\[
\alpha_2(H) \leq n - \frac{k\gamma_2(H) - n}{k - 1}
\]
and after a modification we get
\[
\alpha_2(H) + \gamma_2(H) \leq n + \frac{\alpha_2(H)}{k}.
\]

From this and (8) we get the assertion (14).

For the connected hypergraph \( H \)
\[
\beta_2(H) \leq n - k + 1
\]
\[
\gamma_2(H) \leq n - 1.
\]

After addition we get the assertion (15).

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О ЧИСЛЕ ТОТАЛЬНОЙ НЕЗАВИСИМОСТИ И ТОТАЛЬНОГО ПОКРЫТИЯ
ДЛЯ $k$-УНИФОРМНЫХ ГИПЕРГРАФОВ

František Olejník

Резюме

В этой работе приведены верхние и нижние оценки для суммы числа сильной тотальной независимости, (числа слабой тотальной независимости, числа тотального покрытия) для $k$-униформного гиперграфа $H$ и его дополнения $\bar{H}$.