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OSCILLATION THEOREMS FOR SECOND ORDER NONLINEAR DELAY INEQUALITIES

JÁN SEMAN

Consider the differential inequality

$$x(t)[(r(t)x'(t))' + f(t, x(t), x(g(t)))] \leqslant 0 \tag{1}$$

and the corresponding differential equation

$$(r(t)x'(t))' + f(t, x(t), x(g(t))) = 0 \tag{2}$$

on some $\langle t_0, \infty) \subseteq (0, \infty)$, where the functions r, g, f satisfy the assumptions

(i) $r \in C\langle t_0, \infty)$, $r(t) > 0$ for $t \geqslant t_0$ and $\lim_{t\to\infty} R(t) = \infty$, (3)

where $R(t) = \int_{t_0}^{t} \frac{\mathrm{d}s}{r(s)}$ for $t \geqslant t_0$,

(ii) $f \in C(\langle t_0, \infty) \times R \times R)$, $f(t, x_1, y_1) \geqslant f(t, x_2, y_2) \geqslant 0$ for $t \geqslant t_0$, $x_1 \geqslant x_2 \geqslant 0$, $y_1 \geqslant y_2 \geqslant 0$ and the function $h(t, x, y) = -f(t, -x, -y)$ has the same properties,

(iii) $g \in C\langle t_0, \infty)$, $0 < g(t) \leqslant t$ for $t \geqslant t_0$ and $\lim_{t\to\infty} g(t) = \infty$.

We shall also consider the differential equation

$$(r(t)x'(t))' + a(t)F(x(g(t))) = 0, \tag{4}$$

where the functions r, g satisfy the assumptions (3)(i) and (3)(iii) and the functions a, F satisfy

(i) $a \in C\langle t_0, \infty)$ and $a(t) \geqslant 0$ for $t \geqslant t_0$, (5)

(ii) $F \in C(R)$ is nondecreasing and $xF(x) > 0$ for $x \neq 0$.

We shall consider only these solutions of (1) defined on some $\langle t_1, \infty) \subseteq \subseteq \langle t_0, \infty)$ such that $\sup\{t \geqslant t_1, x(t) \neq 0\} = \infty$. Such solution x of (1) is said to be *oscillatory* if $\sup\{t \geqslant t_1, x(t) = 0\} = \infty$, otherwise it is said to be *nonoscillatory*. The inequality (1) is said to be *oscillatory* if it has only oscillatory solutions, otherwise it is said to be *nonoscillatory*. The same definitions can hold for the equations (2) and (4).

In this paper we shall show that the oscillatoriness of (1) and (2) is equivalent, then we shall prove some analogy of Sturm's comparison theorem and give some oscillatory criteria for the equations (2) and (4). These results will be the generalization of those given in [1] and [2], where the authors assumed $r(t) = 1$ or $r(t)$ to be bounded. Some of our results will be better even in these cases.

In the proofs of the existence of the nonoscillatory solutions of the equation (2) or (4) we shall use one very simple algebraic fixed point theorem.

Fixed point theorem. *Let Y be a complete lattice and $\Phi : Y \to Y$ be an isotonous operator. Then Φ has at least one fixed point in Y.*

Proof. See the theorem II.3.3 in [3].

Lemma 1. *Let the assumptions* (3) *hold and x be the nonoscillatory solution of* (1). *Then there exists $t_1 \geqslant t_0$ such that*

$$x(t)\,x(g(t)) > 0, \quad x(t)\,x'(t) \geqslant 0, \quad x(t)\,x'(g(t)) \geqslant 0 \qquad \textit{for } t \geqslant t_1.$$

The same is valid for the equations (2) *and* (4).

Proof. Suppose that $x(t) > 0$ for all sufficiently large t (the proof for $x(t) < < 0$ is analogous). Then there exists $t_1 \geqslant t_0$ such that $x(t) > 0$ and $x(g(t)) > 0$ for $t \geqslant t_1$. Then by (1) $(r(t)\,x'(t))' \leqslant 0$ and the function $r(t)\,x'(t)$ is nonincreasing in $\langle t_1, \infty)$. If there exists $t_2 \geqslant t_1$ such that $r(t)\,x'(t) \leqslant r(t_2)\,x'(t_2) < 0$ for $t \geqslant t_2$, then dividing this inequality by $r(t)$ and integrating it from t_2 to $t \geqslant t_2$ we get

$$x(t) \leqslant x(t_2) + r(t_2)\,x'(t_2) \int_{t_2}^{t} \frac{\mathrm{d}s}{r(s)} \qquad \text{for } t \geqslant t_2$$

and the assumption (3)(i) leads to the contradiction with the positivity of x. Hence $t_1 \geqslant t_0$ can be chosen so that $x'(t) \geqslant 0$ and $x'(g(t)) \geqslant 0$ for $t \geqslant t_1$. The proof for the equations (2) and (4) is analogous.

Lemma 2. *Let the assumptions* (3) *hold and there exists the function x defined positive and nondecreasing on some $\langle t_1, \infty) \subseteq \langle t_0, \infty)$ and such that*

$$x(t) \geqslant x(t_2) + \int_{t_2}^{t} \frac{1}{r(s)} \int_{s}^{\infty} f(\xi, x(\xi), x(g(\xi)))\,\mathrm{d}\xi\,\mathrm{d}s \qquad \textit{for } t \geqslant t_2, \tag{6}$$

where $t_2 \geqslant t_1$ is such that $g(t) \geqslant t_1$ for $t \geqslant t_2$. Then the equation (2) *has at least one nonoscillatory solution y such that* $0 < y(t) \leqslant x(t)$ *for $t \geqslant t_1$. The same is valid for the equation* (4) *changing the assumption* (3)(ii) *by* (5) *and $f(\xi, x(\xi), x(g(\xi)))$ in* (6) *by $a(\xi)\,F(x(g(\xi)))$.*

Proof. Define Y as the set of all functions y defined, positive and nondecreasing on $\langle t_1, \infty)$ and such that $y(t) = x(t)$ for $t \in \langle t_1, t_2\rangle$ and $y(t) \leqslant x(t)$ for $t \geqslant t_2$, with the obvious point-wise ordering. Then, clearly, Y is the complete lattice. Define the operator Φ by a form

$$(\Phi y)(t) = y(t) \qquad \text{for } t \in \langle t_1, t_2\rangle, \tag{7}$$

$$(\Phi y)(t) = y(t_2) + \int_{t_2}^{t} \frac{1}{r(s)} \int_{s}^{\infty} f(\xi, y(\xi), y(g(\xi)))\, d\xi\, ds \qquad \text{for } t \geqslant t_2 \text{ and } y \in Y.$$

Using the assumptions (3)(ii) and (6) we can easily show that $\Phi Y \subseteq Y$ and Φ is isotonous. Then by the fixed point theorem there exists $y \in Y$ such that $y = \Phi y$. By the definition (7) such y is the neede nonoscillatory solution of (2).

Theorem 1. *Let the assumptions* (3) *hold. Then the inequality* (1) *is oscillatory if and only if the equation* (2) *is.*

Proof. It remains to prove that the existence of the nonoscillatory solution of (1) implies the same for (2). Let x be the nonoscillatory solution of (1) and, without loss of generality, $x(t) > 0$, $x(g(t)) > 0$, $x'(t) \geqslant 0$ and $x'(g(t)) \geqslant 0$ for $t \geqslant t_1 \geqslant t_0$. Then integrating (1) from t to $s \geqslant t \geqslant t_1$ we get

$$r(t)\, x'(t) \geqslant r(s)\, x'(s) + \int_{t}^{s} f(\xi, x(\xi), x(g(\xi)))\, d\xi$$

and from this

$$r(t)\, x'(t) \geqslant \int_{t}^{\infty} f(s, x(s), x(g(s)))\, ds \qquad \text{for } t \geqslant t_1 .$$

Dividing this inequality by $r(t)$ and integrating it from t_2 to $t \geqslant t_2$, where $t_2 \geqslant t_1$ is such that $g(t) \geqslant t_1$ for $t \geqslant t_2$, we can obtain the inequality (6) in lemma 2 and the application of this lemma completes the proof.

Remark 1. With regard to the theorem just proved we can consider in the sequel only the equatins (2) and (4).

Theorem 2. *Let the functions r_i, g_i, f_i satisfy the assumptions* (3) *for $i = 1, 2$ and*

$$r_1(t) \geqslant r_2(t), \quad g_1(t) \leqslant g_2(t), \quad |f_1(t, x, y)| \leqslant |f_2(t, x, y)| \tag{8}$$

for $t \geqslant t_0$ and $xy > 0$.

If the equation

$$(r_1(t)\, x'(t))' + f_1(t, x(t), x(g_1(t))) = 0 \tag{2_1}$$

is oscillatory, then the equation

$$(r_2(t)\, x'(t))' + f_2(t, x(t), x(g_2(t))) = 0 \tag{2_2}$$

is oscillatory, too.

Proof. Suppose to the contrary that (2_2) has the nonoscillatory solution x and, without loss of generality, that $x(t) > 0$, $x(g_1(t)) > 0$, $x'(t) \geqslant 0$ and $x'(g_1(t)) \geqslant 0$ for $t \geqslant t_1$. Let $t_2 \geqslant t_1$ be such that $g_2(t) \geqslant g_1(t) \geqslant t_1$ for $t \geqslant t_2$. In the same way as in the proof of the theorem 1 we can obtain that

$$x(t) \geqslant x(t_2) + \int_{t_2}^{t} \frac{1}{r_2(s)} \int_{s}^{\infty} f_2(\xi, x(\xi), x(g_2(\xi)))\, d\xi\, ds$$

and using the assumptions (3) for (2_2) and (8) that

$$x(t) \geqslant x(t_2) + \int_{t_2}^{t} \frac{1}{r_1(s)} \int_{s}^{\infty} f_1(\xi, x(\xi), x(g_1(\xi)))\,\mathrm{d}\xi\,\mathrm{d}s$$

for $t \geqslant t_2$. Then by lemma 2 the equation (2_1) is nonoscillatory. This contradiction completes the proof.

Theorem 3. *Let the assumptions* (3) *hold. Then the condition*

$$\left| \int_{t_0}^{\infty} R(t) f(t, a, a)\,\mathrm{d}t \right| = \infty \qquad \textit{for any } a \neq 0 \tag{9}$$

is sufficient and neccessary for the equation (2) *not to have any bounded nonoscillatory solution.*

Proof. To prove the sufficiency of (9) suppose to the contrary that there exists the bounded nonoscillatory solution x of (2) and, without loss of generality, that $x(t) > 0$, $x(g(t)) > 0$, $x'(t) \geqslant 0$ and $x'(g(t)) \geqslant 0$ for $t \geqslant t_1 \geqslant t_0$. Then there exists $a > 0$ and $t_2 \geqslant t_1$ such that

$$2a \geqslant x(t) \geqslant x(g(t)) \geqslant a \qquad \text{for } t \geqslant t_2. \tag{10}$$

Multiplying (2) by $R(t)$ and integrating it from t_2 to $t \geqslant t_2$ by parts we have

$$R(t) r(t) x'(t) = R(t_2) r(t_2) x'(t_2) + x(t) - x(t_2) - \\ - \int_{t_2}^{t} R(s) f(s, x(s), x(g(s))\,\mathrm{d}s$$

and using (3)(ii) and (10) we have

$$0 \leqslant R(t) r(t) x'(t) \leqslant K - \int_{t_2}^{t} R(s) f(s, a, a)\,\mathrm{d}s \qquad \text{for } t \geqslant t_2,$$

where $K = R(t_2) r(t_2) x'(t_2) + 2a - x(t_2)$. The last inequality contradicts the assumption (9).

To prove the neccessity part of the theorem suppose that (9) does not hold, i.e. there exists $a \neq 0$, say $a > 0$, (the case $a < 0$ is analogous) such that

$$\int_{t_0}^{\infty} R(t) f(t, a, a)\,\mathrm{d}t < \infty.$$

Then there exists $t_2 \geqslant t_0$ so that $g(t) \geqslant t_0$ for $t \geqslant t_2$ and

$$\int_{t_2}^{\infty} \frac{1}{r(t)} \int_{t}^{\infty} f(s, a, a)\,\mathrm{d}s\,\mathrm{d}t \leqslant \frac{a}{2}.$$

Define the function $x(t) = a/2$ for $t \in \langle t_0, t_2 \rangle$ and $x(t) = a$ for $t > t_2$. Then, clearly, such a function x satisfies the assumptions of lemma 2 and the simple application of this lemma completes the proof.

Remark 2. The condition (9) for the equation (4) will have the form

$$\int_{t_0}^{\infty} R(t)\, a(t)\, \mathrm{d}t = \infty.$$

We shall assume in the sequel that

$$|f(t, x, x)| \geq a(t)\, |F(x)| \qquad \text{for } t \geq t_0,\ x \in R. \tag{11}$$

In the same way as in the proof of theorem 2 we can show that the oscillatoriness of (4) if (11) holds implies the same for (2). Hence we shall consider in the sequel only the equation (4). Moreover we shall assume that

$$g'(t) \geq 0 \quad \text{exists for} \quad t \geq t_0. \tag{12}$$

Theorem 4. *Let* (3) (i), (3) (iii), (5) *and* (12) *hold. If*

$$\int_{t_0}^{\infty} R(g(t))\, a(t)\, \mathrm{d}t = \infty \tag{13}$$

and

$$\int_{\pm\varepsilon}^{\pm\infty} \frac{\mathrm{d}x}{F(x)} < \infty \qquad \textit{for any } \varepsilon > 0, \tag{14}$$

then the equation (4) *is oscillatory.*

Proof. Suppose to the contrary that x is the nonoscillatory solution of (4) and $x(t) > 0$, $x(g(t)) > 0$, $x'(t) \geq 0$ and $x'(g(t)) \geq 0$ for $t \geq t_1 \geq t_0$. Then in the same way as in the proof of theorem 1 we can obtain

$$r(t)\, x'(t) \geq \int_t^{\infty} a(s)\, F(x(g(s)))\, \mathrm{d}s \qquad \text{for } t \geq t_1.$$

Since g, x, F are nondecreasing functions and from (4) $r(t)\, x'(t)$ is nonincreasing we get

$$r(g(t))\, x'(g(t)) \geq F(x(g(t))) \int_t^{\infty} a(s)\, \mathrm{d}s \qquad \text{for } t \geq t_1. \tag{15}$$

Multiplying (15) by $g'(t)$ and dividing it by $r(g(t))\, F(x(g(t)))$ and then integrating it from t_1 to $t \geq t_1$ we have

$$\int_{t_1}^{t} \frac{x'(g(s))\, g'(s)\, \mathrm{d}s}{F(x(g(s)))} \geq \int_{t_1}^{t} \frac{g'(s)}{r(g(s))} \int_s^{\infty} a(\xi)\, \mathrm{d}\xi\, \mathrm{d}s$$

and from this

$$\int_{x(g(t_1))}^{\infty} \frac{dy}{F(y)} \geqslant \int_{x(g(t_1))}^{x(g(t))} \frac{dy}{F(y)} \geqslant \int_{t_1}^{t} \frac{g'(s)}{r(g(s))} \int_{s}^{t} a(\xi)\, d\xi\, ds =$$

$$= \int_{t_1}^{t} [R(g(s)) - R(g(t_1))]\, a(s)\, ds \geqslant \frac{1}{2} \int_{t_2}^{t} R(g(s))\, a(s)\, ds$$

for $t \geqslant t_2$, where $t_2 \geqslant t_1$ is such that $R(g(t_1)) \leqslant \frac{1}{2} R(g(t))$ for $t \geqslant t_2$. The last inequality contradicts the assumptions (13) and (14).

Remark 3. The condition (13) is weaker than the analogous one given in theorem 1 in [2], which for the equation (4) has the form

$$\int_{t_0}^{\infty} g(t)\, a(t)\, dt = \infty. \tag{16}$$

See the following example.

Example 1. The equation

$$\left(\frac{1}{t} x'(t)\right)' + \frac{1}{t^3} x^3(t) = 0$$

does not satisfy the condition (16) but it satisfies the assumptions of theorem 4 and thus this equation is oscillatory.

Theorem 5. *Let the assumptions* (3) (i), (3) (iii), (5) *and* (12) *hold. Let there exist the nondecreasing function* $G \in C(R)$ *such that* $F(x) = |x|\, G(x)$ *for* $x \in R$. *Then, if*

$$\int_{t_0}^{\infty} R^2(g(t))\, a(t) \int_{g(t)}^{\infty} a(s)\, ds\, dt = \infty \tag{17}$$

and

$$\int_{\pm\varepsilon}^{\pm\infty} \frac{dx}{G(x)} < \infty \qquad \text{for any } \varepsilon > 0, \tag{18}$$

the equation (4) *is oscillatory.*

Proof. Suppose to the contrary that x is the nonoscillatory solution of (4), and without loss of generality, that $x(t) > 0$, $x(g(t)) > 0$, $x'(t) \geqslant 0$ and $x'(g(t)) \geqslant 0$ for $t \geqslant t_1 \geqslant t_0$. Then x is the nonoscillatory solution of the equation

$$(r(t)\, x'(t))' + b(t)\, G(x(g(t))) = 0,$$

where $b(t) = a(t)\, x(g(t))$. Then by theorem 4

$$\int_{t_0}^{\infty} R(g(t))\, a(t)\, x(g(t))\, dt < \infty. \tag{19}$$

In the same way as in the proof of theorem 4 we have

$$r(t)\,x'(t) \geqslant F(x(g(t))) \int_t^\infty a(s)\,\mathrm{d}s \geqslant F(x(g(t_1))) \int_t^\infty a(s)\,\mathrm{d}s$$

for $t \geqslant t_1$. Dividing this inequality by $r(t)$ and integrating it from t_1 to $t \geqslant t_1$ we get

$$x(t) \geqslant F(x(g(t_1)) \int_{t_1}^t \frac{1}{r(s)} \int_s^\infty a(\xi)\,\mathrm{d}\xi\,\mathrm{d}s \geqslant$$

$$\geqslant F(x(g(t_1))) \int_{t_1}^t \frac{1}{r(s)} \int_t^\infty a(\xi)\,\mathrm{d}\xi\,\mathrm{d}s = F(x(g(t_1)))\,[R(t) - R(t_1)] \int_t^\infty a(s)\,\mathrm{d}s.$$

Then there exists $t_2 \geqslant t_1$ such that

$$x(g(t)) \geqslant \frac{1}{2}\,F(x(g(t_1)))\,R(g(t)) \int_{g(t)}^\infty a(s)\,\mathrm{d}s \qquad \text{for } t \geqslant t_2.$$

This inequality and (19) contradict the condition (17) and this completes the proof.

Example 2. The equation

$$x''(t) + t^{-3/2}x^3(t^{1/3}) = 0$$

satisfies the assumptions of theorem 5 but the condition (13) of theorem 4 does not hold.

Remark 4. In an analogous way we can show that the condition

$$\int_{t_0}^\infty R^\beta(g(t))\,a(t)\left[\int_{g(t)}^\infty a(s)\,\mathrm{d}s\right]^{\beta-1}\mathrm{d}t = \infty$$

for some $1 \leqslant \beta < \alpha - 1$ is sufficient for the equation

$$(r(t)\,x'(t))' + a(t)\,|x(g(t))|^\alpha \operatorname{sgn}(x(g(t))) = 0$$

with $\alpha > 2$ to be oscillatory.

Finally, we shall consider the case $F(x) = x$, i.e. the linear equation

$$(r(t)\,x'(t))' + a(t)\,x(g(t)) = 0. \tag{20}$$

Lemma 3. *Assume that* (3) (i) *and* (3) (iii) *hold and*

$$\int_{t_0}^{\infty} R(t)\,a(t)\,\mathrm{d}t = \infty. \tag{21}$$

Then for any nonoscillatory solution x of (20) *there exists $t_1 \geqslant t_0$ such that*

$$|x(g(t))| \geqslant \frac{R(g(t))}{R(t)}\,|x(t)| \quad \textit{and} \quad |x'(t)| \leqslant \frac{|x(t)|}{R(t)\,r(t)} \tag{22}$$

for $t \geqslant t_1$.

Proof. Suppose that $x(t) > 0$, $x(g(t)) > 0$, $x'(t) \geqslant 0$ and $x'(g(t)) \geqslant 0$ for $t \geqslant t_2 \geqslant t_0$ (the case $x(t) < 0$ is analogous). Define the function $y(t) = x(t)/R(t)$. Then

$$(R^2(t)\,r(t)\,y'(t))' = R(t)\,(r(t)\,x'(t))' \leqslant 0$$

and the function $R^2(t)\,r(t)\,y'(t)$ is nonincreasing for $t \geqslant t_2$. If $R^2(t)\,r(t)\,y'(t) \geqslant 0$ for all $t \geqslant t_2$, then by the definition of y we have $R(t)\,r(t)\,x'(t) \geqslant x(t)$ for $t \geqslant t_2$. Then multiplying (20) by $R(t)$ and integrating it by parts we get

$$x(t) \leqslant R(t)\,r(t)\,x'(t) \leqslant K + x(t) - x(t_2) - \int_{t_2}^{t} R(s)\,a(s)\,x(g(s))\,\mathrm{d}s,$$

where $K = R(t_2)\,r(t_2)\,x'(t_2)$ and from this we get

$$0 \leqslant K - x(g(t_2)) \int_{t_2}^{t} R(s)\,a(s)\,\mathrm{d}s \qquad \text{for } t \geqslant t_2$$

and the condition (21) leads to the contradiction. Thus there exists $t_1 \geqslant t_2$ such that $R^2(t)\,r(t)\,y'(t) \leqslant 0$ for $t \geqslant t_1$ and this implies (22).

Theorem 6. *Assume that* (3) (i), (3) (iii) *and* (12) *hold. Let one of the following conditions hold, either*

$$\int_{t_0}^{\infty} R^{\lambda}(g(t))\,a(t)\,\mathrm{d}t = \infty \qquad \textit{for some } \lambda \in (0, 1), \tag{23}$$

or

$$\int_{t_0}^{\infty} R^{\lambda}(t)\,a(t)\,\mathrm{d}t = \infty \quad \textit{and} \quad \int_{t_0}^{\infty} \frac{R^{\lambda-1}(t)\,\mathrm{d}t}{R(g(t)\,r(t)} < \infty \tag{24}$$

for some $\lambda \in (0, 1)$, or

$$\int_{t_0}^{\infty} \frac{g'(t)}{R(g(t))\,r(g(t))} \exp\left[-\int_{g(t)}^{t} R(g(s))\,a(s)\,\mathrm{d}s\right] \mathrm{d}t < \infty. \tag{25}$$

Then the equation (20) *is oscillatory.*

P r o o f. Suppose to the contrary that x is the nonoscillatory solution of (20) and $x(t) > 0$, $x(g(t)) > 0$, $x'(t) \geqslant 0$ and $x'(g(t)) \geqslant 0$ for $t \geqslant t_1$. Note that if any of the conditions either (23) or (24) or (25) holds, then (21) and by lemma 3 the condition (22) hold, too.

1. Let the condition (23) hold. Define the function

$$V(t) = \frac{R^{\lambda}(g(t))\, r(t)\, x'(t)}{x(g(t))} \qquad \text{for } t \geqslant t_1 .$$

Then

$$V'(t) \leqslant -R^{\lambda}(g(t))\, a(t) + \frac{\lambda r(t)\, x'(t)\, R^{\lambda-1}(g(t))\, g'(t)}{r(g(t))\, x(g(t))} \leqslant$$

$$\leqslant -R^{\lambda}(g(t))\, a(t) + \frac{\lambda R^{\lambda-2}(g(t))\, g'(t)}{r(g(t))}$$

and integration of this inequality from t_1 to $t \geqslant t_1$ leads to the contradiction with nonegativity of x'.

2. Suppose that the condition (24) holds. Define the function $V(t)$ by the form

$$V(t) = \frac{R^{\lambda}(t)\, r(t)\, x'(t)}{x(g(t))} \qquad \text{for } t \geqslant t_1 .$$

Then we can obtain the contradiction in an analogous way.

3. Finally, let the condition (25) hold. Then $x'(t) > 0$ for $t \geqslant t_1$ and from (20) and (22) we get

$$(r(t)\, x'(t))' + a(t)\, R(g(t))\, r(t)\, x'(t) \leqslant 0 \qquad \text{for } t \geqslant t_1 .$$

Dividing this inequality by $r(t)\, x'(t)$ and integrating it from $g(t)$ to t we have

$$r(t)\, x'(t) \leqslant r(g(t))\, x'(g(t)) \exp\left[-\int_{g(t)}^{t} R(g(s))\, a(s)\, \mathrm{d}s \right]$$

for $t \geqslant t_2$, where $t_2 \geqslant t_1$ is such that $g(t) \geqslant t_1$ for $t \geqslant t_2$. Then we can finish the proof as in the previous cases using the function

$$V(t) = \frac{R(g(t))\, r(t)\, x'(t)}{x(g(t))} .$$

E x a m p l e 3. The equation

$$x''(t) + t^{-4/3} x(t^{1/2}) = 0$$

satisfies (24) for $\lambda = 1/3$ but (23) and (25) do not hold.

E x a m p l e 4. The equation

$$x''(t) + \frac{1}{t} x(\ln t) = 0$$

satisfies (23) for any $\lambda \in (0, 1)$ but (24) and (25) do not hold.

Example 5. The equation

$$x''(t) + \frac{1}{t \ln^2 t} x(\ln t) = 0$$

satisfies (25) but (23) and (24) do not hold.

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ТЕОРЕМЫ КОЛЕБЛЕМОСТИ ДЛЯ НЕЛИНЕЙНЫХ НЕРАВЕНСТВ ВТОРОГО ПОРЯДКА С ОТКЛОНЕНИЕМ

Ján Seman

Резюме

В статье приведены некоторые достаточные условия, при которых дифференциальное неравенство

$$x(t)[(r(t)x'(t))' + f(t, x(t), x(g(t)))] \leqslant 0 \tag{1}$$

является колеблемым. Эти результаты являются расширением результатов, приведенных в [1] и [2], где авторы предполагали, что $r(t) = 1$ или ограничено.