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Some Stability Theorems for Some Iteration Processes

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Abstract

In this paper, we obtain some stability results for Picard and Mann iteration processes in metric space and normed linear space respectively, using two different contractive definitions which are more general than those of Harder and Hicks [4], Rhoades [10, 11], Osilike [8], Osilike and Udomene [9], Berinde [1, 2], Imoru and Olatinwo [5] and Imoru et al [6].

Our results are generalizations of some results of Harder and Hicks [4], Rhoades [10, 11], Osilike [8], Osilike and Udomene [9], Berinde [1, 2], Imoru and Olatinwo [5] and Imoru et al [6].

Key words: Stability results; Picard and Mann iteration processes.

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1 Introduction

Let (X, d) be a complete metric space, $T : X \to X$ a selfmap of X. Suppose that $F_T = \{p \in X \mid Tp = p\}$ is the set of fixed points of T. Let $\{x_n\}_{n=0}^{\infty} \subset X$ be the sequence generated by an iteration procedure involving T which is defined by

$$x_{n+1} = f(T, x_n), \quad n = 0, 1, 2, \dots \tag{1}$$

where $x_0 \in X$ is the initial approximation and f is some function. Suppose $\{x_n\}_{n=0}^{\infty}$ converges to a fixed point p of T. Let $\{y_n\}_{n=0}^{\infty} \subset X$ and set

$$\epsilon_n = d(y_{n+1}, f(T, y_n)), \quad n = 0, 1, 2, \dots$$

Then, the iteration procedure (1) is said to be T-stable or stable with respect to T if and only if $\lim_{n\to\infty} \epsilon_n = 0$ implies $\lim_{n\to\infty} y_n = p$. If in (1),

$$f(T, x_n) = Tx_n, \quad n = 0, 1, 2, \ldots,$$

then we have the Picard iteration process, while we obtain the Mann iteration if

$$f(T, x_n) = (1 - \alpha_n)x_n + \alpha_n Tx_n, \quad n = 0, 1, 2, \ldots, \ \alpha_n \in [0, 1].$$

Several stability results have been obtained by various authors using different contractive definitions. Harder and Hicks [4] obtained interesting stability results for some iteration procedures using various contractive definitions. Rhoades [10,11] generalized the results of Harder and Hicks [4] to a more general contractive mapping. In Osilike [8], a generalization of some of the results of Harder and Hicks [4] and Rhoades [11] was obtained by employing the following contractive definition: there exist a constant $L \geq 0$ and $a \in [0, 1)$ such $\forall x, y \in X$,

$$d(Tx, Ty) \leq Ld(x, Tx) + ad(x, y). \tag{2}$$

Condition(2) is more general than those of Rhoades[11] and Harder and Hicks[4]. As in Harder and Hicks [4], Berinde [1] obtained the same stability results for the same iteration procedures using the same contractive definitions, but applied a different method. The method of Berinde [1] is similar to that employed in Osilike and Udomene [9] .

Recently, Imoru and Olatinwo [5] obtained some stability results for Picard and Mann iteration procedures by using a more general contractive condition than those of Harder and Hicks [4], Rhoades [11], Osilike [8], Osilike and Udomene [9] and Berinde [1]. In the paper [5], the following contractive definition was employed: there exist $a \in [0, 1)$ and a monotone increasing function $\phi : \Re_+ \to \Re_+$, with $\phi(0) = 0$, such that $\forall x, y \in X$,

$$d(Tx, Ty) \leq \phi(d(x, Tx)) + ad(x, y). \tag{3}$$

A function $h : \Re_+ \to \Re_+$ is called a comparison function if:
(i) h is monotone increasing;
(ii) $\lim_{n\to\infty} h^n(t) = 0, \ \forall t \geq 0$.
We remark here that every comparison function satisfies the condition $h(0) = 0$.

It is our purpose in this paper to obtain some stability results by applying two different contractive definitions using again the method of Berinde [1]. We shall use the following contractive definitions:

I) there exist a constant $a \in [0, 1)$ and a monotone increasing function $\Phi : \Re_+ \to \Re_+$ with $\Phi(0) = 1$, such that $\forall x, y \in X$,

$$d(Tx, Ty) \leq ad(x, y)\Phi(d(x, Tx)), \tag{4}$$

II) there exist a constant $L \geq 0$ and a function $\Psi : \Re_+ \to \Re_+$ such that $\forall x, y \in X$,

$$d(Tx, Ty) \leq \Psi(d(x, y))e^{Ld(x,Tx)} \tag{5}$$

where Ψ may be a comparison function or just a monotone increasing function. The contractive conditions (4) and (5) are independent as the right-hand side of (4) cannot be obtained from the right-hand side of (5) or vice-versa.

Condition (4) is more general than (2) in the following sense: If in (4),

$$\Phi(u) = 1 + \frac{ku}{d(x,y)}, \quad k \geq 0,\ d(x,y) \neq 0,\ \forall x, y \in X,\ x \neq y,\ u \in \Re_+,$$

then we obtain the condition (2).

Also, if in (4), we have

$$\Phi(u) = 1 + \frac{\phi(u)}{d(x,y)}, \quad d(x,y) \neq 0,\ \forall x, y \in X,\ x \neq y,\ u \in \Re_+,$$

where ϕ is also a monotone increasing function, then we obtain condition (3). Also, if $\Phi(u) = 1$, $\forall u \in \Re_+$, then we have the strict contraction employed in Harder and Hicks [4], Zeidler [13] and Berinde [1,2].

Similarly, condition (5) is more general than (2) in the sense that if in (5),

$$\Psi(u) = (au + Ld(x,Tx))e^{-Ld(x,Tx)}, \quad a \in [0,1),\ L \geq 0,\ u \in \Re_+,\ \forall x \in X,$$

and if Ψ is monotone increasing, then we obtain the condition(2).

Again, if $\Psi(u) = au$, $a \in [0,1)$, $u \in \Re_+$ and $L = 0$ in (5), then we get the strict contraction employed in Harder and Hicks [4], Berinde [1,2] and also in the classical Banach's contraction mapping principle discussed in Zeidler [13] and other standard texts on the fixed point theory.

Moreover , if in (5),

$$\Psi(u) = (\psi(u) + Ld(x,Tx))e^{-Ld(x,Tx)}, \quad \forall x \in X,\ u \in \Re_+,\ L \geq 0,$$

and if Ψ is monotone increasing and ψ is a comparison function, then we obtain the contractive mapping of Imoru et al [6].

However, we obtain the contractive definition employed in the extension of the Banach's contraction mapping principle due to Berinde [3] if $L = 0$ in (5). See also Berinde [2] for detail on the various generalizations of the Picard–Banach–Caccioppoli theorem.

We shall employ the following Lemmas in the sequel.

Lemma 1 (Berinde [1]) *If δ is a real number such that $0 \leq \delta < 1$, and $\{\epsilon_n\}_{n=0}^{\infty}$ is a sequence of positive numbers such that $\lim_{n\to\infty} \epsilon_n = 0$, then for any sequence of positive numbers $\{u_n\}_{n=0}^{\infty}$ satisfying*

$$u_{n+1} \leq \delta u_n + \epsilon_n, \quad n = 0, 1, \ldots \tag{6}$$

we have

$$\lim_{n\to\infty} u_n = 0.$$

Remark 1 The proof of this lemma is contained in Berinde [1].

Lemma 2 *If* $\psi : \Re_+ \to \Re_+$ *is a subadditive comparison function and* $\{\epsilon_n\}_{n=0}^{\infty}$ *is a sequence of positive numbers such that* $\lim_{n\to\infty} \epsilon_n = 0$*, then for any sequence of positive numbers* $\{u_n\}_{n=0}^{\infty}$ *satisfying*

$$u_{n+1} \leq \sum_{m=0}^{s} \delta_m \psi^m(u_n) + \epsilon_n, \quad n = 0, 1, 2, \ldots, \tag{7}$$

where $\sum_{m=0}^{s} \delta_m = 1$*,* $\delta_0, \delta_1, \ldots, \delta_s \in [0, 1]$*, we have*

$$\lim_{n\to\infty} u_n = 0.$$

Remark 2 The proof of this Lemma is contained in Imoru et al [6].

Remark 3 If $\delta_k = 0$ in (7), $k = 1, 2, \ldots, s$, then we obtain the Lemma 1 of Berinde [1] with $0 \leq \delta_o < 1$.

Remark 4 If $\delta_1 = 1$ and $\delta_o = \delta_2 = \delta_3 = \cdots = \delta_{s-1} = \delta_s = 0$ in (7), then we obtain a stability result for the Picard iteration process.

Remark 5 We have a stability result for the Krasnoselskij iteration procedure if $\delta_o = \delta_1 = 1/2$ and $\delta_2 = \delta_3 = \cdots = \delta_s = 0$ in (7).

Remark 6 We obtain stability results for the Mann and Schaefer's iteration processes if $\delta_0 + \delta_1 = 1$, $\delta_2 = \delta_3 = \cdots = \delta_s = 0$ in (7).

Remark 7 If $\delta_0 + \delta_1 + \delta_2 = 1$, $\delta_3 = \delta_4 = \cdots = \delta_s = 0$ in (7), then we obtain a stability result for the Ishikawa iteration procedure.

Remark 8 If $\sum_{m=0}^{k} \delta_m = 1$ (i.e. $s = k$) in (7),then we have a stability result for the Kirk's iteration process.

2 Main Results

The following are stability results for the Picard iteration process.

Theorem 1 *Let* (X, d) *be a complete metric space and* $T : X \to X$ *a selfmap of* X *satisfying (4). Suppose* T *has a fixed point* p*. Let* $x_0 \in X$ *and let*

$$x_{n+1} = f(T, x_n) = Tx_n, \quad n = 0, 1, \ldots$$

be the Picard iteration associated to T*. Suppose also that* $\Phi : \Re_+ \to \Re_+$ *is a monotone increasing function such that* $\Phi(0) = 1$*. Then, the Picard iteration is T-stable.*

Proof Let $\epsilon_n = d(y_{n+1}, Ty_n)$, $n = 0, 1, \dots$ and suppose $\lim_{n\to\infty} \epsilon_n = 0$. Then, we shall establish that $\lim_{n\to\infty} y_n = p$. using (4) and the triangle inequality. Therefore,

$$\begin{aligned} d(y_{n+1}, p) &\leq d(y_{n+1}, Ty_n) + d(Ty_n, p) \\ = \epsilon_n + d(Ty_n, Tp) &= d(Tp, Ty_n) + \epsilon_n \leq ad(p, y_n)\Phi(d(p, Tp)) + \epsilon_n \\ &= ad(y_n, p)\Phi(0) + \epsilon_n = ad(y_n, p) + \epsilon_n. \end{aligned} \tag{8}$$

Since $a \in [0, 1)$, using Lemma 1 in (8) yields $\lim_{n\to\infty} y_n = p$.

Conversely, let $\lim_{n\to\infty} y_n = p$. Then, by (4) and the triangle inequality, we have

$$\begin{aligned} \epsilon_n = d(y_{n+1}, Ty_n) &\leq d(y_{n+1}, p) + d(p, Ty_n) = d(y_{n+1}, p) + d(Tp, Ty_n) \\ &\leq d(y_{n+1}, p) + ad(p, y_n)\Phi(d(p, Tp)) = d(y_{n+1}, p) + ad(y_n, p)\Phi(0) \\ &= d(y_{n+1}, p) + ad(y_n, p) \to 0 \quad \text{as } n \to \infty. \end{aligned}$$

□

Theorem 2 *Let (X, d) be a complete metric space and $T : X \to X$ a selfmap of X satisfying (5). Suppose that T has a fixed point p. Let $x_0 \in X$ and let*

$$x_{n+1} = f(T, x_n) = Tx_n, \quad n = 0, 1, \dots,$$

be the Picard iteration associated to T. Suppose that $\Psi : \mathbb{R}_+ \to \mathbb{R}_+$ is a comparison function (or just a monotone increasing function) which is continuous. Then, the Picard iteration is T-stable.

Proof Let $\epsilon_n = d(y_{n+1}, Ty_n)$, $n = 0, 1, \dots$, and suppose that $\lim_{n\to\infty} \epsilon_n = 0$. Then, we shall establish that $\lim_{n\to\infty} y_n = p$, using (5) and the triangle inequality. Therefore,

$$\begin{aligned} d(y_{n+1}, p) &\leq d(y_{n+1}, Ty_n) + d(Ty_n, p) \\ = \epsilon_n + d(Ty_n, Tp) &= d(Tp, Ty_n) + \epsilon_n \leq \Psi(d(p, y_n))e^{Ld(p,Tp)} + \epsilon_n \\ &= \Psi(d(y_n, p)) + \epsilon_n. \end{aligned} \tag{9}$$

Applying Lemma 2 in (9) yields $\lim_{n\to\infty} y_n = p$.

Conversely, let $\lim_{n\to\infty} y_n = p$. Then, by (5)and the triangle inequality, we obtain

$$\begin{aligned} \epsilon_n = d(y_{n+1}, Ty_n) &\leq d(y_{n+1}, p) + d(p, Ty_n) = d(y_{n+1}, p) + d(Tp, Ty_n) \\ &\leq d(y_{n+1}, p) + \Psi(d(p, y_n))e^{Ld(p,Tp)} \\ &= d(y_{n+1}, p) + \Psi(d(y_n, p)) \to 0 \quad \text{as } n \to \infty. \end{aligned}$$

□

Remark 9 Theorem 1 is a generalization of Theorem 3.1 of Imoru and Olatinwo [5], while Theorem 2 is a generalization of both Theorems P1 and P2 of Imoru et al [6]. Also, each of the Theorem 3.1 of [5] and Theorems P1 and P2 of [6] is itself a generalization of Theorem 2 of Harder and Hicks [4], Theorem 1 of Rhoades[10, 11], Theorems 1 and 2 of Berinde [1], Theorem 1 of Osilike [8] as well as Theorem 4 of Osilike and Udomene [9].

We now establish some stability results for the Mann iteration process.

Theorem 3 *Let $(X, ||\cdot||)$ be a normed linear space, and $T : X \to X$ a selfmap of X satisfying (4). Suppose T has a fixed point p. Let $x_o \in X$ and let*

$$x_{n+1} = f(T, x_n) = (1-\alpha_n)x_n + \alpha_n T x_n, \quad \alpha_n \in [0,1],\ n = 0, 1, \dots,$$

be the Mann iteration process such that $0 < \alpha \leq \alpha_n$, $n = 0, 1, 2, \dots$ Let $\Phi : \mathbb{R}_+ \to \mathbb{R}_+$ be a monotone increasing function such that $\Phi(0) = 1$. Then, the Mann iteration process is T-stable.

Proof Let $\epsilon_n = ||y_{n+1} - (1-\alpha_n)y_n - \alpha_n T y_n||$, $n = 0, 1, \dots$ and suppose that $\lim_{n\to\infty} \epsilon_n = 0$. Then,we shall prove that $\lim_{n\to\infty} y_n = p$, by (4) and the triangle inequality: Therefore,

$$\begin{aligned}
||y_{n+1} - p|| &\leq ||y_{n+1} - (1-\alpha_n)y_n - \alpha_n T y_n|| + ||(1-\alpha_n)y_n + \alpha_n T y_n - p|| \\
&= \epsilon_n + ||(1-\alpha_n)y_n + \alpha_n T y_n - (1-\alpha_n+\alpha_n)p|| \\
&= ||(1-\alpha_n)(y_n - p) + \alpha_n(T y_n - p)|| + \epsilon_n \\
&\leq (1-\alpha_n)||y_n - p|| + \alpha_n||T y_n - p|| + \epsilon_n \\
&= (1-\alpha_n)||y_n - p|| + \alpha_n||T y_n - Tp|| + \epsilon_n \\
&\leq (1-\alpha_n)||y_n - p|| + \alpha_n a||p - y_n||\Phi(||p - Tp||) + \epsilon_n \\
&= (1-\alpha_n)||y_n - p|| + \alpha_n\ a||y_n - p||\Phi(0) + \epsilon_n \\
&= (1-\alpha_n)||y_n - p|| + a\alpha_n||y_n - p|| + \epsilon_n \\
&= [1 - (1-a)\alpha_n]||y_n - p|| + \epsilon_n \\
&\leq [1 - (1-a)\alpha]||y_n - p|| + \epsilon_n. \qquad (10)
\end{aligned}$$

Using Lemma 1 in (10) since $0 \leq 1 - (1-a)\alpha < 1$, we obtain

$$\lim_{n\to\infty} y_n = p.$$

Conversely, let $\lim_{n\to\infty} y_n = p$. Then, by (4) and the triangle inequality, we get

$$\begin{aligned}
\epsilon_n &= ||y_{n+1} - (1-\alpha_n)y_n - \alpha_n T y_n|| \\
&\leq ||y_{n+1} - p|| + ||p - (1-\alpha_n)y_n - \alpha_n T y_n|| \\
&= ||y_{n+1} - p|| + ||(1-\alpha_n+\alpha_n)p - (1-\alpha_n)y_n - \alpha_n T y_n|| \\
&= ||y_{n+1} - p|| + ||(1-\alpha_n)(p - y_n) + \alpha_n(p - T y_n)|| \\
&\leq ||y_{n+1} - p|| + (1-\alpha_n)||y_n - p|| + \alpha_n||p - T y_n|| \\
&= ||y_{n+1} - p|| + (1-\alpha_n)||y_n - p|| + \alpha_n||Tp - T y_n|| \\
&\leq ||y_{n+1} - p|| + (1-\alpha_n)||y_n - p|| + \alpha_n a||p - y_n||\Phi(||p - Tp||) \\
&= ||y_{n+1} - p|| + (1-\alpha_n)||y_n - p|| + \alpha_n a||y_n - p||\phi(0) \\
&= ||y_{n+1} - p|| + [1 - (1-a)\alpha_n]||y_n - p|| \\
&\leq ||y_{n+1} - p|| + [1 - (1-a)\alpha]||y_n - p|| \to 0 \quad \text{as } n \to \infty.
\end{aligned}$$

□

Theorem 4 *Let $(X, ||\cdot||)$ be a normed linear space and $T : X \to X$ a selfmap of X satisfying (5). Suppose that T has a fixed point p. Let $x_0 \in X$ and let $x_{n+1} = f(T, x_n) = (1-\alpha_n)x_n + \alpha_n T x_n$, $\alpha_n \in [0,1]$, $n = 0,1,\ldots,$ be the Mann iteration process such that $0 < \alpha \leq \alpha_n$, $n = 0,1,2,\ldots$ Suppose that $\Psi : \mathbb{R}_+ \to \mathbb{R}_+$ is a comparison function (or just a monotone increasing function) which is continuous. Then, the Mann iteration is T-stable.*

Proof Let $\epsilon_n = ||y_{n+1} - (1-\alpha_n)y_n - \alpha_n T y_n||$, $n = 0,1,\ldots,$ and suppose that $\lim_{n\to\infty} \epsilon_n = 0$. Then, we shall prove that $\lim_{n\to\infty} y_n = p$, by using (5) and the triangle inequality. Therefore,

$$\begin{aligned} ||y_{n+1} - p|| &\leq ||y_{n+1} - (1-\alpha_n)y_n - \alpha_n T y_n|| + ||(1-\alpha_n)y_n + \alpha_n T y_n - p|| \\ &= \epsilon_n + ||(1-\alpha_n)y_n + \alpha_n T y_n - (1-\alpha_n+\alpha_n)p|| \\ &= ||(1-\alpha_n)(y_n - p) + \alpha_n(T y_n - p)|| + \epsilon_n \\ &\leq (1-\alpha_n)||y_n - p|| + \alpha_n||T y_n - p|| + \epsilon_n \\ &= (1-\alpha_n)||y_n - p|| + \alpha_n||T y_n - Tp|| + \epsilon_n \\ &= (1-\alpha_n)||y_n - p|| + \alpha_n||Tp - T y_n|| + \epsilon_n \\ &\leq (1-\alpha_n)||y_n - p|| + \alpha_n \Psi(||p - y_n||)e^{L||p-Tp||} + \epsilon_n \\ &= (1-\alpha_n)||y_n - p|| + \alpha_n \Psi(||y_n - p||) + \epsilon_n \\ &\leq (1-\alpha)||y_n - p|| + \alpha \Psi(||y_n - p||) + \epsilon_n. \qquad (11) \end{aligned}$$

By applying Lemma 2 in (11), we obtain

$$\lim_{n\to\infty} y_n = p.$$

Conversely, let $\lim_{n\to\infty} y_n = p$. Then, by using (5) and the triangle inequality, we have

$$\begin{aligned} \epsilon_n &= ||y_{n+1} - (1-\alpha_n)y_n - \alpha_n T y_n|| \\ &\leq ||(y_{n+1} - p|| + ||p - (1-\alpha_n)y_n - \alpha_n T y_n|| \\ &= ||y_{n+1} - p|| + ||(1-\alpha_n+\alpha_n)p - (1-\alpha_n)y_n - \alpha_n T y_n|| \\ &= ||y_{n+1} - p|| + ||(1-\alpha_n)(p - y_n) + \alpha_n(p - T y_n)|| \\ &\leq ||y_{n+1} - p|| + (1-\alpha_n)||y_n - p|| + \alpha_n \Psi(||p - y_n||)e^{L||p-Tp||} \\ &= ||y_{n+1} - p|| + (1-\alpha_n)||y_n - p|| + \alpha_n \Psi(||y_n - p||) \to 0 \\ &\leq ||y_{n+1} - p|| + (1-\alpha)||y_n - p|| + \alpha \Psi(||y_n - p||) \to 0 \quad \text{as } n \to \infty. \end{aligned}$$

Remark 10 Theorem 3 is a generalization of Theorem 3.2 of Imoru and Olatinwo [5], while Theorem 4 is a generalization of Theorem M of Imoru et al [6]. Moreover, each of both Theorem 3.2 of [5] and Theorem M of [6] is itself a generalization of Theorem 3 of Harder and Hicks [4] ,Theorem 2 of Rhoades [10, 11] and Theorem 3 of Berinde [1].

Remark 11 If in (4), $\Phi(u) = e^{Lu}$, $L \geq 0$, or, in (5), $\Psi(u) = au$, $a \in [0,1)$, $u \in \Re_+$, then we obtain the following contractive definition: there exist $a \in [0,1)$ and a constant $L \geq 0$ such that $\forall x, y \in X$,

$$d(Tx, Ty) \leq ad(x,y)e^{Ld(x,Tx)}. \tag{12}$$

By Remark 11, we obtain the following corollary to Theorems 1 and 2.

Corollary 1 *Let (X, d) be a complete metric space and $T : X \to X$ a selfmap of X satisfying (12). Suppose that T has a fixed point p. Let $x_o \in X$ and let*

$$x_{n+1} = f(T, x_n) = Tx_n, \quad n = 0, 1, 2, \ldots$$

be the Picard iteration. Then, the Picard iteration is T-stable.

In a similar manner, we obtain the following corollary to Theorems 3 and 4.

Corollary 2 *Let $(X, ||\cdot||)$ be a normed linear space and $T : X \to X$ a selfmap of X satisfying (12). Suppose that T has a fixed point p. Let $x_0 \in X$ and let*

$$x_{n+1} = f(T, x_n) = (1 - \alpha_n)x_n + \alpha_n Tx_n, \quad \alpha_n \in [0,1], \; n = 0, 1, \ldots$$

be the Mann iteration process such that $0 < \alpha \leq \alpha_n$. Then, the Mann iteration process is T-stable.

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