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BOOK REVIEWS

C. Faber, G. van der Geer, F. Oort: MODULI OF ABELIAN VARIETIES. Birkhäuser, 2001, ISBN 3-7643-6517-X, 536 pages, DM 196.–.

The interest in the study of the moduli spaces of abelian varieties dates back to the 19th century through the work on moduli of elliptic curves. The corresponding modular curves and their function theory are interesting not only as Riemann surfaces, but mainly from the point of view of their arithmetic properties via the Kronecker-Weber theorem. The result that all motives coming from elliptic curves defined over Q are present in the motives defined by modular curves (Shimura-Taniyama-Weil conjecture) recently culminated in the proof of Fermat's last theorem. Moduli spaces of higher dimensional varieties started to attract attention due to the work of Siegel, Satake, Igusa and others in the complex analytic case. In 1960's D. Mumford proved the existence of (complex) moduli spaces for polarized abelian varieties in the sense of Grothendieck. Good compactifications of moduli spaces of abelian varieties over the ring of integers have been constructed by Faltings and Chai. The book presents a collection of 17 refereed articles of leading experts in the field, originating from the 3rd "Texel Conference" held in 1999.

Petr Somberg, Praha

Dmitry Burago, Yuri Burago, Sergei Ivanov: A COURSE IN METRIC GEOMETRY. Graduate Studies in Mathematics, vol. 33, AMS, 2001, 420 pages, USD 44.–.

The book consists of 10 chapters. The first chapter is devoted to the standard and advanced introduction to the metric spaces. Hausdorff measure and dimension is explained, and the Banach-Hausdorff-Tarski paradox is also mentioned in an informal way.

The second chapter deals with the very general "length structure" defined on a topological space and later studied on metric spaces. Examples are taken from a broad spectrum ranging from the Euclidean geometry to the Finsler geometry. The famous theorem by Hopf and Rinow is generalized from the Riemannian case to the general case of a length space.

In the third chapter, the glueings of length spaces, the quotient metrics, polyhedral spaces, metric graphs (including so-called Cayley graphs), direct products and warped products on length spaces are considered. The abstract study of angles concludes this chapter.

The fourth chapter is devoted to spaces of bounded curvature (Alexandrov spaces). Here the curvature is not defined explicitly but instead, spaces of non-positive curvature and those of non-negative curvature are characterized by the geometric behaviour of their distance functions when compared with the Euclidean case. The first variation formula (known from differential geometry) is derived for Alexandrov spaces.

The fifth chapter is devoted to the Riemannian metrics, Finsler metrics, sub-Riemannian metrics and to the detailed study of the hyperbolic plane. The Besicovitch inequality is proved at the end.

The sixth chapter deals with the curvature of Riemannian metrics and with the related comparison theorems.

The seventh chapter investigates the "space of metric spaces" (Lipschitz distance, Gromov-Hausdorff distance and convergence).

The eighth chapter with the title “Large-scale geometry” deals mainly with Gromov hyperbolic spaces and with periodic metrics.

The ninth chapter is devoted to the spaces of curvature bounded from above, mainly to the Hadamard spaces (again in the most general setting). As “example”, the theory of semi-dispersing billiards is treated.

The last section treats spaces of curvature bounded from below. The main results are the general version of the Toponogov theorem and the Gromov-Bishop inequality.

The book is written in both an informal and precise style, with many exercises. This is a really excellent book which should be read by every expert in geometry, and which can be warmly recommended even to undergraduate students. In the second plan, a tribute is paid here to the fundamental contributions of mathematicians of Russian origin to many parts of geometry and topology.

Oldřich Kowalski, Praha

Liviu I. Nicolaescu: NOTES ON SEIBERG-WITTEN THEORY. Graduate Studies in Mathematics, vol. 28, AMS, Providence, Rhode Island, 2000, 484 pages, USD 59.–.

The book presents an introduction into various aspects of a system of PDE’s on 4-manifolds, called the Seiberg-Witten equations. Having their origin in theoretical physics, these equations—invented in 1994 by physicists Seiberg and Witten—typically represent interplay between 4-dim. gauge theories studied in physics on the one hand and differential resp. algebraic topology of 4-dim. manifolds on the other.

The second chapter introduces the space of solutions of S-W theory (= monopoles) and the action of the infinite-dimensional Abelian group acting on this space. As an application, the proof of Thom conjecture given by Mrowka and Kronheimer is presented.

In the third chapter monopoles are studied on algebraic surfaces. Special attention is paid to Kähler manifolds, K3 and elliptic surfaces. The fourth and last Chapter present the most technical parts such as cut-and-paste techniques for computing Seiberg-Witten invariants.

This monography contains many explicit examples and provides enough background for the interested reader to be able to continue the Seiberg-Witten journey on her/his own.

Petr Somberg, Praha

Thierry Aubin: A COURSE IN DIFFERENTIAL GEOMETRY. Graduate Studies in Mathematics, vol. 27, American Mathematical Society, Providence, Rhode Island, 2001, ISBN 0-8218-2709-X, USD 35.–.

The book is a very detailed textbook of modern differential geometry. On the first sight its content is very traditional. The introduction recalls the basic notions of topology, operations with tensors and some theorems from Calculus. Chapters, ordered quite naturally, follow: on differentiable manifolds, tangent spaces, linear connections and Riemann metrics. Inserted among them is a chapter on integration of vector fields and systems of differentiable forms (Frobenius Theorem). Whitney Theorem on imbedding a variety in the Euclidean space, Sard’s Theorem on the critical values of a differential map and of course also the Stokes Formula are also included. The character of the last chapter is quite different: it is devoted to a special problem of existence of Riemann metric with constant scalar curvature, which stems from the Japanese mathematician H. Yamabe and was also studied by the author of the book.

A great number of exercises adjoint to each chapter represent a great advantage of the book. Just a minor part of them are simple, while most of them require a good knowledge of both differential geometry and Differential Calculus, first of all of differential equations. Some exercises, although not by far all of them, are provided with the solution. The book can be strongly recommended to all who engage in studying differential geometry of Riemann manifolds. Some noncorrect figures are the only flaw of the book.

Leo Boček, Praha

Hershel M. Farkas, Irvin Kra: THETA CONSTANTS, RIEMANN SURFACES AND THE MODULAR GROUP. Graduate Studies in Mathematics, vol. 37, AMS, Providence, Rhode Island, 2001, 531 pages, USD 69.–.

The book serves as an introduction with applications to uniformization theorems for Riemann surfaces and combinatorial number theory (e.g. partition identities).

The first chapter deals with modular curves obtained by the quotient of the upper half plane $\mathbb{H}$ by congruence subgroups $\Gamma(\mathbb{N}) \subset PSL(2, \mathbb{Z})$, and gives an adequate treatment of their uniformizations.

The theta functions with (rational) characteristics are defined in the second chapter. Many properties of them, e.g. their transformation laws under the action of $PSL(2, \mathbb{Z})$, the Jacobi triple product formula etc. are discussed in detail, and the identities for theta functions are consequently identified with Ramanujan identities. Chapter 3 lies in the heart of this monograph. The modular and cusp forms are constructed for principal congruence subgroups and using them, imbeddings of Riemann surfaces into projective spaces of reasonably low dimensions are obtained. In Chapter 4, the theta function identities are related to the underlying combinatorial number-theoretical identities.

Chapters 5, 6, 7 present some parts of all previous considerations in a more technical and detailed fashion.

The monograph offers an explicit and detailed account for readers ranging from the beginning graduate students to professional mathematicians, whose interests are either in combinatorial number theory or function theory of (modular) Riemann surfaces. A very useful feature of the book is the treatment of many examples in detail.

Petr Somberg, Praha

M. M. Rao, Z. D. Ren: APPLICATIONS OF ORLICZ SPACES. Pure and Applied Mathematics Series, vol. 250, Marcel Dekker, New York, 2002, xi+464 pages, ISBN 0-8247-0730-3/hbk, USD 185.–.

Orlicz spaces are of great importance in modern mathematical analysis and applications. Despite of that there was not any synthetic large work devoted to the Orlicz spaces themselves since the well known classical monograph by Krasnosel'skii and Rutitskii and their more general counterpart developed by the Poznań school (the monograph on modular spaces by J. Musielak). There are several books on integral operators in Orlicz spaces and some shorter expositions in several well-known monographs. An immense amount of material is scattered in a huge number of papers.

The monograph under review fills this gap in important areas and follows the first monograph by the same authors (Theory of Orlicz Spaces, Marcel Dekker, New York 1991). Its main emphasis is put on various geometric aspects of the Orlicz spaces theory but there is also a lot of material from other areas. The authors study concepts from the general Banach space theory in the particular setup of Orlicz spaces, connections with Fourier analysis, prediction and stochastic analysis, there are also some applications to PDEs and to some other fields of analysis.

Contents of the book: Chapter I. Introduction and backward material. Chapter II. Nonsquare and von Neumann-Jordan constants. Chapter III. Normal structure and WCS coefficients. Chapter IV. Jung constants of Orlicz spaces. Chapter V. Packing in Orlicz spaces. Chapter VI: Fourier analysis in Orlicz spaces. Chapter VII. Applications to prediction analysis. Chapter VIII. Applications to stochastic analysis. Chapter IX. Nonlinear PDEs and Orlicz spaces. Chapter X. Miscellaneous applications. References, Notation and Index. There are bibliographical comments at the end of each chapter.

Let us survey the book in more detail. In Chapter I some additional material of introductory nature not contained in the first volume is collected, above all some further indices of N-functions important in the sequel are studied (note that relations of some of these indices to the standard Boyd indices have been established by the A. Fiorenza and the reviewer in 1997); further one finds basic results on convolutions and interpolation in Orlicz spaces.

In Chapter II the authors recall the concept of a uniformly nonsquare space, nonsquare constant, and von Neumann-Jordan constant in Banach spaces and derive lower and upper bounds in Orlicz spaces. Similarly Chapter III gives results about geometric properties of Orlicz spaces in terms of particular properties of further general Banach spaces concepts, namely, of N coefficients (normal structure), BS coefficients (bounded sequence), and (WCS) coefficients (weak convergent sequence). In Chapter IV the Jung constants are studied, attention is paid to lower and upper estimates for them in general Orlicz spaces, and exact values are found for some special classes. The packing constants in Orlicz sequence spaces, Orlicz function spaces, and especially in reflexive Orlicz spaces are studied in Chapter V, there is also a survey for spaces with finite dimension, for Hilbert and Lebesgue spaces. In Chapter VI the reader will find results on the convergence of Fourier series in Orlicz spaces on compact abelian groups. Special attention is paid to the conjugate function for the torus and to Ryan's characterization of its boundedness in terms of reflexivity of the space in question. Next two chapters, VII and VIII, will be useful for people in the prediction and stochastic analysis, showing the usefulness of Young functions in the theory (the convex loss function and processes with values in Orlicz spaces). The authors deal with the existence and uniqueness of best predictors, consider both the linear and nonlinear prediction operators in nonreflexive spaces, clarify the natural role of Orlicz spaces in study of large deviations, and then tackle the regularity of stochastic functions and study martingales in Orlicz spaces. Chapter IX contains some of the numerous applications of Orlicz spaces in the PDEs theory and basic information on classical operators in Orlicz spaces. The remarkable Shapiro's result (1977) on removable sets is also included in this chapter. The choice here is rather difficult to make; to give a more complete picture it would be perhaps useful to mention at least some older crucial and fundamental results in the area (problems leading to critical imbeddings (due to N. Trudinger and others) existence theorems in nonreflexive spaces (due to J.-P. Gossez, V. Mustonen and others)), whose further development is a long-term challenge. The concluding Chapter X contains various applications on Beurling-Orlicz algebras, embedding theorems for Orlicz sequence spaces, Riesz angles of Orlicz spaces, differentiability properties of Orlicz spaces.

The book will be very useful both for experts and students. It has been written by experienced authors, belonging to the top in the field and has been prepared with great skill. It fills a big and unpleasant gap in literature.

Miroslav Krbec, Praha