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Book Reviews

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BOOK REVIEWS

Neal Madras, Gordon Slade: THE SELF-AVOIDING WALK. Probability and Its Applications, Birkhäuser, Boston 1996, xiv+425 pages, ISBN 3-7643-3891-1, price DM 68,–.

Self-avoiding walks have been investigated in chemistry and statistical physics for several decades. Their definition is simple: An N-step self-avoiding walk ω (beginning at x) on the d-dimensional discrete lattice $\mathbb{Z}^d$ is a path $(\omega(0),\ldots,\omega(N))$ such that $\omega(0)=x$ and $\omega(i)\neq\omega(j)$ for $i\neq j$. However, to quote authors' Preface, "in spite of this simple definition, many of the most basic questions about this model are difficult to resolve in mathematically rigorous fashion. ... Important questions about the self-avoiding walk remain unsolved ..., although the physics and chemistry communities have reached consensus on the answers by a variety of nonrigorous methods".

The aim of the book is to survey the progress in the mathematical theory of the self-avoiding walk which has been reached recently, by means of methods originating in combinatorics and mathematical physics rather than in the classical probability theory.

Let us list briefly some of the topics considered in the book. In the first two chapters, the notion of critical exponents is discussed and the non-rigorous results on their values, that can be viewed as a motivation for the whole subsequent theory, are explained. The next chapter is an account of classical Hammersley-Welsh's and Kesten's upper bounds on the number of N-step self-avoiding walks beginning at the origin. A rigorous argument (the method of lace expansions) leading to critical exponents is available only in high dimensions, this is treated in Chapters 5 and 6. The lengthy ninth chapter is devoted to a thorough analysis of Monte Carlo algorithms used to obtain estimates of critical exponents and related numerical values.

The monograph is a very useful (and accessible) source of information on the mathematical understanding of the self-avoiding walk model. This is also testified by the fact that the book, issued for the first time in 1993, is now, three years later, reprinted as a paperback.

Bohdan Maslowski

Helge Holden, Bernt Øksendal, Jan Ubøe, Tusheng Zhang: STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS. A MODELING, WHITE NOISE FUNCTIONAL APPROACH. Probability and Its Applications, Birkhäuser, Boston 1996, x+230 pages, ISBN 0-8176-3928-4, price DM 118,–.

At first, let us recall the basic example motivating the theory developed in this monograph, as described by the authors in the first chapter of the book. A flow of fluid injected into a dry, porous rock at the injection rate $f_t(x)$ is to be described. Assume that the saturation $\theta(t,x)$ at a point x at time t is either zero or we have a complete saturation $\theta_0(x)>0$. Let $p_t(x)$ denote the pressure, then we arrive at the following moving boundary problem

$$(1)\qquad \begin{cases} \operatorname{div}\bigl(k(x)\nabla p_t(x)\bigr) = -f_t(x), \quad x\in D_t,\\ p_t(x)=0 \quad \text{on } \partial D_t,\\ \theta_0(x)\dfrac{\mathrm{d}}{\mathrm{dt}}\bigl(\partial D_t\bigr) = -k(x)\nabla p_t(x), \quad x\in\partial D_t, \end{cases}$$

where D_t stands for the wet region at time t, $D_t = \{x;\ \theta(t,x) = \theta_0(x)\}$. Experimental data indicate that the permeability k of the rock may fluctuate in an irregular way and it seems natural to model it as a random field. However, the standard theory of stochastic partial differential equations (SPDE's for short), as presented e.g. by J. B. Walsh in his well known lecture notes, is not easily applicable to problems of the type (1). According to this theory, the solution to (1) should be a random process with values in a certain Sobolev space of a negative order and it remains unclear how to interpret the multiplication involved in (1). Moreover, the noise k should be positive, hence it cannot be the white noise (the derivative of the Brownian sheet) that is usually used in the SPDE's theory. The authors, motivated by Hida's theory of white noise, adopt a different point of view. Instead of looking for a distribution valued random process, they consider solutions to (1) as functions $x \longmapsto p_t(x)$ from a domain D in $\mathbb{R}^d$ to an appropriate space of generalized random variables (stochastic distributions). These spaces are equipped with a multiplication (called the Wick product), therefore replacing the ordinary products with the Wick ones makes it possible to treat equations like (1) in the usual strong sense (in the space of stochastic distributions).

The treatise under review is devoted to a careful and relatively self-contained account of the new approach to SPDE's. After a short Introduction, in Chapter 2 the authors develop the tools from the white noise analysis needed in the sequel: first, the Wiener-Itô expansions are described, and the stochastic test functions and stochastic distributions are introduced, with a particular emphasis on the Kondratiev space $(\mathcal{S})_{-1}$. Then the Wick product is dealt with and it is shown that the Skorokhod integral can be reinterpreted as a Pettis integral in the space $(\mathcal{S})_{-1}$. Finally, the Hermite transform (and related operators) are investigated. Chapter 3 is about applications to ordinary (but, in general, anticipative) stochastic differential equations; the next chapter is devoted to stochastic partial differential equations (including, e.g., the viscous Burgers' equation with a stochastic source and the stochastic pressure equation). A unified method based on the Hermite transform is employed to find solution to stochastic equations, leading often to explicit formulae of the Feynman-Kac type.

The whole theory of Wick type SPDE's has been created only recently, to a great extent due to the effort of the authors of this monograph. A systematic yet accessible presentation of the results hitherto obtained as provided in the book will certainly attract further attention to this promising field of research.

Jan Seidler

STOCHASTIC DIFFERENTIAL AND DIFFERENCE EQUATIONS. (Eds.: I. Csiszár and Gy. Michaletzky.) Progress in Systems and Control Theory, Vol. 23, Birkhäuser, Boston 1997, xvii+353 pages, ISBN 0-8176-3971-3, price DM 238,–.

The Conference on Stochastic Differential and Difference Equations was held in August 1996 in Győr (Hungary) as a satellite meeting to the fourth world congress of the Bernoulli Society in Vienna. The conference was organized in two parallel sessions, devoted to stochastic partial differential equations and to ARMA models and related topics, respectively. Therefore, these two subjects prevail in the proceedings under review, but interesting contributions addressing other problems in stochastic analysis are also included (let us mention e.g. a paper on invariant measures for wave equations on Riemannian manifolds by A. B. Cruzeiro and Z. Haba). One can only regret the rather high price that might prevent this useful book from wider distributing.

Ivo Vrkoč