BOOK REVIEWS

M. de Gosson: SYMPLECTIC GEOMETRY AND QUANTUM MECHANICS. Birkhäuser-Verlag, Basel, 2006. ISBN 3-7643-7574-4, 367 pages, price EUR 138,–.

This book covers various topics in symplectic geometry with emphasis on applications to quantum mechanics—symplectic treatment of quantum mechanics in semi-classical and operator-theoretical formulation.

Part I starts with rigorous presentation of the basics of symplectic geometry—symplectic spaces and the symplectic group. The principal topic of Part I is the Maslov index and its generalization (Arnold-Leray-Maslov index).

Part II starts with Lagrangian manifolds and their semiclassical quantization. Then the Heisenberg group and construction of Weyl pseudodifferential operators are discussed. The last chapter of Part II is devoted to the study of the metaplectic group and the associated Maslov indices.

The third and last part focuses on Quantum Mechanics in Phase space. For example the uncertainty principle is interpreted in terms of Wigner ellipsoids in phase space. Then Hilbert-Schmidt and trace-class operators are used in treatment of density matrices. In the final chapter Weyl pseudo-differential calculus is extended to phase space.

Some classical topics (Classical Lie Groups, Covering spaces, Pseudodifferential operators and Probability theory) are reviewed in Appendices.

The book is predominantly self-contained, however, it seems that in order to fully appreciate all parts of the book, solid background in quantum theory is required. Pure mathematicians working in geometry and topology would find many of the discussed topics of genuine interest.

Vojtěch Pravda

E. Koelink, J. van Neerven, B. de Pagter, G. Sweers, eds.: PARTIAL DIFFERENTIAL EQUATIONS AND FUNCTIONAL ANALYSIS. Operator Theory: Advances and Applications, Vol. 168. Birkhäuser-Verlag, Basel, 2006. ISBN 3-7643-7600-7, x+294 pages, price Eur 128,–.

These proceedings are dedicated to Philippe Clément on the occasion of his retirement. They originate from the workshop "Partial Differential Equations and Functional Analysis" held at Delft, November 29–December 1, 2004.

Besides the preface of the editors, a portrait of Philippe Clément, and his curriculum vitae, the book contains sixteen contributions from the theory of partial differential and integral equations, operator theory, stochastic differential equations, functional analysis, and numerical analysis. All these articles are listed below together with their brief reviews. The topics of the presented papers reflect very well the wide interests and active contributions of Philippe Clément.

G. Caristi, E. Mitidieri: *Harnack inequality and applications to solutions of biharmonic equations*, pp. 1–26.

The authors prove the local boundedness, continuity, and Harnack type inequalities for the weak solution of the equation $\Delta^2 u = Vu$ in a domain $\Omega \subseteq \mathbb{R}^N$, where V belongs to a natural Kato class of potentials associated to the biharmonic operator. A comment on Green functions for Schrödinger biharmonic operators and applications of the results obtained are presented as well.

C. Carstensen: *Clément interpolation and its role in adaptive finite element error control*, pp. 27–43.

This contribution shows the Clément type interpolations in an abstract setting, it proves their fundamental properties and presents their usage in explicit residual a posteriori error estimators for elliptic problems. The paper is concluded by several short comments on interesting tasks of a posteriori error control.

S. Cerrai: *Ergodic properties of reaction-diffusion equations perturbed by a degenerate multiplicative noise*, pp. 45–59.

This is a generalization of the author's previous results. The paper proves uniqueness, ergodicity, and strongly mixing property of a class of evolution stochastic reaction-diffusion equations of one spatial variable with Dirichlet or Neumann boundary conditions. The results are obtained by random time changes and comparison arguments with Bessel processes.

G. Da Prato, A. Lunardi: *Kolmogorov operators of Hamiltonian systems perturbed by noise*, pp. 61–71.

The article studies a Hamiltonian system with friction perturbed by the Brownian noise and proves that the corresponding Kolmogorov operator possesses a realization that is m-dissipative and generates an analytic semigroup.

A. F. M. ter Elst, D. W. Robinson, A. Sikora, Y. Zhu: *Dirichlet forms and degenerate elliptic operators*, pp. 73–95.

The authors analyse the second-order divergence-form elliptic operators with measurable coefficients by the quadratic form techniques. They prove regularity and locality of the corresponding Dirichlet form and characterize the evolution semigroup by a function over pairs of measurable subsets of $\mathbb{R}^d$.

O. van Gaans: *On R-boundedness of unions of sets of operators*, pp. 97–111.

The paper proves a sufficient condition for the R-boundedness of the union of a sequence of R-bounded sets of operators. The R-boundedness is equivalent to the maximal L^p regularity of the abstract Cauchy problem and the results presented can be used as tools in R-boundedness proofs. In addition, a couple of illustrative examples is given.

M. Geißert, H. Heck, M. Hieber: *On the equation* $\operatorname{div} u = g$ *and Bogovskiĭ's operator in Sobolev spaces of negative order*, pp. 113–121.

Two approaches to prove the existence result for the divergence problem with homogeneous Dirichlet data on a Lipschitz domain are presented. The first approach is based on Bogovskiĭ's solution operator and the Calderón-Zygmund theory, whereas the second one relies on the inhomogeneous Stokes problem.

F. den Hollander: *Renormalization of interacting diffusions: A program and four examples*, pp. 123–136.

The paper analyzes a system of coupled 1D stochastic differential equations which model population dynamics of large colonies. The general system is well described and the stochastic and analytic parts of renormalization are carried out for four examples.

T. P. Hytönen: *Reduced Mihlin-Lizorkin multiplier theorem in vector-valued L^p spaces*, pp. 137–151.

A Fourier multiplier theorem is proved for operator-valued symbols in UMD spaces with property (α). The obtained sufficient condition intersects the known Mihlin-Lizorkin and Hörmander type assumptions.

S.-O. Londen: *Interpolation spaces for initial values of abstract fractional differential equations*, pp. 153–168.

The abstract parabolic evolutionary equations with fractional time derivative are considered in the article. The trace spaces are analyzed and the existence, uniqueness, and regularity results are proven.

N. Okazawa: *Semilinear elliptic problems associated with the complex Ginzburg-Landau equation*, pp. 169–187.

The author considers the stationary version of the complex Ginzburg-Landau equation and proves the existence, uniqueness, and a priori estimates for the strong solution.

J. Prüss, G. Simonett: *Operator-valued symbols for elliptic and parabolic problems on wedges*, pp. 189–208.

The authors characterize the spectrum of the parameter-dependent operators $\lambda e^{sx} + P(\partial_x)$ and $\partial_t e^{sx} + P(\partial_x)$, where P stands for a quadratic polynomial. They show applications to free boundary problems with moving contact lines and they study the diffusion equation in an angle or a wedge domain with dynamic boundary conditions.

J. Prüss, M. Wilke: *Maximal L_p-regularity and long-time behaviour of the non-isothermal Cahn-Hilliard equation with dynamic boundary conditions*, pp. 209–236.

The paper proves the existence, uniqueness, and regularity for the nonlinear Cahn-Hilliard equation with nonconstant temperature and dynamic boundary conditions.

J. Rappaz: *Numerical approximation of PDEs and Clément interpolation*, pp. 237–250.

The author explains a formalism of consistency and stability of the finite element methods for the numerical approximation of nonlinear partial differential equations of elliptic and parabolic type. He proves the a priori and a posteriori error estimates of the residual type and shows the role of the inf-sup condition.

E. G. F. Thomas: *On Prohorov's criterion for projective limits*, pp. 251–261.

A generalization of the Prohorov's theorem is proven and a consequent application to projective limits of Radon measures for direct construction of the Wiener measure on the space of continuous functions is shown.

L. Weis: *The H^∞ holomorphic functional calculus for sectorial operators—a survey*, pp. 263–294.

This is a thorough, selfconsistent, and well written review of the topic. The paper describes the construction of H^∞-functional calculus, examples, some applications, it extends the calculus form Hilbert to Banach spaces, and shows the related techniques.

Tomáš Vejchodský

B. Fine, G. Rosenberger: NUMBER THEORY, AN INTRODUCTION VIA THE DISTRIBUTION OF PRIMES. Birkhäuser-Verlag, Boston, 2007. ISBN 0-8176-4472-7, 342 pages, price EUR 48,–.

This book provides a good introduction to contemporary number theory. It covers the standard topics in elementary number theory as well as giving a basic introduction to analytic number theory and algebraic number theory. As the title of the book suggests, the authors emphasize the properties and distribution of the primes in their treatment of the topics in the book. This book is suitable as a textbook for both upper level undergraduates and beginning graduate students.

Of particular interest are the authors' several proofs of the infinitude of primes. Especially nice are a variation of Euclid's standard proof, two proofs involving Fibonacci numbers, proofs based on the properties of Mersenne numbers, Fermat numbers, and the Euler phi-function, a proof involving continued fractions, a proof based on properties of polynomials with integer coefficients, a proof using codes, and a topological proof. The book also gives fine proofs of several special cases of Dirichlet's theorem on the infinitude of primes in arithmetic progressions.

After carefully laying the groundwork, Fine and Rosenberger also present complete proofs using complex analysis of both the prime number theorem and Dirichlet's theorem referred to earlier. The book also gives a sketch of an elementary proof of the prime number theorem.

The authors also provide applications of number theory to cryptography. In particular, Fine and Rosenberger discuss the RSA public-key cryptosystem and the discrete log problem. In addition, they give a good account of primality testing. The authors especially provide a nice discussion of the recent breakthrough by Agrawal, Kayal, and Saxena in discovering a deterministic polynomial-time algorithm for primality testing.

Lawrence Somer