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OPTIMAL MULTIVARIABLE PID REGULATOR[1]

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A continuous version of optimal LQG design under presence of Wiener disturbances is solved for MIMO controlled plant. Traditional design tools fail to solve this problem due to unstability of the augmented plant. A class of all optimality criteria, which guarantee existence of an asymptotical solution, is defined using a plant deviation model. This class is utilized in design of an optimal state and an error feedback regulator which is presented here. The resultant optimal error regulator is interpreted as an optimal multivariable matrix PID regulator.

1. INTRODUCTION

This paper deals with structure design and parameter setting of an optimal multivariable matrix PID regulator using LQG optimization. This regulator is a generalization of a classical PID regulator often used in industry control applications.

According to alternative system theory [14], the optimization problem is described as a design of an autonomous causal control system composed from an augmented plant and a regulator. The augmented plant is a controlled system comprising all surroundings relevant to the given problem. Components of the control system are mutually connected only via informational relations and they are not influenced by the environment.

There is a large range of literature about the output regulation problem, e. g. [4] for the latest one. However, they mostly deal with deterministic models. We consider a non-astatic stochastic linear t-invariant system, where the nominal output and external plant disturbances are modeled by Wiener process represented by a system of parallel integrators. According to internal model principle [5], an integration feedback must be included in the system in order to guarantee the error to be asymptotically zero.

In this paper, we study a connection between the solution of LQ/LQG optimization and design of PID regulators. We use the results of [7, 13] to derivate a plant deviation model. This model is used for defining a class of such optimality criteria that allows to design a state space feedback regulators. Even if the augmented plant is unstable, the standard approach of LQG optimization can be used.

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In cases, where whole plant state is not measured, the optimal regulator uses its optimal estimation. However, this can be complicated by undetectability of some states of the augmented plant. Then the estimator is designed using a fact that the optimal control law is a specific combination of the linear state variables.

The resultant optimal error feedback regulator can be interpreted as a matrix MIMO PID regulator. We would like to point out that the LQG optimization gives both optimal regulator structure and optimal parameter setup.

2. PROBLEM FORMULATION

Consider an augmented plant of the controlled plant and a Wiener model of the surroundings described by

$$\dot{x} = \boldsymbol{A} \cdot x + \boldsymbol{B} \cdot u + \boldsymbol{G} \cdot w + \boldsymbol{\Gamma} \cdot \xi \quad (1)$$

$$\dot{w} = \boldsymbol{\Delta} \cdot \xi \quad (2)$$

$$y = \boldsymbol{C} \cdot x \quad (3)$$

$$y_M = \boldsymbol{C} \cdot x + \boldsymbol{H} \cdot w, \quad (4)$$

where $x \in \mathbb{R}^n$ is state of the controlled plant (1) and $u \in \mathbb{R}^r$ $y \in \mathbb{R}^p$ are its control input and output, respectively. Further, $w \in \mathbb{R}^m$ is vector of Wiener disturbances, and $\xi \in \mathbb{R}^q$ is an absolutely random fictitious process with zero mean and known covariance which models all randomness in the controlled plant. The additional variable $y_M \in \mathbb{R}^p$ represents a measured output available for control. Block scheme of the augmented plant is shown in Figure 1.

It is supposed that the dimensions of control input and controlled output are equal. Furthermore, nonsingularity of the dynamic matrix $\boldsymbol{A}$ and full rank of the gain matrix $\boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{B}$ are assumed.

The following text deals in detail with both the full information feedback design, where y_M corresponds to the augmented state (x, w), and the error feedback design, where y_M is the control error $e = y - y_R$.

We look for such a controller that realizes a causal control law

$$u(t) = \phi(t, y_M[0, t], u[0, t)) \quad (5)$$

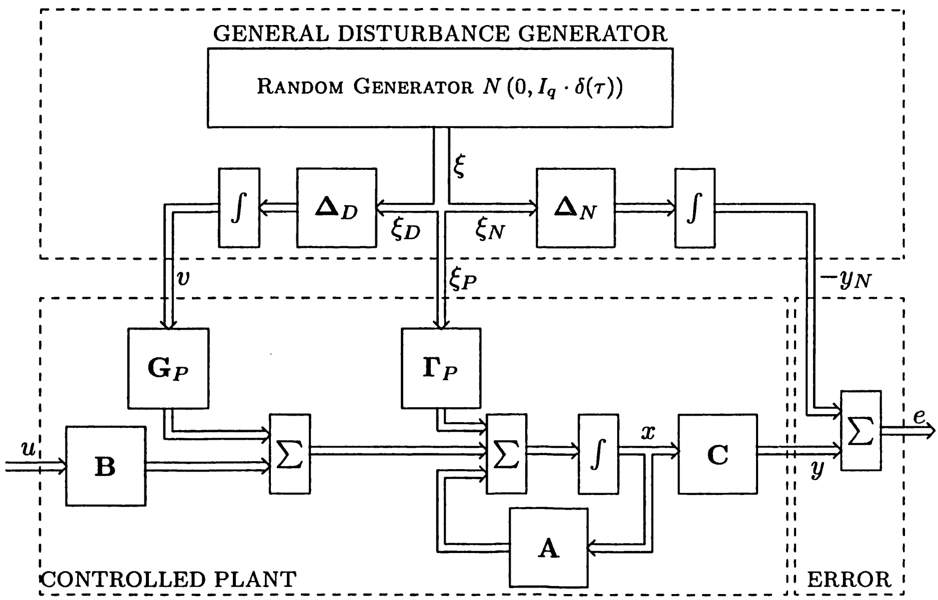
and guarantees the best behavior of the control system for given conditions. Asymptotic solution of the LQG optimization is not feasible, because the controlled augmented plant is not stabilizable due to presence of the generator of Wiener disturbance (2). In most cases variance of the error e grows to infinity and value of the standard quadratic optimality criteria becomes unlimited.

The need to define another class of quadratic optimality criteria which removes this unpleasant property is simplified by definition of a deviation model of the augmented plant [13]. The deviation model is obtained by the following transformation of the plant state

$$x_E = x + \boldsymbol{A}^{-1} \cdot (\boldsymbol{G} - \boldsymbol{B} \cdot \boldsymbol{L}) \cdot w, \quad (6)$$

where

$$\boldsymbol{L} = -\left(\boldsymbol{C} \cdot \boldsymbol{A}^{-1} \cdot \boldsymbol{B}\right)^{-1} \cdot \left(\boldsymbol{H} - \boldsymbol{C} \cdot \boldsymbol{A}^{-1} \cdot \boldsymbol{G}\right). \quad (7)$$



$$\xi' = [\ \xi_N' \quad \xi_P' \quad \xi_N'\ ],\ w' = [\ -y_N' \quad v'\ ]$$

Fig. 1. Block scheme of the augmented plant.

Then the plant deviation model is given as

$$\dot{x}_E = \boldsymbol{A} \cdot x_E + \boldsymbol{B} \cdot u_E + \boldsymbol{\Gamma}_E \cdot \xi \tag{8}$$

$$e = \boldsymbol{C} \cdot x_E. \tag{9}$$

The input u_E is defined as deviation

$$u_E = u - u_N \tag{10}$$

of input u from its nominal trajectory

$$u_N = -\boldsymbol{L} \cdot w. \tag{11}$$

An optimal state feedback regulator for the plant deviation model can be obtained by applying standard tools of LQG optimization for optimality criterion

$$J = \lim_{t_F \to +\infty} \mathsf{E}\left\{ \frac{1}{t_F} \int_0^{t_F} (x_E' \cdot \boldsymbol{Q} \cdot x_E + u_E' \cdot \boldsymbol{R} \cdot u_E)\, \mathrm{d}t \right\}, \tag{12}$$

where $\boldsymbol{Q} \geq 0$ and $\boldsymbol{R} > 0$.

A class of quadratic optimality criteria which guarantee existence of a solution of the LQG problem for the augmented plant (1)–(4) is defined by utilizing equivalence of the plant deviation model and the augmented plant. This class is obtained by substitution of the transformations (6) and (10) to the criterion (12). Then the criterion evaluating performance of the augmented plant can be expressed in a form

$$J = \lim_{t_F \to +\infty} \mathsf{E} \left\{ \frac{1}{t_F} \int_0^{t_F} (e' \cdot \boldsymbol{Q}_e \cdot e + (u + \boldsymbol{L} \cdot w)' \cdot \boldsymbol{R} \cdot (u + \boldsymbol{L} \cdot w) \, \mathrm{d}t \right\}, \tag{13}$$

where $\boldsymbol{Q}_e = \boldsymbol{C}'\boldsymbol{Q}\boldsymbol{C}$. Since we have assumed controllability and observability of the controlled plant, providing $\boldsymbol{Q}_e > 0$ the solution of the optimization for the criterion (13) yields an optimal state feedback regulator. Note that a similar way of the selection optimality criterion was used for solving SISO deterministic tracking problem, see e. g. [12].

3. OPTIMAL LQG REGULATOR

Due to unstability of the augmented plant, the standard design tools can not be used directly. However, the solution can be obtained using equivalence of the deviation model and the augmented plant.

Assume that solution of the algebraic Riccati equation relevant to optimality criterion (12) exists. Then the optimal state feedback regulator for deviation model (8) – (9) is obtained using standard tools [3] as

$$u_E^* = -\boldsymbol{L}_E \cdot x_E. \tag{14}$$

Design of the optimal feedback regulator for measured output $y_M = (x, w)$ follows from transformations (10) and (6) and has a form

$$u^* = -\left(\boldsymbol{L} + \boldsymbol{L}_E \cdot \boldsymbol{A}^{-1} (\boldsymbol{G} - \boldsymbol{B} \cdot \boldsymbol{L})\right) \cdot w - \boldsymbol{L}_E \cdot x. \tag{15}$$

Now, we discuss the error feedback regulator in detail. From the separation theorem, the optimal error feedback regulator for measured output $y_M = e$ is given as

$$u^* = \hat{u}_N - \boldsymbol{L}_E \cdot \hat{x}_E, \tag{16}$$

where

$$\hat{u}_N(t) = \mathsf{E}\{u_N(t) \,|\, e\langle 0, t), u\langle 0, t)\} \tag{17}$$

$$\hat{x}_E(t) = \mathsf{E}\{x_E(t) \,|\, e\langle 0, t), u\langle 0, t)\} \tag{18}$$

are estimations of u_N and x_E produced by an optimal estimator. This design of an estimator removes the problem of nondetectability of some states of the augmented plant. The estimator produces estimations of the nominal output u_N and the deviation state x_E which are observable through error e.

Since we assume $(\boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{B})$ to be nonsingular, the output matrix $\boldsymbol{C}$ must be of full row rank. This allows us to find such a regular transformation that the output matrix of the system has a form

$$\boldsymbol{C} = [\boldsymbol{I}_p \;\; \boldsymbol{0}], \tag{19}$$

where $\boldsymbol{I}_p$ is an identity matrix with dimension p.

Hence we can rewrite the plant deviation model (8) for $u_E = u - u_N$ as

$$\begin{bmatrix} \dot{e} \\ \dot{x}_e \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{1,1} & \boldsymbol{A}_{1,2} \\ \boldsymbol{A}_{2,1} & \boldsymbol{A}_{2,2} \end{bmatrix} \begin{bmatrix} e \\ x_e \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_1 \\ \boldsymbol{B}_2 \end{bmatrix} u - \begin{bmatrix} \boldsymbol{B}_1 \\ \boldsymbol{B}_2 \end{bmatrix} u_N + \begin{bmatrix} \boldsymbol{\Gamma}_{E,1} \\ \boldsymbol{\Gamma}_{E,2} \end{bmatrix} \xi. \quad (20)$$

The dynamic model of the nominal control u_N is obtained by differentiation of equation (11). After substitution (2) into (11), the behavior of the nominal control can be described as

$$\dot{u}_N = -\boldsymbol{L} \cdot \boldsymbol{\Delta} \cdot \xi. \quad (21)$$

Equations (20) and (21) represent a suitable model for estimation of x_e and u_N. From the assumption of observability of $(\boldsymbol{C}, \boldsymbol{A})$ and nonsingularity of matrix $\boldsymbol{A}$, it can be concluded that the state of the estimation model is observable.

Using the estimation model, we define a fictitious measurement

$$z = \dot{e} - \boldsymbol{A}_{1,1} \cdot e - \boldsymbol{B}_1 \cdot u. \quad (22)$$

After substitution of $\dot{e}$ from (20) into (22), the variable z can be expressed as

$$z = \boldsymbol{A}_{1,2} \cdot x_e - \boldsymbol{B}_1 \cdot u_N + \boldsymbol{\Gamma}_{E,1} \cdot \xi. \quad (23)$$

Using (20), (21) and (23), the estimation model can be rewritten as

$$\begin{aligned} \begin{bmatrix} \dot{x}_e \\ \dot{u}_N \end{bmatrix} &= \begin{bmatrix} \boldsymbol{A}_{2,2} & -\boldsymbol{B}_2 \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} x_e \\ u_N \end{bmatrix} + \begin{bmatrix} \boldsymbol{A}_{2,1} & \boldsymbol{B}_2 \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} e \\ u \end{bmatrix} + \bar{\xi}_1 \quad (24) \\ z &= \begin{bmatrix} \boldsymbol{A}_{1,2} & -\boldsymbol{B}_1 \end{bmatrix} \begin{bmatrix} x_e \\ u_N \end{bmatrix} + \bar{\xi}_2, \quad (25) \end{aligned}$$

where $\bar{\xi}_1$ and $\bar{\xi}_2$ are absolutely random processes with zero mean and covariance matrices as

$$\begin{aligned} \text{cov}_{\bar{\xi}_1,\bar{\xi}_1} &= \begin{bmatrix} \boldsymbol{\Gamma}_{E,2}\boldsymbol{\Gamma}'_{E,2} & \boldsymbol{\Gamma}_{E,2}\boldsymbol{\Delta}'\boldsymbol{L}' \\ \boldsymbol{L}\boldsymbol{\Delta}\boldsymbol{\Gamma}'_{E,2} & \boldsymbol{L}\boldsymbol{\Delta}\boldsymbol{\Delta}'\boldsymbol{L}' \end{bmatrix}, \quad \text{cov}_{\bar{\xi}_2,\bar{\xi}_2} = \boldsymbol{\Gamma}_{E,1}\boldsymbol{\Gamma}'_{E,1}, \\ \text{cov}_{\bar{\xi}_1,\bar{\xi}_2} &= \begin{bmatrix} \boldsymbol{\Gamma}_{E,2}\boldsymbol{\Gamma}_{E,1} \\ \boldsymbol{L}\boldsymbol{\Delta}\boldsymbol{\Gamma}'_{E,1} \end{bmatrix}. \end{aligned} \quad (26)$$

An optimal estimator for the estimation model (24) and (25) and the noise covariance matrices (26) are obtained by standard design tools in a form

$$\begin{aligned} \begin{bmatrix} \dot{\hat{x}}_e \\ \dot{\hat{u}}_N \end{bmatrix} &= \begin{bmatrix} \boldsymbol{A}_{2,2} & -\boldsymbol{B}_2 \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \hat{x}_e \\ \hat{u}_N \end{bmatrix} + \begin{bmatrix} \boldsymbol{A}_{2,1} & \boldsymbol{B}_2 \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} e \\ u \end{bmatrix} + \boldsymbol{K} \cdot (z - \hat{z}) \quad (27) \\ \hat{z} &= \boldsymbol{A}_{1,2} \cdot \hat{x}_e - \boldsymbol{B}_1 \cdot \hat{u}_N, \quad (28) \end{aligned}$$

where the innovation $(z - \hat{z})$ was derived from (23) and (28) as

$$(z - \hat{z}) = \dot{e} - \boldsymbol{A}_{1,1} \cdot e - \boldsymbol{B}_1 \cdot u - \boldsymbol{A}_{1,2} \cdot \hat{x}_e + \boldsymbol{B}_1 \cdot \hat{u}_N. \quad (29)$$

The optimal regulator (16) is given by the gain matrix $\boldsymbol{L}_E$. If we denote the first p columns of the state regulator gain matrix $\boldsymbol{L}_E$ as $\boldsymbol{L}_e$ and the remaining block as $\boldsymbol{L}_x$, the regulator can be rewritten as

$$u^* = \hat{u}_N - \boldsymbol{L}_x \cdot \hat{x}_e - \boldsymbol{L}_e \cdot e. \tag{30}$$

The optimal error feedback regulator with inputs u and e and output u^* is represented by equations (27) and (30). The recommended control u^* is optimal in open–loop control. Thus the LQG regulator can be used in an open–loop as an advisor in the control system with variable structure of the plant (see Figure 2a). This can be used for elimination of wind–up effect, as e. g. in [2].

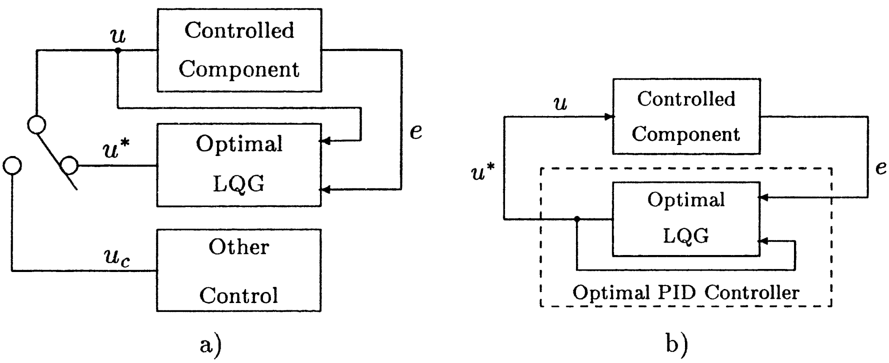


Fig. 2. Structure of LQG and PID control.

4. OPTIMAL PID REGULATOR

Here, we assume that every recommended control is realized, which means that

$$u = u^*. \tag{31}$$

Then, after substitution (31) into (27), we obtain the optimal feedback regulator as shown in Figure 2b. The following text deals with a relation between this and a PID regulator.

Denoting the last p rows of the innovation gain matrix $\boldsymbol{K}$ as $\boldsymbol{K}_2$ and the remaining block as $\boldsymbol{K}_1$, the optimal regulator (27) and (30) can after substitution of (31) into (27) and some calculations be rewritten in a form

$$\dot{\hat{x}}_e = \boldsymbol{A}_{R,1} \cdot \hat{x}_e + \boldsymbol{B}_{R,1} \cdot e + \boldsymbol{K}_1 \cdot \dot{e} \tag{32}$$

$$\dot{\hat{u}}_N = \boldsymbol{A}_{R,2} \cdot \hat{x}_e + \boldsymbol{B}_{R,2} \cdot e + \boldsymbol{K}_2 \cdot \dot{e} \tag{33}$$

$$u^* = -\boldsymbol{L}_x \cdot \hat{x}_e + \hat{u}_N - \boldsymbol{L}_e \cdot e, \tag{34}$$

where

$$\begin{aligned} \boldsymbol{A}_{R,1} &= \boldsymbol{A}_{2,2} - \boldsymbol{K}_1 \cdot \boldsymbol{A}_{1,2} - (\boldsymbol{B}_2 - \boldsymbol{K}_1 \cdot \boldsymbol{B}_1) \cdot \boldsymbol{L}_x \\ \boldsymbol{A}_{R,2} &= -\boldsymbol{K}_2 \cdot \boldsymbol{A}_{1,2} + \boldsymbol{K}_2 \cdot \boldsymbol{B}_1 \cdot \boldsymbol{L}_x \\ \boldsymbol{B}_{R,1} &= \boldsymbol{A}_{2,1} - \boldsymbol{K}_1 \cdot \boldsymbol{A}_{1,1} - (\boldsymbol{B}_2 - \boldsymbol{K}_1 \cdot \boldsymbol{B}_1) \cdot \boldsymbol{L}_e \\ \boldsymbol{B}_{R,2} &= -\boldsymbol{K}_2 \cdot \boldsymbol{A}_{1,1} + \boldsymbol{K}_2 \cdot \boldsymbol{B}_1 \cdot \boldsymbol{L}_e. \end{aligned} \tag{35}$$

If the matrix $\boldsymbol{A}_{R,1}$ is nonsingular, then p eigenvalues of the regulator (32)–(34) are zero. They can be interpreted as p parallel integrators in the error feedback. Such regulator, according the internal model principle [5], guarantees robust servomechanism of control system.

The regulator (27) and (30) can be easily written as a matrix MIMO PID regulator, after using following substitutions

$$\hat{x}_e = x_D - \boldsymbol{A}_{R,1}^{-1} \cdot \boldsymbol{B}_{R,1} \cdot e \tag{36}$$

$$\hat{u}_N = u_I + \boldsymbol{A}_{R,2} \cdot \boldsymbol{A}_{R,1}^{-1} \cdot \hat{x}_e - \left(\boldsymbol{A}_{R,2} \cdot \boldsymbol{A}_{R,1}^{-1} \cdot \boldsymbol{K}_1 - \boldsymbol{K}_2\right) \cdot e, \tag{37}$$

where $\boldsymbol{A}_{R,1}$ is assumed to be a nonsingular matrix. After (36) and (37) are substituted into (32)–(34) and some simplifications we obtain the matrix MIMO PID regulator as

$$u_P = \boldsymbol{K}_P \cdot e \tag{38}$$

$$\boldsymbol{T}_I \cdot \dot{u}_I = e \tag{39}$$

$$\boldsymbol{T}_D \cdot \dot{x}_D + x_D = \boldsymbol{B}_D \cdot \dot{e} \tag{40}$$

$$u_D = \boldsymbol{C}_D \cdot x_D \tag{41}$$

$$u^* = u_P + u_I + u_D, \tag{42}$$

where u_P is proportional, u_I integrational and u_D derivative control component and

$$\begin{aligned} \boldsymbol{K}_P &= -\boldsymbol{L}_e - \boldsymbol{A}_{R,2} \cdot \boldsymbol{A}_{R,1}^{-1} \cdot \boldsymbol{K}_1 + \boldsymbol{K}_2 + \boldsymbol{L}_x \cdot \boldsymbol{A}_{R,1}^{-1} \cdot \boldsymbol{B}_{R,1} \\ &\quad -\boldsymbol{A}_{R,2} \cdot \boldsymbol{A}_{R,1}^{-1} \cdot \boldsymbol{A}_{R,1}^{-1} \cdot \boldsymbol{B}_{R,1} \\ \boldsymbol{T}_I &= \left(\boldsymbol{B}_{R,2} - \boldsymbol{A}_{R,2} \cdot \boldsymbol{A}_{R,1}^{-1} \cdot \boldsymbol{B}_{R,1}\right)^{-1} \\ \boldsymbol{T}_D &= -\boldsymbol{A}_{R,1}^{-1} \\ \boldsymbol{B}_D &= -\boldsymbol{A}_{R,1}^{-1} \cdot \left(\boldsymbol{K}_1 + \boldsymbol{A}_{R,1}^{-1} \cdot \boldsymbol{B}_{R,1}\right) \\ \boldsymbol{C}_D &= \boldsymbol{A}_{R,2} \cdot \boldsymbol{A}_{R,1}^{-1} - \boldsymbol{L}_x. \end{aligned} \tag{43}$$

Here, the matrix $\boldsymbol{K}_P$ is a proportional and $\boldsymbol{K}_D = \boldsymbol{C}_D \boldsymbol{B}_D$ a derivative feedback gain matrix. Matrices $\boldsymbol{T}_I$, $\boldsymbol{T}_D$ are integrational and derivative matrix time constant, respectively.

This structure of the optimal error feedback regulator was expected. However, the structure of the matrix derivation block is unusual. If we rewrite it in a Jordan form, we obtain $(n-p)$ scalar derivators. Time constants of these derivators are given by the eigenvalues of $\boldsymbol{T}_D$. Input of the derivators is a linear weighted combination of the error. The derivative control action is then given by a linear combination of the individual output derivator components.

5. EXAMPLE

Here, we shown an example of the proposed LQG optimal control design. Consider a stable plant of a third order with two inputs and two outputs, where

$$\boldsymbol{A}=\begin{bmatrix}-1 & 0 & 0\\ 0 & -\frac{1}{2} & 0\\ 0 & 0 & -1\end{bmatrix},\quad \boldsymbol{B}=\begin{bmatrix}1 & 0\\ 0 & \frac{1}{2}\\ 0 & 1\end{bmatrix},\quad \boldsymbol{G}=\begin{bmatrix}0 & 0 & -1\\ 0 & 0 & -\frac{1}{2}\\ 0 & 0 & -1\end{bmatrix},$$

$$\boldsymbol{C}=\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix},\ \boldsymbol{H}=\begin{bmatrix}-1 & 0 & 0\\ 0 & -1 & 0\end{bmatrix},\ \boldsymbol{\Gamma}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\ \boldsymbol{\Delta}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.$$

The generalized disturbance w represents reference signals of outputs y and exo-disturbances of the controlled plant. Diagonality of the disturbance generator matrix $\boldsymbol{\Delta}$ guarantees a mutual independence of the components of the disturbance vector w.

In this case, the solution of equation (7), which determines the nominal control (11), is given by matrix

$$\boldsymbol{L}=\begin{bmatrix}-1 & 0.5 & -1\\ 0 & -0.5 & -1\end{bmatrix}.$$

Defining the optimality criterion (12) as

$$J=\lim_{t_F\to\infty}\mathsf{E}\left\{\frac{1}{t_F}\int_0^{t_F}\left((y-y_R)'\cdot(y-y_R)+(u-u_N)'\cdot(u-u_N)\right)\,\mathrm{d}t\right\}.$$

the gain matrix of the optimal regulator (15) for measured state of the augmented plant is given as

$$\boldsymbol{L}_e=\begin{bmatrix}0.4825 & -0.0404 & -0.0798\\ 0.3425 & 0.5369 & -0.1618\end{bmatrix}.$$

Finally, the calculation of parameters of the matrix MIMO PID regulator (38)–(42) yields

$$\boldsymbol{K_P}=\begin{bmatrix}-1.8991 & -0.2793\\ -0.6739 & -1.8811\end{bmatrix},\quad \boldsymbol{T_I}=\begin{bmatrix}-0.4340 & -0.0181\\ 0.2849 & -0.6222\end{bmatrix},$$

$$\boldsymbol{T_D}=1.4609,\quad \boldsymbol{B}=\begin{bmatrix}-0.0003 & 0.5023\end{bmatrix},\quad \boldsymbol{C_D}=\begin{bmatrix}0.8233\\ 0.7855\end{bmatrix}.$$

As there is less information available for the PID regulator then for the state regulator, its control quality is lower. In this example, the optimality criterion value of the state regulator is $J^*_{\mathrm{state}}=0.831$, and of the PID regulator is $J^*_{\mathrm{PID}}=1.503$. Responses of the plant variables u, y to a unit step of w are shown in Figure 3. Both regulators, state and PID one, were used.

Difference between control quality of the state and the error feedback control increases when the plant is not stable. This fact will be demonstrated in the next example with an unstable plant. The plant has the same quadratic form [10] of all transfer functions between outputs and inputs as the stable plant. Two transfer functions have the same quadratic form if and only if the absolute values of their zeros and poles and gains are identical. The most different results are obtained for a "conjugated" plant with zeros and poles of all transfer functions which lie opposite to poles and zeros of the stable plant in the complex plane.

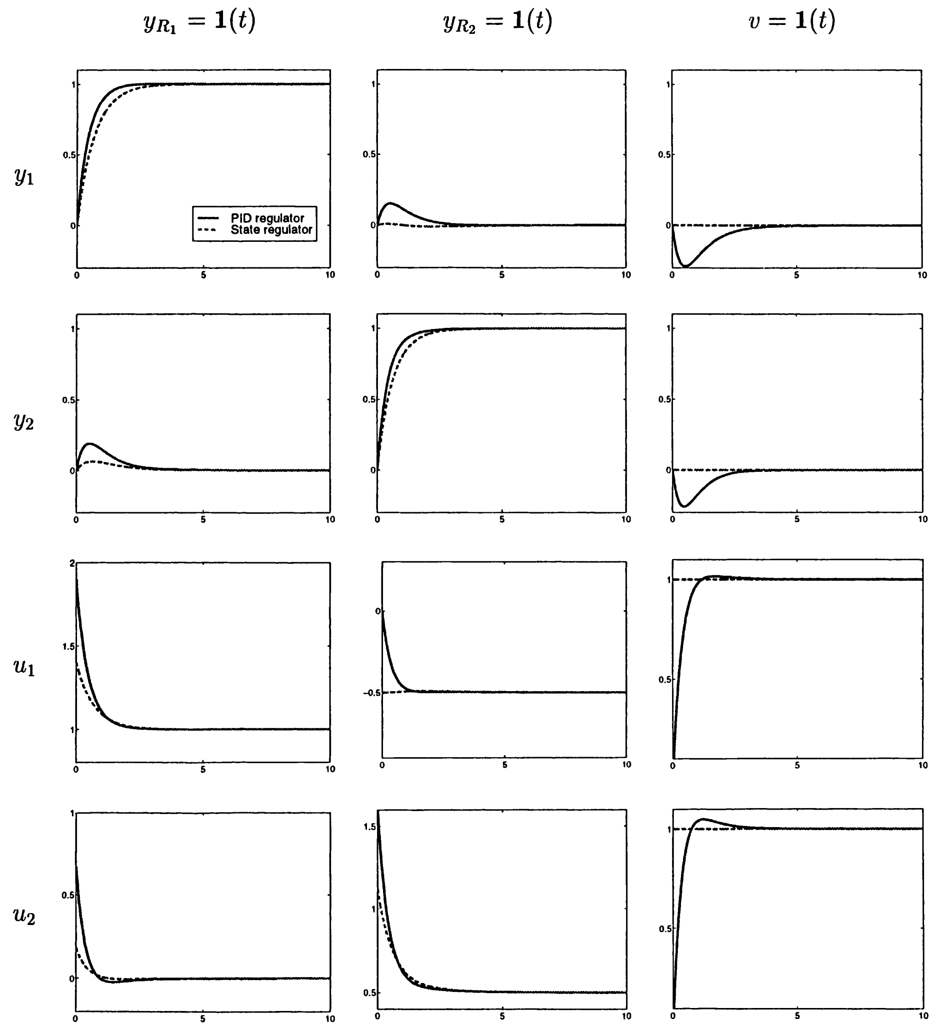


Fig. 3. Responses of control system to an input step on exo-disturbances.

"Conjugated" plant is created by changing signs of matrices $\boldsymbol{A}$ and $\boldsymbol{C}$. Then the value of the optimality criterion of the state regulator is $J_{\text{state}} = 7.997$ and of the error regulator $J_{\text{PID}} = 20050.830$. The difference in quality between control of a stable and an unstable plant with state regulator is shown in Figure 4.

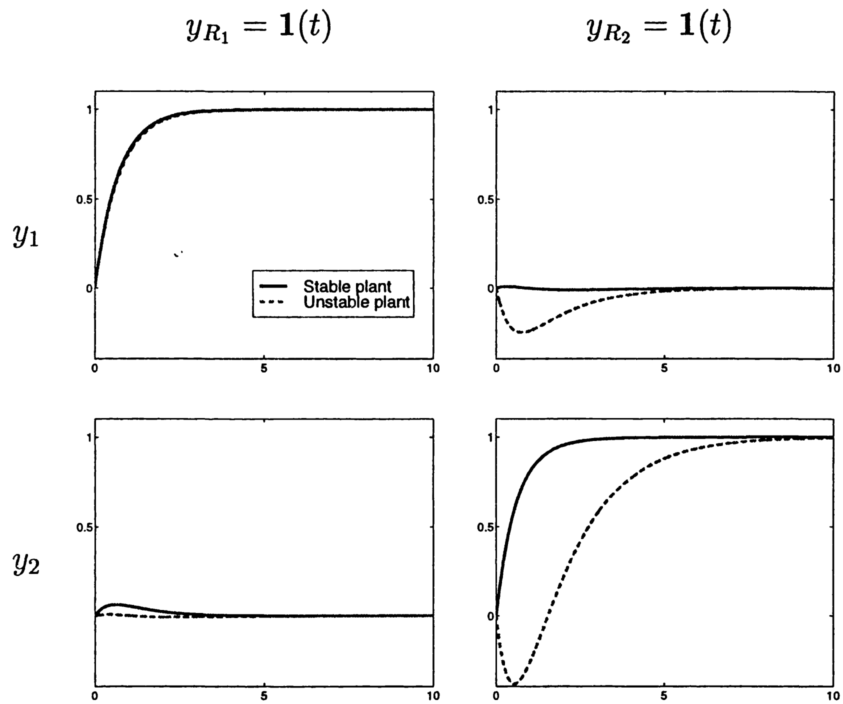


Fig. 4. Responses of control system with state regulator and stable/unstable plant to an input step on exo-disturbances.

6. CONCLUSIONS

In this paper, solution of a continuous version of the LQG problem under presence of Wiener disturbances was shown. As a result of the optimization for error regulation, a structure and a parameter setup of a matrix MIMO PID regulator was obtained. It can be proven that the matrix MIMO PID regulator guarantees a servo robustness of the control system.

Authors experience [8] shows that in most cases the complexity of the error feedback regulator structure can be reduced without a significant loss of quality. Parametric optimization of the regulator structure can also provides information about the quality loss for the reduced regulator structure.

Discrete version of the presented problem can be solved the same way [9].

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