

## Book reviews

*Kybernetika*, Vol. 38 (2002), No. 6, [761]--764

Persistent URL: <http://dml.cz/dmlcz/135501>

## Terms of use:

© Institute of Information Theory and Automation AS CR, 2002

Institute of Mathematics of the Academy of Sciences of the Czech Republic provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.



This paper has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library*  
<http://project.dml.cz>

IVAN KRAMOSIL

## Probabilistic Analysis of Belief Functions

ISFR International Series on Systems Science and Engineering, Volume 16.  
Kluwer Academic / Plenum Publishers. New York – Boston – Dordrecht –  
– London – Moscow 2001.  
xvii + 214 pages.  
ISBN 0-306-46702-X.

The adequate processing of indeterminism in the modelled phenomena belongs to the most urgent topics of the (not only) applied mathematics of the second half of the 20th century, and it will be probably also one of the most inspiring tasks of the mathematics in the beginning of a new century. Good management of indeterminism of any kind (randomness, vagueness, uncertainty and others) is a necessary condition for further development of some applications of mathematical tools in such branches like optimization, decision-making, economic models, sociological models and many others, in which the indeterminism represents a significant component of the modelled and investigated reality.

The referred book contributes to this field in a very important way. It is focused on various aspects of the Dempster–Shafer theory and namely of its fundamental concept of the belief function. As far as the author of this review knows, the Kramosil's book is the first general monograph summarizing the probabilistic views on the concept of the belief function, its properties, and its wider consequences for the modelling of uncertainty. Of course, the Dempster–Shafer theory and belief functions can be treated even without using the probabilistic concepts. On the other hand, including these concept in the original model means some advantages. It supports the realism of the description of the optimization problems and their solutions. Even more, it opens the way to qualified utilization of the classical and well developed probabilistic apparatus for an inspiring approach to qualitatively new types of problems.

The book is written in a lucid and mathematically perfect way. The explanation of the topic is divided into 14 relatively short parts. It simplifies the orientation of the reader in the presented text and in the relation between its particular parts. These 14 parts can be, with certain degree of simplification, divided into three groups, which are not strictly separated but which fluently pass over the each other.

The first group contains chapters of rather introductory character (*Introduction, Preliminaries on Axiomatic Probability Theory, Probabilistic Model of Decision Making Under Uncertainty* and in certain degree also *Basic Elements of Dempster–Shafer Theory*). These chapters introduce and discuss the background of the investigated problems and of the mathematical tools prepared for their analysis.

In the second group, we can include a few chapters oriented to the classical model of belief functions and to its stochastic version based on the standard probabilistic concepts. The last chapter mentioned in the previous paragraph is a sort of transition between the previous and this part. Its other chapters are *Elementary Properties of Belief Functions, Probabilistic Analysis of Dempster Combination Rule* and also *Nonspecificity Degrees of Basic Probability Assignments*. The last group contains specific modifications of the standard model, using some advanced or generalized counterparts of the classical probabilistic and generally mathematical concepts used in the chapters from the previous part. It contains chapters devoted to *Belief Functions Induced by Partial Compatibility Relations, Belief Functions Over Infinite State Spaces, Boolean Combinations of Set-Valued Random Variables, Belief Functions With Signed and Nonstandard Values, Jordan Decomposition of Signed Belief Functions* and also rather special modifications of the investigated con-

cepts and their processing in specific models, namely *Monte-Carlo Estimations for Belief Functions*, and *Boolean-Valued and Boolean-Like Processed Belief Functions*.

The monograph is completed by a list of references (113 items) which representatively covers the topics mentioned in the book, and by an Index.

The referred book offers a complete survey of the methods and results regarding the relevant branches. Moreover, the exact mathematical presentations of the theoretical concepts is completed by a heuristic and methodological comments and discussions of their mutual relations, as well as of their wider consequences for the human understanding of the real world. Even this feature turns the book to something more than “only” perfect mathematical monograph. Some of the readers may, perhaps, miss some numerical examples illustrating the theoretical explanations. Nevertheless, it is to be stated that such numerical illustrations could break the homogeneity of the referred work, and that they can be found, if necessary, in the referred literature.

The Kramosil's book can be recommended to readers who have managed the basic mathematical culture and who are interested in a representative explanation of the topic mentioned in its heading.

*Milan Mareš*

JAROSLAV RAMÍK, MILAN VLACH

## **Generalized Concavity in Fuzzy Optimization and Decision Analysis**

Kluwer's International Series in Operations Research and Management Sciences.  
Kluwer Academic Publishers. Boston – Dordrecht – London 2002.

xv + 296 pages.

ISBN 0-7923-7495-9.

The optimization and decision-making procedures and their modifications belong to the fundamental topics of the advanced applied mathematics. It follows from the essential properties of these problems that convex set of feasible alternatives and concave (eventually also convex) objective functions play an exceptional role in the formulation and solvability of the mentioned optimization problems. There exist numerous important results based on the pure convexity and concavity. One of the significant problems to be solved is to extend these results to a wider class of models in which the convexity and concavity are substituted by some rather modified and, hence, generalized concepts in some sense similar to the classical ones. The presented book represents an interesting and useful contribution to this trend, and it is to be stressed that it contributes to the topic in an excellent way.

The text of the book is divided into ten chapters which are grouped into two parts. The first one includes, besides *Preliminaries*, the theoretical models and results. They start with several approaches to the *Generalized Convex Sets*, including sections devoted to starshaped sets and their modification. Chapter on *Generalized Concave Functions* follows and it is focused on such topics like starshaped functions, quasiconnected functions, differentiable functions, pseudoconcave functions and some others. These chapters are followed by a brief but essential survey of *Triangular Norms and T-Quasiconcave Functions*, their properties and representation. The next chapter is focused on *Aggregation Operators*, their basic properties and continuity, averaging aggregation operators, Sugeno and Choquet integrals and also the concept of aggregation of functions. The last chapter of the first part deals

with the presentation of *Fuzzy Sets* and other concepts related to them, like fuzzy relations, fuzzy numbers and others.

The second part of the book is devoted to four chapters on the applications of the above theoretical concepts. They deal with *Fuzzy Multi-Criteria Decision Making* especially focused on fuzzy criteria, their aggregation, their extreme properties, compromise decisions and some other related topics. The next chapters deal with *Fuzzy Mathematical Programming* and its properties and *Fuzzy Linear Programming* including duality, properties of solution and several special models. The last chapter of the second part is oriented to *Fuzzy Sequencing and Scheduling* including its deterministic and stochastic versions. The book is completed by the list of symbols, the Index and a representative (139 items) list of references.

Both, the theoretical and the applied parts are very well written with full respect to mathematical exactness as well as to the readability and understandability of the text. The organization of chapters and sections is logical, and it supports the reader's understanding of the relations between particular topics.

The referred book represents a valuable contribution to mathematical processing of uncertainty of different type existing in the optimization and decision-making models. It covers certain gap existing in this field of study, and it can be recommended as a useful source of knowledge and inspiration for everybody who is interested in a precise survey of the approaches and results achieved in the relevant part of mathematics.

Milan Mareš

H. W. KUHN, S. NASAR, Eds.:

### **The Essential John Nash**

Princeton University Press, Princeton 2002.

xxv + 244 pages.

ISBN 0-691-09527.

The referred book aims to characterize the person of John Forbes Nash, an outstanding mathematician whose contribution to the theory of game (and not only to it) is extraordinary. Only these professional qualities and the winning of Nobel Prize for economy (1994, with Reinhard Selten and John Harsanyi) themselves could deserve attention enough for edition of the referred volume. In the case of John Nash, they are connected with an impressive life and personal experience which is not usual not only within the scientific community. The referred volume offers a kind and well informed views on the main milestones of the life and scientific work of John Nash.

The volume aiming to cover such wide field of facts must necessarily be rather heterogeneous. After introductory *Preface* and *Introduction* written by the editors, it includes biographically oriented section, namely *Press Release* of the Royal Swedish Academy of Sciences informing about the winners of the Nobel Prize 1994, *Autobiography* by John F. Nash, and also a *Photo Essay* offering a lot of pictures connected with Nash's life and work.

The following ten sections are devoted to particular topics of the Nash's scientific work. The first several of them are oriented to the game theory, namely *the Game of Hex*, *The Bargaining Problem* (these two sections are introduced by editors), *Equilibrium Points in n-Person Games*, *Non-Cooperative Games* (which is published as a facsimile of Ph. D. Thesis),

paper on *Non-Cooperative Games* (the last three sections are introduced and commented by the author), and, finally, *Two-Person Cooperative Games*. The remaining four sections are devoted to *Parallel Control* (with editors' introduction), *Real Algebraic Manifolds*, *The Imbedding Problem for Riemannian Manifolds* (with author's note), and *Continuity of Solutions of Parabolic and Elliptic Equations*. The book is completed by a brief *Afterword* and important list of *Sources* presenting precise references to the publications introduced in the previous sections.

The volume is composed carefully with very good knowledge of both, the person of John Nash and his work. Regarding the formal ordering of the text, it is natural and respecting the essential relations between the referred topics. The reader can only ask, why the exact references of the papers are summarized at the end of the volume and not attached to particular quoted contributions. Such representative presentation of an outstanding scientist could be also completed by his complete (or extensive) bibliography. But these marginal comments do not change the fact that the volume is done with responsibility and care.

The contribution of John Forbes Nash to the modern mathematics is really remarkable, and everybody who is interested in it can find all essential about it in the referred volume.

*Milan Mareš*