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A FEEDFORWARD COMPENSATION SCHEME FOR PERFECT DECOUPLING OF MEASURABLE INPUT FUNCTIONS

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In this paper the exact decoupling problem of signals that are accessible for measurement is investigated. Exploiting the tools and the procedures of the geometric approach, the structure of a feedforward compensator is derived that, cascaded to a linear dynamical system and taking the measurable signal as input, provides the control law that solves the decoupling problem and ensures the internal stability of the overall system.

Keywords: geometric control theory, disturbance decoupling, measurable input functions, model matching, unknown input observation

AMS Subject Classification: 93C35

1. INTRODUCTION

In recent years, much attention has been devoted to the localization and rejection of input functions, which can be either disturbances or references. Necessary and sufficient conditions for the solvability of the perfect decoupling of inaccessible signals by state-feedback with stability were first presented by Basile and Marro in [2] and proved by Schumacher in [9]. These conditions involve the relevant concept of self-bounded controlled invariance, that has two important advantages. On the one hand, it enables these conditions to be expressed in a simple and concise form. On the other, self-bounded controlled invariant subspaces involve the minimum number of fixed poles.

The *measurable signal decoupling problem* (MSDP) by state-feedback and algebraic feedforward was first presented by Bhattacharyya in [5] in strict structural terms, and then extended by Basile, Marro and Piazzi in [4] in order to ensure internal stability by adding a suitable stabilizability condition, expressed in terms of self-bounded controlled invariant subspaces. Hence, a pair of necessary and sufficient conditions are obtained, that directly extend the ones concerning inaccessible input functions.

In this paper, a full feedforward compensation scheme is proposed for the solution of the MSDP. In fact, if the signal to be localized is accessible for measurement and if the geometric conditions for its rejection are satisfied, perfect decoupling can be achieved by means of a suitable feedforward unit, that also guarantees internal stability of the overall system. Hence, all the free poles of the internally stabilizable controlled invariant subspace on which the trajectory lies are properly chosen in the design of the feedforward unit, and the concept of self-boundedness is used to derive a compensator of minimum dimension.

The procedure presented is based on a detailed analysis of the internal free and fixed eigenstructure of a generic controlled invariant subspace. This approach is alternative to that presented in [3, p. 217], where the assignment of the internal and external eigenvalues of a controlled invariant subspace is carried out through a change of coordinates in the state and input space. Conversely, the method herein presented is particularly convenient for computational purposes. In fact, it can be easily exploited to derive new and efficient algorithms for the assignment of the free internal and external eigenvalues of a controlled invariant subspace by means of a state-feedback, as well as an alternative way to compute the invariant zeros of a linear system.

Furthermore, in recent years it has been pointed out that different tracking and filtering problems can be recast as measurable signal decoupling problems. For example, it is an easily established fact that the *unknown input observation* problem is exactly dual to the MSDP (see to this purpose [7] and references therein). Hence, the structure of the feedforward unit herein presented can be dualized so as to obtain the matrices of an observer whose input is the sole informative output of the given system and whose output is an estimation of a linear combination of the state variables of the system, which is exact if the geometric conditions are satisfied. These are in their turn dual to the ones presented in [4]. Moreover, in [8] it has been shown that the feedforward *model matching* problem can be reformulated as an extended MSDP. Hence, the signal to be decoupled is, in this case, a tracking reference.

The paper is organized as follows: in the second section the statement of the problem, its geometric solvability conditions and the motivations are presented, while the third section deals with the assignment of the free poles of a generic controlled invariant subspace by state-feedback. This is the preliminary result for the fourth section, where the main theorem is presented, which provides a way of deriving the matrices of the feedforward compensator if the conditions of exact solvability are met. In the fifth section, these results are extended to the case of non-purely dynamical systems. The last section presents all the steps for the design of the feedforward unit as an algorithm, that can be used as a trace for software implementation.

Notation. Throughout this paper, the symbol $\mathbb{R}^{n \times m}$ denotes the space of $n \times m$ real matrices; matrices and linear maps are denoted by slanted capitals. The image and the null-space of matrix A are denoted by $\operatorname{im} A$ and $\ker A$, whereas A^T and A^+ denote the transpose and the Moore–Penrose pseudo-inverse of A respectively. The symbol I_n stands for the $n \times n$ identity matrix, while 0_n denotes the origin of the vector space $\mathbb{R}^n$.

2. STATEMENT OF THE PROBLEM

Consider a linear, time-invariant continuous-time system Σ, described by

$$\begin{cases} \dot{x}(t) = A\,x(t) + B\,u(t) + H\,d(t) \quad x(0)=0 \\ y(t) = C\,x(t) \end{cases} \tag{1}$$

where, for all $t \geq 0$, $x(t) \in \mathbb{R}^n$ denotes the state, $u(t) \in \mathbb{R}^m$ the control input, $d(t) \in \mathbb{R}^s$ a measurable input signal, $y(t) \in \mathbb{R}^p$ the output. Matrix $A \in \mathbb{R}^{n\times n}$ is assumed to be stable[1]. Without loss of generality, assume that matrices $B \in \mathbb{R}^{n\times m}$ and $H \in \mathbb{R}^{n\times s}$ have linearly independent columns and $C \in \mathbb{R}^{p\times n}$ has linearly independent rows.

The measurable signal decoupling problem (MSDP) herein considered is stated as follows.

Problem 1. Find, if possible, a feasible control law $u|_{[0,+\infty)}$ ensuring

$$y(t) = 0 \quad \text{for all } t \geq 0 \tag{2}$$

for any piecewise continuous and bounded $d|_{[0,+\infty)}$ and such that the state-trajectory is bounded.

The same problem can also be formulated for a discrete-time system

$$\begin{cases} x(k+1) = A\,x(k) + B\,u(k) + H\,d(k) \quad x(0) = 0 \\ y(k) = C\,x(k). \end{cases}$$

It is well-known (see for example [3, p. 212]) that the necessary and sufficient conditions for Problem 1 to be solvable can be expressed in geometric terms as

$$\text{(C1)} \qquad \operatorname{im} H \subseteq \mathcal{V}^* + \operatorname{im} B$$

$$\text{(C2)} \qquad \mathcal{V}_m \text{ is internally stabilizable}$$

where $\mathcal{V}^* := \max \mathcal{V}(A, \operatorname{im} B, \ker C)$ denotes the largest $(A,\,B)$-controlled invariant subspace contained in $\ker C$, and $\mathcal{V}_m := \mathcal{V}^* \cap \min \mathcal{S}(A, \ker C, \operatorname{im} B + \operatorname{im} H)$ is the smallest $(A,\,B)$-controlled invariant subspace self-bounded with respect to $\ker C$ and containing $\operatorname{im} H$.

The structural condition (C1) was first presented in [5], and then extended in [4] to include the stability condition (C2), stated in terms of self-bounded controlled invariant subspaces.

The aim of this paper is that of finding the exact structure of a feedforward dynamical unit whose input is d and whose output is the control input u that solves Problem 1 as shown in Figure 1. in terms of a quadruple of matrices $(A_c,\, B_c,\, C_c,\, D_c)$.

Notice that Problem 1 can be reformulated as follows.

[1] Note that this condition is necessary as long as a pure feedforward solution is sought. However, it can be easily relaxed to the stabilizability of the pair (A, B). In fact, in this case, a preliminary stabilizing state-feedback can be performed, and what follows will be applied to the system thus obtained.

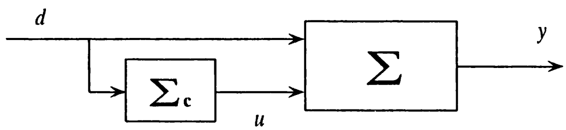

Fig. 1. Feedforward measurable signal decoupling scheme.

Problem 2. Consider Figure 1, and let conditions (C1) and (C2) be satisfied. Find an LTI compensator Σ_c such that (2) holds, and such that the overall system is stable.

3. SOME PRELIMINARY RESULTS

We begin by presenting an algorithm that, for a given h-dimensional $(A,\ B)$-controlled invariant subspace $\mathcal{V}$, enables a matrix $F \in \mathbb{R}^{n\times m}$ to be found such that $\mathcal{V}$ is $(A+BF)$-invariant, while assigning all the free poles, i.e., all the internal eigenvalues of $\mathcal{R}_{\mathcal{V}}$, the reachable subspace on $\mathcal{V}$, defined as the minimum $(A+BF)$-invariant subspace containing $\mathcal{V}\cap \operatorname{im} B$ (see [3, p. 216] and [10, p. 84]).

Consider a basis matrix V of the controlled invariant subspace $\mathcal{V}$. It is well-known that two matrices $X \in \mathbb{R}^{h\times h}$ and $U \in \mathbb{R}^{m\times h}$ exist such that

$$A\,V = V\,X + B\,U \tag{3}$$

(see [3, p. 207]). By definition of controlled invariance

$$A\,\mathcal{V} \subseteq \mathcal{V} + \operatorname{im} B$$

(see [3, p. 204]). As a result, it is found that $A\,V \subseteq \operatorname{im}\begin{bmatrix} V & B \end{bmatrix}$. Hence, the set of solutions of (3) can be parametrized as

$$\begin{bmatrix} X \\ U \end{bmatrix} = \begin{bmatrix} V & B \end{bmatrix}^{+} A\,V + K\,\Phi \tag{4}$$

where K is a basis matrix of $\ker\begin{bmatrix} V & B \end{bmatrix}$, whose dimension is denoted by g, and Φ is an arbitrary $g\times h$ matrix. Now, notice that the following identities are equivalent:

1. $\mathcal{V}\cap \operatorname{im} B = 0_n$
2. $\ker[\,V\;\;B\,] = 0_{n+m}$
3. $\mathcal{R}_{\mathcal{V}} = 0_n$.

Therefore, if $\mathcal{R}_{\mathcal{V}}$ is zero, then the pair of matrices $(X,\ U)$ satisfying (3) is unique. Once two matrices X and U satisfying (3) are determined with (4), a matrix F such that $U = -\,F\,V$ can be computed. Hence equation (3) yields the identity

$$(A + B\,F)\,V = V\,X \tag{5}$$

which points out that the eigenvalues of X are the poles of $\mathcal{V}$. If $\mathcal{R}_\mathcal{V}$ is not zero, then X and U can be expressed in a form that enables a matrix F to be derived, thus assigning all the free poles of $\mathcal{V}$.

To this purpose, consider a basis matrix $V = [\; R_\mathrm{V} \quad V_\mathrm{C} \;]$ of $\mathcal{V}$ adapted to $\mathcal{R}_\mathcal{V}$, i. e., such that R_V is a basis matrix of $\mathcal{R}_\mathcal{V}$; denote by r the dimension of $\mathcal{R}_\mathcal{V}$. With respect to this basis, the matrices in (4) can be partitioned as

$$\begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \\ U_1 & U_2 \end{bmatrix} = [\; R_\mathrm{V} \quad V_\mathrm{C} \quad B \;]^+ A \, [\; R_\mathrm{V} \quad V_\mathrm{C} \;] + \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} [\; \Phi_1 \quad \Phi_2 \;] \tag{6}$$

with $X_{11} \in \mathbb{R}^{r \times r}$, $X_{21} \in \mathbb{R}^{(h-r) \times r}$, $U_1 \in \mathbb{R}^{m \times r}$, $K_1 \in \mathbb{R}^{r \times g}$, $\Phi_1 \in \mathbb{R}^{g \times r}$.

It is easy to show that $K_2 = 0$: in fact, if one chooses $\Phi = 0$, the pair (X, U) solving (3) is such that $X_{21} = 0$ since, if F is such that $U = -F\,V$, from

$$(A + B\,F) \, [\; R_\mathrm{V} \quad V_\mathrm{C} \;] = [\; R_\mathrm{V} \quad V_\mathrm{C} \;] \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} \tag{7}$$

it follows that $X_{21} = 0$, since $\mathcal{R}_\mathcal{V}$ is $(A + BF)$-invariant. Any other choice of Φ cannot modify X in a way that X_{21} differs from zero because, since $\mathcal{R}_\mathcal{V}$ is $(A + BF)$-invariant for any choice of Φ, then X_{21} is zero for any choice of Φ. Owing to the arbitrariness of H this is possible only if $K_2 = 0$.

Since $X_{21} = 0$, from (7) it follows that the eigenvalues of X_{11} are the internal eigenvalues of $\mathcal{R}_\mathcal{V}$, while those of X_{22} are the fixed poles of $\mathcal{V}$.

By suitably partitioning $[R_\mathrm{V} \; V_\mathrm{C} \; B]^+ A [R_\mathrm{V} \; V_\mathrm{C}]$ likewise, equation (6) can be written as

$$\begin{bmatrix} X_{11} & X_{12} \\ O & X_{22} \\ U_1 & U_2 \end{bmatrix} = \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ O & \Xi_{22} \\ Y_1 & Y_2 \end{bmatrix} + \begin{bmatrix} K_1 \\ O \\ K_3 \end{bmatrix} [\; \Phi_1 \quad \Phi_2 \;]\,. \tag{8}$$

Note that the eigenvalues of Ξ_{22} cannot be modified by any choice of Φ. The pair (Ξ_{11}, K_1) is controllable, since the internal eigenvalues of $\mathcal{R}_\mathcal{V}$ are all arbitrarily assignable. If our aim is that of assigning the free poles of $\mathcal{V}$, then Φ_2 can be taken equal to zero, because its value does not modify the internal eigenstructure of $\mathcal{R}_\mathcal{V}$.

Matrix F can be computed with the relation

$$F = -U\,(V^T\,V)^{-1}\,V^T. \tag{9}$$

This choice, in fact, ensures that $U = -F\,V$, and yields an important property: with respect to a basis adapted to $\mathcal{V}$ through its basis matrix V, F assumes the structure

$$F = [\; -U \quad O \;]\,. \tag{10}$$

Hence, F does not modify any of the external eigenvalues of $\mathcal{V}$.

4. MAIN RESULT

In this section, the geometric approach is applied to derive the linear dynamical system Σ_c that solves the decoupling problem with internal stability. To this purpose, we introduce three important lemmas.

Lemma 1. The subspace $\mathcal{V}_m$ is an $(A,\, B)$-controlled invariant subspace contained in $\ker C$.

Lemma 2. $\mathcal{R}_{\mathcal{V}^*}$, the reachable set on $\mathcal{V}^*$, is a subspace of $\mathcal{V}_m$.

Lemma 3. Let $\operatorname{im} H \subseteq \mathcal{V}^* + \operatorname{im} B$. Then

$$\operatorname{im} H \subseteq \mathcal{V}_m + \operatorname{im} B. \tag{11}$$

These results are proved in [1] and [3]. Inclusion (11) ensures that two matrices $\Pi_1 \in \mathbb{R}^{h \times s}$ and $\Pi_2 \in \mathbb{R}^{m \times s}$ exist such that

$$H = V\,\Pi_1 + B\,\Pi_2$$

where V is a basis matrix of $\mathcal{V}_m$, whose dimension is denoted by h. The matrices Π_1 and Π_2 project the subspace $\operatorname{im} H$ on $\mathcal{V}_m$ and $\operatorname{im} B$ respectively, and by virtue of (11), they can be computed by

$$\begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix} = \begin{bmatrix} V & B \end{bmatrix}^+ H + K\,\Psi \tag{12}$$

where $K \in \mathbb{R}^{(h+m) \times g}$ is a basis matrix of the subspace $\ker \begin{bmatrix} V & B \end{bmatrix}$, whose dimension is denoted by g, and Ψ is an arbitrary $g \times s$ matrix.

Let $H_V := V\,\Pi_1$ and $H_B := B\,\Pi_2$. It follows that

$$H\,d(t) = H_V\,d(t) + H_B\,d(t) \quad \forall\, t \geq 0$$

with $H_V\,d(t) \in \mathcal{V}_m$ and $H_B\,d(t) \in \operatorname{im} B$ for all $t \geq 0$. Since the pair of projecting matrices $(\Pi_1,\, \Pi_2)$ computed by means of (12) is parametrized on $\ker \begin{bmatrix} V & B \end{bmatrix}$, the projection of $d(t)$ on $\mathcal{V}_m$ and $\operatorname{im} B$ is not unique in general, unless $\mathcal{V}_m \cap \operatorname{im} B = 0_n$, i.e., unless the system is left-invertible.

The following theorem provides the matrices of the dynamical compensator Σ_c in Figure 1 that solves the decoupling problem.

Theorem. Let A be stable and suppose that conditions (C1) – (C2) hold; let V be a basis matrix of $\mathcal{V}_m$ adapted to $\mathcal{R}_{\mathcal{V}^*}$, the reachable subspace on $\mathcal{V}^*$. Let h be the dimension of $\mathcal{V}_m$, and $X \in \mathbb{R}^{h \times h}$ and $U \in \mathbb{R}^{m \times h}$ be matrices satisfying (3) referred to $\mathcal{V}_m$ and such that, with F defined as in (9), $\mathcal{V}_m$ is an internally stable $(A + BF)$-invariant subspace.

Let $(\Pi_1,\, \Pi_2)$ be the projecting pair of $\operatorname{im} H$ on $\mathcal{V}_m$ and $\operatorname{im} B$ respectively. A dynamical compensator Σ_c, whose input is d and whose output is u solving Problem 2 is described by the quadruple $(A_c,\, B_c,\, C_c,\, D_c) = (X,\, \Pi_1,\, -U,\, -\Pi_2)$. If $\mathcal{V}_m = 0_n$ it reduces to an algebraic unit $D_c = -\,B^+ H$.

Proof. First consider the discrete-time case. Denoting by z the state of Σ_c, the equations of the compensator are

$$\begin{cases} z(k+1) = X\,z(k) + \Pi_1\,d(k) \qquad z(0)=0 \\ u(k) = -U\,z(k) - \Pi_2\,d(k). \end{cases}$$

By virtue of Lemma 2, a basis matrix of $\mathcal{V}_m$ adapted to $\mathcal{R}_{\mathcal{V}^*}$ exists. The choice of matrix F is such that the output equation of the compensator is

$$u(k) = F\,V\,z(k) - \Pi_2\,d(k)$$

which, once substituted in the state equation of Σ, leads to

$$x(k+1) = A\,x(k) + B\,F\,V\,z(k) + V\,\Pi_1\,d(k). \tag{13}$$

The state functions $x(k)$ and $z(k)$ are linked by the relation

$$x(i) = V\,z(i) \quad \forall\, i \geq 0 \tag{14}$$

that can be proved by induction. Equation (14) holds for $i=0$ since the initial conditions of both Σ and Σ_c are supposed to be null. Suppose that (14) holds for $i=k$; then, from (13)

$$\begin{aligned} x(k+1) &= A\,V\,z(k) + B\,F\,V\,z(k) + V\,\Pi_1\,d(k) \\ &= V\,X\,z(k) + V\,\Pi_1\,d(k) = V\,z(k+1) \end{aligned}$$

by virtue of (5). As a consequence, equation

$$x(k+1) = (A + B\,F)\,x(k) + V\,\Pi_1\,d(k)$$

describes a motion on $\mathbb{R}^n$ which is all contained in $\mathcal{V}_m$, hence invisible on the output.

If $\mathcal{V}_m = 0_n$, the control law

$$u(k) = -B^+\,H\,d(k)$$

cancels the part of the disturbance on $\operatorname{im} B$.

Now consider the continuous-time case. The dynamics of Σ_c are described by

$$\begin{cases} \dot{z}(t) = X\,z(t) + \Pi_1\,d(t) \qquad z(0)=0 \\ u(t) = -U\,z(t) - \Pi_2\,d(t). \end{cases}$$

Then $u(t) = F\,V\,z(t) - \Pi_2\,d(t)$. This leads to

$$\dot{x}(t) = A\,x(t) + B\,F\,V\,z(t) + V\,\Pi_1\,d(t).$$

We prove that $x(t) = V\,z(t)$ for each positive t if $x(0)=0$. Let t be such that $x(t) = V\,z(t)$:

$$\begin{aligned} \dot{x}(t) &= (A + B\,F)\,V\,z(t) + V\,\Pi_1\,d(t) \\ &= V\,X\,z(t) + V\,\Pi_1\,d(t) = V\,\dot{z}(t) \end{aligned}$$

It follows that the whole state-trajectory lies on $\mathcal{V}_m$.

Both in the discrete and in the continuous time-domain, the closed-loop system with the compensator described by the quadruple $(X, \Pi_1, -U, -\Pi_2)$ has state matrix

$$A_{cl} = \begin{bmatrix} A & -BU \\ 0 & X \end{bmatrix}$$

which is strictly stable. □

Notice that the order of the feedforward unit Σ_C is minimum. In fact, it is not necessary to reproduce the state components corresponding to $(A+BF)_{\mathbb{R}^n/\mathcal{V}_m}$ in Σ_C since they are not influenced by input d.

Internal stabilizability of $\mathcal{V}_m$ is ensured if the plant is minimim-phase, since the internal fixed eigenvalues of $\mathcal{V}_m$ are part of those of $\mathcal{V}^*$.
Observe that if the classical structural condition for decoupling of inaccessible signals

$$\operatorname{im} H \subseteq \mathcal{V}^*$$

holds, the compensator is purely dynamical, i. e. $D_C = -\Pi_2 = 0$. In fact, in this case, there is no need for a component of the control input u that cancels the projection of d on $\operatorname{im} B$.

5. NON STRICTLY PROPER SYSTEMS

The procedures of the previous sections can be easily extended to systems characterized by an algebraic feedthrough between the output y and the inputs u and d (see for example [3], pp. 245–247).

First, consider a discrete-time linear system Σ_d with null initial state

$$\begin{cases} x(k+1) = A\,x(k) + B\,u(k) + H\,d(k) \\ y(k) = C\,x(k) + D\,u(k) + G\,d(k) \end{cases}$$

where $D \in \mathbb{R}^{p\times m}$ and $G \in \mathbb{R}^{p\times s}$ are such that the matrices $\begin{bmatrix} B^T & D^T \end{bmatrix}$ and $\begin{bmatrix} H^T & G^T \end{bmatrix}$ are full row-rank.

It is possible to define a new variable z satisfying $z(k+1) = y(k)$ for all k, and consider it a state extension of Σ_d:

$$\begin{cases} \begin{bmatrix} x(k+1) \\ z(k+1) \end{bmatrix} = \widehat{A} \begin{bmatrix} x(k) \\ z(k) \end{bmatrix} + \widehat{B}\,u(k) + \widehat{H} d(k) \\ z(k) = \widehat{C} \begin{bmatrix} x(k) \\ z(k) \end{bmatrix} \end{cases}$$

where

$$\begin{aligned} \widehat{A} &= \begin{bmatrix} A & O \\ C & O \end{bmatrix}, \quad \widehat{B} = \begin{bmatrix} B \\ D \end{bmatrix}, \quad \widehat{H} = \begin{bmatrix} H \\ G \end{bmatrix}, \\ \widehat{C} &= \begin{bmatrix} O & I_n \end{bmatrix}. \end{aligned} \tag{15}$$

The new variable z can be interpreted as the state of a unit delay connected at the output of Σ_d. Note that $z(0)=y(-1)=0$: the conditions and the standard procedures to solve the decoupling problem of a measurable signal can be applied to the extended system above since it is purely dynamical and characterized by null initial conditions, thus ensuring

$$z(k)=0 \quad \forall\, k>0$$

and, as a consequence, $y(k)=0$ for each $k \geq 0$.

The same artifice can be adopted for the continuous-time system

$$\begin{cases} \dot{x}(t) = A\,x(t) + B\,u(t) + H\,d(t) & x(0)=0 \\ y(t) = C\,x(t) + D\,u(t) + G\,d(t). \end{cases}$$

In this case define $\dot{z}(t)=y(t)$ as the state of an integrator stage connected in cascade at the output y. Since z is a primitive of y, it is continuous on $[0,+\infty)$: it follows that, since $y(t)=0$ for all $t<0$, it is also $z(0)=0$. The extended system with null initial conditions thus obtained is of the kind of system (1). This allows the application of the conditions (C1) – (C2) and of the results of Theorem, thus ensuring $y(t)=0$ for any $t \geq 0$.

6. AN ALGORITHMIC PROCEDURE

The results expounded in the previous sections are collected here as an algorithm for the calculation of the matrices of the feedforward unit that solves Problem 2.

Step 1. If Σ is non purely dynamical, a state extension has to be performed as pointed out in Section 5, by re-defining matrices A, B, H and C according to (15).

Step 2. If $\mathcal{V}_m$ differs from zero, a basis matrix V of $\mathcal{V}_m$ is computed, and the two conditions (C1) – (C2) are tested: if they are not satisfied, the algorithm stops.

Step 3. A basis matrix of $\ker\,[\,V \;\; B\,]$ is computed; if that subspace is zero, then the matrices X and U such that (4) holds are directly determined by (5).

Step 4. If on the contrary $\mathcal{R}_{\mathcal{V}^*}$ differs from zero, the basis matrix of $\mathcal{V}_m$ can be chosen in a way that its first columns are a basis matrix of $\mathcal{R}_{\mathcal{V}^*}$. The matrix $[\,V \;\; B\,]^+ A\,V$ is then computed and, by defining the submatrices Ξ_{11} and K_1 as in (9), a matrix H can be derived that arbitrarily places the poles of $\Xi_{11} + K_1 H_1$, the internal assignable eigenvalues of $\mathcal{V}_m$. Choosing $H_2=0$, then X and U follow from (8).

Step 5. Taking for example $\Psi=0$ in (12), the matrices Π_1 and Π_2 are determined.

Step 6. If $\mathcal{V}_m$ differs from zero, then the quadruple of matrices $(A_c,\, B_c,\, C_c,\, D_c)$ is obtained by simply assigning them the values of $(X,\, \Pi_1,\, -U,\, -\Pi_2)$; if, on the contrary, $\mathcal{V}_m = 0_n$, then the feedforward compensator reduces to an algebraic unit with a gain $B^+ H$.

7. CONCLUDING REMARKS

It has been shown that the geometric approach provides a simple way to derive a linear compensator that solves the measurable signal decoupling problem. The exact design of the feedforward unit for this problem can also be applied for the solution of another fundamental control problem, the unknown-input observation of a linear function of the state, which is the dual of the problem herein considered, as shown in [7], and the model matching problem, both feedforward and feedback, as considered in [8].

The theory is supported by simple software routines for MATLAB: see in particular the functions `effesta.m`, `gazero.m` and `hud.m`, which can be freely downloadable with the toolbox `ga` at www3.deis.unibo.it/Staff/FullProf/GiovanniMarro/geometric.htm.

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