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THE CANCELLATION LAW FOR PSEUDO-CONVOLUTION

Andrea Stupňanová

Cancellation law for pseudo-convolutions based on triangular norms is discussed. In more details, the cases of extremal t-norms $T_M$ and $T_D$, of continuous Archimedean t-norms, and of general continuous t-norms are investigated. Several examples are included.

Keywords: cancellation law, t-norm, pseudo-convolution

AMS Subject Classification: 03E72, 28E10

1. INTRODUCTION

In algebraic structures, a commutative binary operation $*$ is said to be cancellative if for all elements $g, h, v$ it holds

$$g * v = h * v \Rightarrow g = h.$$ 

The cancellation law ensures for example the uniqueness of solution of equation $x * v = u$ (if a solution exists).

The aim of this paper is investigation of the cancellativity of pseudo-convolutions introduced in [16]. Recall that the standard probabilistic convolution of distribution functions is cancellative.

The paper is organized as follows. In the next section, pseudo-convolutions are introduced. In Section 3, cancellation law for pseudo-convolutions based on boundary t-norms is discussed. Section 4 and Section 5 are devoted to the study of cancellation law in the case of continuous Archimedean t-norms and more general continuous t-norms based pseudo-convolutions.

2. PSEUDO–CONVOLUTIONS

2.1. Pseudo-convolution of real functions

Let $[a, b]$ be a closed subinterval of the extended real line (sometimes also semiclosed subintervals are taken into account).

\hspace{1cm} \footnote{Presented at the 7th FSTA international conference held in Liptovský Mikuláš, Slovakia, on January 26–30, 2004.}
Definition 1. A binary operation $\oplus$ on $[a, b]$ is called a pseudo-addition on $[a, b]$ if it is commutative, nondecreasing, associative, continuous (possibly up to the points $(a, b)$, $(b, a)$) and with a neutral element, denoted by 0, i.e., for each $x \in [a, b]$ $0 \oplus x = x$ holds.

So, $\oplus$ is either a t-norm, or a t-conorm or a uni-norm on $[a, b]$, see [4]. Because of the duality, it is sufficient to deal with t-conoms and uninorms only. Denote $[a, b]_+ = \{ x; x \in [a, b], x \geq 0 \}$.

Definition 2. A binary operation $\otimes$ on $[a, b]$ is called a pseudo-multiplication with respect to $\oplus$ if it is commutative, associative, distributive with respect to $\oplus$, positively nondecreasing (i.e., $x < y \Rightarrow x \otimes z \leq y \otimes z$ if $z \in [a, b]_+$) with a unit element, denoted by 1, (i.e., for each $x \in [a, b]$ 1 $\otimes$ x = x holds). We suppose, further, 0 $\otimes$ x = 0, i.e., 0 is annihilator.

The structure $([a, b], \oplus, \otimes)$ is called a semiring, see e.g., [2].

Let $([a, b], \oplus, \otimes)$ be a semiring with continuous operations (possibly up to the continuity of $\otimes$ in points $(0, a)$, $(0, b)$, $(a, 0)$ and $(b, 0)$). The standard building up of an integral with respect to $\otimes$-decomposable measures based on the pseudo-addition and pseudo-multiplication leads to the definition of a pseudo-integral [12]. The pseudo-convolution of the functions defined on $[0, \infty)$ with values in $[a, b]$ was introduced in [16], see also [12, 14], by means of the corresponding pseudo-integral,

$$g \ast h(z) = \sup_{x \in [0, z]} T(g(z-x), h(x)),$$

(1)

In our paper we will deal with the special semiring only, so we will not describe some details here. (It is possible to find them in [14, 16].)

2.2. Pseudo-convolution with respect to the semiring $([0, 1], \lor, T)$

One of typical examples of a semiring is $([0, 1], \lor, T)$, where $\lor = \text{sup}$ and $T$ is a t-norm, see [4]. This is the semiring with 0 = 0 and 1 = 1. In this case the formula for convolution (1) can be rewritten to

$$g \ast h(z) = \int_{[0, z]} g(z-x) \otimes h(x)dx.$$

(2)

where $T$ is a t-norm.

Observe that the pseudo-convolution $\ast$ is commutative due to the commutativity of $T$, however, it need not be associative, in general. Nevertheless, for t-norms continuous on $[0, 1]^2$, $\ast$ is also associative.

Note that the kernel of a function $g : [0, \infty[ \to [0, 1]$ is defined as

$$\ker(g) = \{ x \in [0, \infty[; g(x) = 1 \}.$$
Denote by $\mathcal{D}$ the class of all continuous distribution functions on $[0, \infty]$ and by $\mathcal{S}$ the subclass of $\mathcal{D}$ such that the restriction of $g$ on $]a_g, b_g[ := \text{supp} g \setminus \ker(g)$ (if $\ker(g) = \emptyset$ then $b_g = \infty$) is strictly increasing, i.e.,

$$\mathcal{S} = \{ g : [0, \infty[ \to [0, 1]; g(0) = 0, g|_{a_g, b_g} \to ]0, 1[ \text{ is increasing bijection} \}.$$ 

**Lemma 1.** Let $\ker(v) \neq \emptyset$ for a function $v \in \mathcal{D}$. Then for all $g, h \in \mathcal{S}$ the following implication holds:

$$g * v = h * v \Rightarrow \ker(g) = \ker(h), \quad \text{i.e., } b_g = b_h. \quad (3)$$

**Proof.** Let $\ker(v) \neq \emptyset$. We can get the formula (3) from the property $\ker(g * v) = \ker(g) + \ker(v)$. First we suppose that $b_g < \infty$.

- Let $z \geq b_g + b_v$. Then

$$g * v(z) = \sup_{x \in [0, z]} T(g(z - x), v(x))$$

$$= \max \left\{ \sup_{x \in [0, z - b_g]} T(g(z - x), v(x)), \sup_{x \in [z - b_v, z]} T(g(z - x), v(x)) \right\},$$

- if $0 \leq x < z - b_g$, i.e., $z - x > b_g$

$$g * v(z) = \sup_{x \in [0, z - b_g]} T(1, v(x)) = \sup_{x \in [0, z - b_g]} v(x) = 1,$$

- if $b_v \leq z - b_g \leq x \leq z$, i.e., $z - x \leq b_g$

$$g * v(z) = \sup_{x \in [z - b_g, z]} T(g(z - x), 1) = \sup_{x \in [z - b_g, z]} g(z - x) = g(b_g) = 1.$$

- Let $0 \leq z < b_g + b_v$. Then

$$g * v(z) = \max \left\{ \sup_{x \in [0, z - b_g]} T(g(z - x), v(x)), \sup_{x \in [z - b_g, b_v]} T(g(z - x), v(x)) \right\},$$

$$\sup_{x \in [b_v, z]} T(g(z - x), v(x)) \right\},$$

- if $0 \leq x \leq z - b_g$, i.e., $z - x \geq b_g$

$$g * v(z) = \sup_{x \in [0, z - b_g]} T(1, v(x)) = \sup_{x \in [0, z - b_g]} v(x) = v(z - b_g) < 1,$$

$$\sup_{< b_v} v(x) = v(z - b_g) < 1,$$
- if $z - b_g < x < b_v$, i.e., $z - x < b_g$

$$g * v(z) = \sup_{x \in [z - b_g, b_v]} T(g(z - x), v(x)) < 1,$$

- if $b_v \leq x \leq z$

$$g * v(z) = \sup_{x \in [b_v, z]} T(g(z - x), 1) = \sup_{x \in [b_v, z]} g(z - x) = g(z - b_v) < 1.$$

It is easy to see that if $b_v < \infty$ then $b_g = \infty$ if and only if $b_g v = \infty$, i.e., if $b_g = \infty$ then $b_h = \infty$ too.

3. THE CANCELLATION LAW FOR PSEUDO-CONVOLUTION BASED ON THE SEMIRING $([0,1], \lor, T_M)$ AND $([0,1], \lor, T_D)$, RESPECTIVELY

Recall that $T$ is a t-norm if it is associative, commutative, non-decreasing binary operation on $[0,1]$ with neutral element 1. For more details we recommend [4]. For any t-norm $T$ it holds $T_D \leq T \leq T_M$, where the strongest t-norm $T_M = \min$ and the weakest t-norm $T_D$ (the drastic product) is given by

$$T_D(x, y) = \begin{cases} 
\min(x, y) & \text{if } \max(x, y) = 1, \\
0 & \text{elsewhere.}
\end{cases}$$

**Theorem 1.** Consider the strongest t-norm $T_M$. Let $g, h, v \in D$. Then the cancellation law holds, i.e.,

$$g * v = h * v \Rightarrow g = h.$$

**Proof.** We denote $g^{(c)}$ the $c$-cut of function $g$, i.e.,

$$g^{(c)} = \{x; g(x) \geq c\} \text{ for } c \in [0,1].$$

Then for convolution based on the $T_M$ it holds

$$(g * h)^{(c)} = g^{(c)} + h^{(c)} \text{ for any } c \in [0,1].$$

An arbitrary $c$-cut of function $g$ from $D$ is interval $[a_g^{(c)}, \infty]$. Suppose $g * v = h * v$. Then

$$[a_g^{(c)}, \infty] + [a_v^{(c)}, \infty] = [a_h^{(c)}, \infty] + [a_v^{(c)}, \infty] \text{ for all } c \in [0,1].$$

Thus $[a_g^{(c)} + a_v^{(c)}, \infty] = [a_h^{(c)} + a_v^{(c)}, \infty] \Rightarrow a_g^{(c)} = a_h^{(c)} \Rightarrow g^{(c)} = h^{(c)} \text{ for all } c \in [0,1] \Rightarrow g = h,$

i.e., the cancellation law holds.

**Remark 1.** The cancellation law with respect to $T_M$ fails if sup $v < 1$ or inf $v > 0$ or if we deal with non-monotone functions. See Example 1.
Example 1. Consider the t-norm $T_M$. Let $v(x) = \begin{cases} x, & x \in [0, 1] \\ 1, & x \in ]1, \infty[ \\ x, & x \in ]1, \frac{1}{2}[ \\ \frac{1}{2}, & x \in ]\frac{1}{2}, 1[ \\ -x + \frac{3}{2}, & x \in ]1, \frac{3}{2}[ \\ \frac{1}{2}, & x \in ]\frac{3}{2}, 2[ \\ x - \frac{3}{2}, & x \in ]2, \frac{5}{2}[ \\ 1, & x \in ]\frac{5}{2}, \infty[ \end{cases}$ and $h(x) = \begin{cases} 1, & x \in ]0, 1[ \\ \frac{1}{2}, & x \in ]1, \frac{3}{2}[ \\ \frac{3}{4}, & x \in ]\frac{3}{2}, \frac{5}{2}[ \\ 1, & x \in ]\frac{5}{2}, \infty[ \end{cases}$

The function $h$ is not a monotone function. Then pseudo-convolutions of functions $g, v$ and $h, v$ based on semiring $([0, 1], V, T_M)$ are the same, i.e.

$$g * v(x) = h * v(x) = \begin{cases} \frac{1}{2}x, & x \in [0, 1] \\ \frac{1}{2}, & x \in ]1, \frac{3}{2}[ \\ \frac{3}{4}, & x \in ]\frac{3}{2}, \frac{5}{2}[ \\ 1, & x \in ]\frac{5}{2}, \infty[ \end{cases}$$

On the other hand, consider the weakest t-norm $T_D$. Then the cancellation law holds only in special cases.

Theorem 2. Consider the pseudo-convolution based on the $T_D$. Let $g, h, v \in D$. Moreover, let

$$v(b_v - x) \leq \min(g(b_g - x), h(b_h - x)) \quad \text{for all } x \in [0, b],$$

where $b := \min\{b_v, b_g, b_h\}$. Then $g * v = h * v \iff g = h$.

Proof. Applying the formula for sum of fuzzy quantities based on the drastic product from [9], we get

$$g * v(x) = \max\{g(x - b_v), v(x - b_g)\}$$
for all $x \in [0, \infty[$, where $|x/ = \min \{\max \{0, x\}, 1\}$. Now we can easily get condition for cancellativity.

4. THE CANCELLATION LAW FOR PSEUDO-CONVOLUTION BASED ON $([0, 1], \vee, T)$, WHERE $T$ IS AN ARCHIMEDEAN CONTINUOUS $t$-NORM

In this section at first we describe some Zagrodny’s results [20]. Further we will apply them for investigation of validity of cancellation law for pseudo-convolution of functions based on a strict $t$-norm. Finally, the case of nilpotent $t$-norms will be discussed.

4.1. The cancellation law for inf-convolution – Zagrodny’s results

Definition 3. Let $g, h : \mathbb{R} \to \mathbb{R} \cup \{\infty\}$. The inf-convolution of $g$ and $h$ at $x \in \mathbb{R}$ is defined by

$$g \Box h(x) := \inf_{y + z = x} (g(y) + h(z)).$$

Definition 4. Let $h : \mathbb{R} \to \mathbb{R} \cup \{\infty\}$. The function $h$ is said to be uniformly convex if for $\forall \varepsilon \geq 0 \exists \delta > 0$

$$|a - b| \geq \varepsilon \Rightarrow h \left( \frac{a + b}{2} \right) \leq \frac{h(a) + h(b) - \delta |a - b|}{2}, \forall a, b \in \text{dom } h.$$ 

Note that the domain of functions $g, h$ can be restricted to some intervals. Zagrodny in [20] deal with more general functions on Banach space.

Theorem 3. Let $X$ be a reflexive Banach space. If $g, h : X \to \mathbb{R} \cup \{\infty\}$ are proper lower semicontinuous convex functions such that $h$ is strictly convex and $\lim_{\|x\| \to \infty} \frac{h(x)}{\|x\|} = \infty$ then $g \Box h = g \Box h$ implies $q = g$.

Theorem 4. Let $X$ be a Banach space and $g, h : X \to \mathbb{R} \cup \{\infty\}$ be proper lower semicontinuous convex functions. Moreover, suppose $h$ is uniformly convex. Then $q \Box h = g \Box h$ implies $q = g$.

4.2. The cancellation law for pseudo-convolution based on a strict $t$-norm

Recall that the pseudo-convolution of functions based on semiring $([0, 1], \vee, T)$ with some Archimedean continuous $t$-norm $T$ can be expressed by

$$g \star h(x) = f[-1] \left( \inf_{y + z = x} (f(g(y)) + f(h(z))) \right) = f^{-1} (f \circ (g \Box h (x))), x \in [0, \infty[,$$
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where \( f \) is additive generator of t-norm \( T \), i.e., \( f : [0,1] \rightarrow [0,\infty] \) is continuous strictly decreasing mapping verifying \( f(1) = 0 \), and pseudo-inverse \( f^{-1} : [0,\infty] \rightarrow [0,1] \) of \( f \) is defined by

\[
f^{-1}(x) = f^{-1}(\min(f(0), x)).
\]

Archimedean continuous t-norms can be divided into two classes: strict and nilpotent. An additive generator of a strict t-norm is unbounded, and then \( f^{-1} = f^{-1} \).

**Theorem 5.** Consider a strict t-norm \( T \) with an additive generator \( f \). Let \( g, h, v \in S \) such that \( f \circ g \) and \( f \circ h \) are convex on \( [a_g, b_g] \) and \( [a_h, b_h] \), respectively and \( f \circ v \) is either

(i) uniformly convex on \( [a_v, b_v] \) or

(ii) strictly convex on \( [a_v, b_v] \)

and if \( b_v = \infty \) then \( \lim_{x \to \infty} \frac{f \circ v(x)}{x} = \infty \).

Then \( g \ast v = h \ast v \) implies \( g = v \).

**Proof.** Assume \( f \circ g, f \circ h \) and \( f \circ v \) verify conditions from theorem. Let \( g \ast v = h \ast v \). This imply \( f \circ g \sqcup f \circ v = f \circ h \sqcup f \circ v \) and by Zagrodny’s results \( f \circ g = f \circ h \Rightarrow g = h \), i.e., the cancellativity is valid.

**4.3. The cancellation law for pseudo-convolution based on a nilpotent t-norm**

The case of nilpotent t-norm is more complicated. Conditions from Theorem 5 are deficient. See Example 2.

**Example 2.** Consider the Lukasiewicz t-norm \( T_L \) with additive generator

\[
f(x) = 1 - x
\]

and functions

\[
g(x) = \begin{cases} 
  x, & x \in [0,1] \\
  1, & x \in ]1,\infty[,
\end{cases}
\]

\[
h(x) = \begin{cases} 
  0, & x \in [0,0.1] \\
  2x - 0.2, & x \in ]0.1,0.2] \\
  x, & x \in ]0.2,1] \\
  1, & x \in ]1,\infty[
\end{cases}
\]

and \( v(x) = \begin{cases} 
  1 - (x - 1)^2, & x \in [0,1] \\
  1, & x \in ]1,\infty[.
\end{cases} \)

The interval \( [a_v, b_v] = [0,1] \) and \( f \circ v \) is given by formula \( f \circ v(x) = 1 - (x - 1)^2 \) (i.e., strictly convex function).

The interval \( [a_g, b_g] = [0,1] \) too and \( f \circ g(x) = 1 - x \) on \( [0,1] \) (i.e., convex function).

Finally, \( [a_h, b_h] = [0.1,1] \) and

\[
f \circ h = \begin{cases} 
  1.2 - 2x, & x \in [0.1,0.2] \\
  1 - x, & x \in ]0.2,1[.
\end{cases}
\]
(i.e., convex function).

However, the pseudo-convolution based on \(([0,1], \lor, T_L)\) of functions \(v\) and \(g\) is the same as pseudo-convolution (based on the same semiring) of functions \(v\) and \(h\).

\[
g \ast v(x) = h \ast v(x) = \begin{cases} 
0, & x \in [0, \frac{3}{4}] \\
x - \frac{3}{4}, & x \in ]\frac{3}{4}, \frac{3}{2}] \\
1 - (x - 2)^2, & x \in ]\frac{3}{2}, 2] \\
1, & x \in ]2, \infty[.
\end{cases}
\]

Thus Theorem 5 is not valid in the case when \(T\) is a nilpotent \(t\)-norm, in general.

For nilpotent \(t\)-norms, we have only the following special cancellation theorems.

**Theorem 6.** Consider a nilpotent \(t\)-norm \(T\) with normed additive generator \(f\). Let \(g, h, v \in S\), such that \(f \circ g, f \circ h\) and \(f \circ v\) are concave on the interval \([a_g, b_g]\), \([a_h, b_h]\) and \([a_v, b_v]\) respectively. Moreover,

\[
v(b_v - x) \leq \min (g(b_g - x), h(b_h - x)) \quad \text{for all } x \in [0, b],
\]

where \(b := \min\{b_v, b_g, b_h\}\). Then \(g \ast v = h \ast v \iff g = h\).

The proof follows from the fact that under requirements of the theorem, the pseudo-convolution of function based on semiring \(([0,1], \lor, T)\) with some nilpotent \(t\)-norm \(T\) behaves as the pseudo-convolution of function based on semiring \(([0,1], \lor, T_D)\), see [7, 9]. Note that the same claim is true also for strict \(t\)-norms. However then \(b_g = b_h = b_v = \infty\).

Consider \((a,b) \in \mathbb{R}^2, a \neq b\), then \(\phi(a,b)\) is the linear transformation defined by

\[
\phi(a,b)(x) = \frac{x - a}{b - a}.
\]

Note that the inverse mapping \(\phi^{-1}(a,b)\) of \(\phi(a,b)\) is given by \(\phi^{-1}(a,b)(x) = a + (b - a)x\). \(\square\)
Theorem 7. Consider a nilpotent t-norm $T$ with normed additive generator $f$. Let $g, h, v \in S$, such that $b_g, b_h, b_v < \infty$ and $f \circ v \circ \phi_v^{-1}(x) = f \circ g \circ \phi_g^{-1}(x) = f \circ h \circ \phi_h^{-1}(x) = 1 - (1 - x)^p$ on the interval $(0,1)$ for some $p \in (1,\infty)$, where $\phi_v = \phi_{(a_v,b_v)}$ and similarly for functions $g, h$. Then $g * v = h * v \Rightarrow g = h$.

Proof. Following [8], under requirements of the theorem,

$$f \circ (g * v) \circ \phi_{g* v}^{-1}(x) = 1 - (1 - x)^p,$$

where $b_{g* v} = b_g + b_v$ and

$$(b_{g* v} - a_{g* v})^{1/p - 1} = (b_g - a_g)^{1/p - 1} + (b_v - a_v)^{1/p - 1}.$$

Similarly,

$$f \circ (h * v) \circ \phi_{h* v}^{-1}(x) = 1 - (1 - x)^p,$$

where $b_{h* v} = b_h + b_v$ and

$$(b_{h* v} - a_{h* v})^{1/p - 1} = (b_h - a_h)^{1/p - 1} + (b_v - a_v)^{1/p - 1}.$$

Now, it is evident that $g * v = h * v$ if and only if $a_g = a_h, b_g = b_h$, i.e., $g = h$. \qed

5. THE CANCELLATION LAW FOR PSEUDO-CONVOLUTION BASED ON $([0,1], V, T)$, WHERE $T$ IS A CONTINUOUS t-NORM

5.1. Ordinal sums of t-norms

Definition 5. Consider a family $(T_k)_{k \in K}$ of t-norms and a family $(\alpha_k, \beta_k)_{k \in K}$ of pairwise disjoint open non-degenerate subintervals of $[0,1]$. The $[0,1]^2 \to [0,1]$ mapping $T$ defined by

$$T(x, y) = \begin{cases} 
\phi_k^{-1}(T_k (\phi_k(x), \phi_k(y))), & \text{if } (x, y) \in [\alpha_k, \beta_k]^2 \\
T_M(x, y), & \text{elsewhere},
\end{cases}$$

where $\phi_k = \phi_{(\alpha_k, \beta_k)}$, is a t-norm. $T$ is called the ordinal sum of the summands $\langle \alpha_k, \beta_k, T_k \rangle$, and is denoted by $T \equiv \langle (\alpha_k, \beta_k, T_k) \mid k \in K \rangle$.

Note that in the foregoing proposition the case of an empty index set is also allowed, and obviously leads to the minimum operator $T_M$. The notion 'ordinal sum' has led to the following important characterization of continuous t-norms.

Theorem 8. A $[0,1]^2 \to [0,1]$ mapping $T$ is a continuous t-norm if and only if it is an ordinal sum of continuous Archimedean t-norms.

5.2. Cancellation law for pseudo-convolution

Theorem 8 and the results from [1] allow to transform the cancellation law for pseudo-convolution based on a continuous t-norm $T$ to the cases discussed in the previous sections.
Definition 6. Consider a real function $g$ and $(a, b) \in [0, 1]^2$, $a < b$.

(i) The function $g^{[a,b]}$ is defined as
\[ g^{[a,b]} = \phi_{(a,b)} \circ g, \]
i.e. $g^{[a,b]}(x) = \frac{g(x) - a}{b - a}$, where $\phi(x) = \min\{\max\{0, x\}, 1\}$

(ii) The function $g_{[a,b]}$ is defined by
\[ g_{[a,b]}(x) = \begin{cases} \phi^{-1}_{(a,b)}(g(x)), & \text{if } g(x) > 0 \\ 0, & \text{elsewhere}. \end{cases} \]

Theorem 9. Consider an ordinal sum $T \equiv \left\{ \langle a_i, b_i, T_i \rangle \mid i \in I \right\}$ written in such a way that $\bigcup_{i \in I} [a_i, b_i] = [0, 1]$, and functions $g$, $h \in S$, then the pseudo-convolution based on the semiring $([0,1], \lor, T)$ is given by
\[ g * h(x) = \sup_{i \in I} \left( g^{[a_i,b_i]} *_{T_i} h^{[a_i,b_i]} \right)_{[a_i,b_i]}(x), \]
where $*_{T_i}$ is pseudo-convolution based on semiring $([0,1], \lor, T_i)$.

Theorem 10. Let $T$ be a continuous t-norm represented as an ordinal sum of Archimedean continuous t-norms, $T \equiv \left\{ \langle a_i, b_i, T_i \rangle \mid i \in I \right\}$ and let $g$, $h$, $v \in S$. Then cancellation law for pseudo-convolution based on the semiring $([0,1], \lor, T)$ is valid iff for $\forall i \in I$ holds
\[ g^{[a_i,b_i]} *_{T_i} v^{[a_i,b_i]} = h^{[a_i,b_i]} *_{T_i} v^{[a_i,b_i]} \Rightarrow g^{[a_i,b_i]} = h^{[a_i,b_i]}. \]

Example 3. Consider the continuous t-norm $T = \{ \langle 0, \frac{1}{2}, T_P \rangle \}$ and $g$, $h$, $v \in S$. Let $- \ln v(x)$ be strictly convex on the interval $[a_v, v^{-1}(\frac{1}{2})]$ and $- \ln g(x)$ and $- \ln h(x)$ be convex on $[a_g, g^{-1}(\frac{1}{2})]$ and $[a_h, h^{-1}(\frac{1}{2})]$ respectively. Then $g * v = h * v \Rightarrow g = h.$

6. CONCLUSION

We have discussed the cancellation law for pseudo-convolutions based on triangular norms. While for the case of $T_M$ the cancellation law is valid without special requirements, in all other cases it holds only under special restrictions. Note that $T$-based pseudo-convolutions acting on (continuous) distribution functions are special triangle functions, see e.g. [4, Chapter 9 ], and thus our results provide a partial answer to an open problem of V. Hohle posed in [3, Problem 13]. As a continuation of our work, we aim to discuss the cancellation law for another types of triangle functions.
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