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Mathematica Slovaca, Vol. 29 (1979), No. 3, 293--303

Persistent URL: http://dml.cz/dmlcz/136215

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ON THE ASYMPTOTIC BEHAVIOUR OF A SOLUTION OF A DIFFERENTIAL EQUATION IN A HILBERT SPACE

IGOR BOCK

1. Introduction

We shall be dealing with the initial value problem

\[ A_0(t) \frac{d^m u}{dt^m} + \ldots + A_{m-1}(t) \frac{du}{dt} + A_m(t)u = f(t) \]  
(1.1)

\[ \left. \frac{du}{dt} \right|_{t=0} = u_r, \quad r = 0, 1, \ldots, m - 1 \]  
(1.2)

with the abstract functions \( u: (R^+ \rightarrow X) \), \( f: (R^+ \rightarrow X^*) \), the operator functions \( A_r(\cdot): (R^+ \rightarrow L(X, X^*)) \) and the elements \( u_r \in X \), where \( R^+ = [0, \infty) \), \( X \) is a Hilbert space, \( X^* \) is a dual space to \( X \) and \( L(X, X^*) \) is a space of all linear bounded operators mapping \( X \) into \( X^* \).

We shall analyse the behaviour of a solution of (1.1), (1.2) for \( t \rightarrow \infty \). Due to the results obtained in this paper the solution behaves in the same way as the deflection of a viscoelastic plate made of aging material. These results generalize the results of paper [1], where the problem (1.1), (1.2) with the stationary operator functions \( A_r(t) = A_r \) was considered.

First we shall introduce some results from the theory of differential equations in a Banach space proved in [3].

Let \( X \) be a complex Banach space, \( R^+ = [0, \infty) \). We denote by \( C(R^+, X) \) the space of all continuous functions mapping \( R^+ \) into \( X \) and by \( C^{(m)}(R^+, X) \) the space of all \( m \)-times continuously differentiable functions mapping \( R^+ \) into \( X \).

Consider the initial value problem for the differential equation in the space \( X \)

\[ \frac{du}{dt} = A(t)u + f(t), \quad t \in R^+ \]  
(1.3)

\[ u(0) = u_0 \]  
(1.4)
Theorem 1.1 ([3], III. 1.2). Let \( f \in C(R^+, X) \), \( A(\cdot) \in C(R^+, L(X, X)) \), \( u_0 \in X \). Then there exists a unique solution \( u \in C^0(R^+, X) \) of the problem (1.3), (1.4).

A solution \( u \) of the problem (1.3), (1.4) can be expressed with the help of a solution \( U \in C^0(R^+, L(X, X)) \) of the homogeneous operator differential equation in the space \( L(X, X) \)

\[
\frac{dU}{dt} = A(t)U
\]

\[
U(0) = I \quad \text{(the identical operator)}
\]

There exists for each \( t \in R^+ \) the inverse operator \( U^{-1}(t) \). The operator function \( V(.) \) is a solution of the problem

\[
\frac{dV}{dt} = -VA(t)
\]

\[
V(0) = I
\]

A solution \( u \) of (1.3), (1.4) can be expressed in the form

\[
u(t) = U(t)u_0 + \int_0^t U(t, \tau)f(\tau) \, d\tau,
\]

where

\[
U(t, \tau) = U(t)U^{-1}(\tau).
\]

The following theorem plays an important role in our further considerations of the asymptotic behaviour of a solution of the problem (1.1), (1.2).

Theorem 1.2 ([3], III. 6.3). Let \( A(.) \in C(R^+, L(X, X)) \), \( A_\infty \in L(X, X) \), \( \lim_{t \to \infty} \|A(t) - A_\infty\| = 0 \). Re \( \lambda < -\nu_0 < 0 \) for all \( \lambda \in \sigma(A_\infty) \), where \( \sigma(A_\infty) \) is the spectrum of the operator \( A_\infty \), \( \|\cdot\| \) is the norm in the space \( L(X, X) \).

Then there exist such constants \( \nu > 0 \), \( N \) depending only on \( A(t) \) that

\[
\|U(t, \tau)\| \leq Ne^{-\nu(t-\tau)}, \quad \forall t \geq \tau, \forall \tau \in R^+
\]

2. The existence and the uniqueness of a solution

Let \( X \) be a complex Hilbert space with the scalar product \((.,.)\) and the norm \( \|\cdot\| \) and \( X^* \) with the norm \( \|\cdot\|_* \) the antidual space of all linear bounded functionals over \( X \).

We formulate a theorem of the existence and the uniqueness of a solution of the problem (1.1), (1.2)

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**Theorem 2.1.** Let \( f \in C(\mathbb{R}^+, X^*) \), \( A_i(.) \in C(\mathbb{R}^+, L(X, X^*)) \), \( i = 0, 1, ..., m; \ u_r \in X, r = 0, 1, ..., m - 1 \). If there exists such a real positive and continuous on \( \mathbb{R}^+ \) function \( \alpha(t) \) that

\[
\alpha(t) \|x\|^2 \leq \|\langle A_0(t)x, x \rangle\|, \quad \forall x \in X, t \in \mathbb{R}^+
\]

then there exists a unique solution \( u \in C^{(m)}(\mathbb{R}^+, X) \) of the problem (1.1), (1.2).

**Proof.** Due to (2.1) the operators \( A(t) \) and \( A(t)^* \) (the adjoint operator to \( A(t) \)) satisfy the inequalities

\[
\alpha(t) \|x\| \leq \|A(t)x\|, \quad \forall x \in X, t \in \mathbb{R}^+
\]

Using (2.2) and the theorem on the solvability of the operator equations ([6], VII. 5) we obtain that there exists the inverse operator \( A_0^{-1}(t) \in L(X^*, X) \) satisfying

\[
\|A_0^{-1}(t)\|_{L(X^*, X)} \leq \alpha(t)^{-1}, \quad \forall t \in \mathbb{R}^+
\]

where the function \( \alpha(t)^{-1} \) is continuous on \( \mathbb{R}^+ \). Using the relation

\[
A_0^{-1}(t) - A_0^{-1}(t_0) = A_0^{-1}(t_0)(A_0(t_0) - A_0(t))A_0^{-1}(t)
\]

we can verify easily that the operator-function \( A_0^{-1}(.) \) is continuous in each point \( t_0 \in \mathbb{R}^+ \) and hence

\[
A_0^{-1}(.) \in C(\mathbb{R}^+, L(X^*, X)).
\]

Consider the initial value problem in the Hilbert product space \( \chi = [X]^m \)

\[
\frac{du}{dt} = \mathcal{A}(t)u + F(t)
\]

(2.6)

\( u(0) = u_0 \)

with the operator function \( \mathcal{A}(.) : (\mathbb{R}^+ \rightarrow L(\chi, \chi)) \), the function \( F(.) : (\mathbb{R}^+ \rightarrow \chi) \) and the element \( u_0 \in \chi \) defined by

\[
\mathcal{A}(t) = \begin{pmatrix}
0, & I, & 0, & \ldots, & 0 \\
0, & 0, & I, & \ldots, & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
0, & 0, & 0, & \ldots, & I \\
-A_0^{-1}(t)A_m(t), & \ldots, & -A_0^{-1}(t)A_1(t)
\end{pmatrix}
\]

(2.7)

\[
F(t) = (0, 0, \ldots, A_0^{-1}(t)f(t))^T,
\]

(2.8)

\[
u_0 = (u_0, u_1, \ldots, u_{m-1})^T
\]

(2.9)

Using (2.4) we obtain \( \mathcal{A}(.) \in C(\mathbb{R}^+, L(\chi, \chi)), F(.) \in C(\mathbb{R}^+, \chi) \). There exists, due to
Theorem 1.1, a unique solution \( u \in C^1(R^+, \chi) \) of (2.5), (2.6) which has the form

(2.10) \[ u(t) = (u(t), u'(t), ..., u^{(m-1)}(t))^T. \]

The function \( u \in C^{(m)}(R^+, X) \) is then a unique solution of the problem (1.1), (1.2).

3. On the asymptotic behaviour of a solution

Using the result of Theorem 1.2 we shall investigate the asymptotic behaviour of a solution of the problem (1.1), (1.2).

**Theorem 3.1.** Assume that the assumptions of Theorem 2.1 are fulfilled. Assume, moreover, that there exist such a constant \( \alpha_0 > 0 \) and the operators \( A_i \in L(X, X^*), i = 0, 1, ..., m \), that

(3.1) \[ \alpha_0 \|x\|^2 \leq \langle A_0(t)x, x \rangle, \quad \forall x \in X, t \in R^+, \]

(3.2) \[ \lim_{t \to \infty} \|A_i(t) - A_i,\| = 0, \quad i = 0, 1, ..., m \]

and the polynomial operator

(3.3) \[ D(\lambda) = \lambda^m A_{0,\infty} + ... + \lambda A_{m-1,\infty} + A_m, \lambda \in C \]

possesses the inverse operator \( D(\lambda)^{-1} \in L(X^*, X) \) for all \( \lambda \in C \) with \( \text{Re} \lambda \geq 0 \). Then the estimate

(3.4) \[ \sum_{i=0}^{m-1} \|u^{(i)}(t)\| \leq M e^{-\nu t} \left( \sum_{i=0}^{m-1} \|u_i\| + \int_0^t e^\nu \|f(\tau)\| d\tau \right) \]

of a solution \( u \in C^{(m)}(R^+, X) \) of (1.1), (1.2) holds with the constants \( M, \nu > 0 \) depending only on \( A_i(t), i = 0, 1, ..., m \).

If there exists such a functional \( f \in X^* \), that

(3.5) \[ \lim_{t \to \infty} \|f(t) - f_\infty\| = 0, \]

then

(3.6) \[ \lim_{t \to \infty} \|(u(t) - A_{m,\infty}^{-1} f_\infty\| + \sum_{i=0}^{m-1} \|u^{(i)}(t)\| = 0 \]

**Proof.** Consider the problem (1.1), (1.2) as the problem (2.5), (2.6) in the space \( \chi = [X]^m \). Using (3.1), (3.2) we can see that there exists the inverse operator \( A_{0,\infty}^{-1} \in L(X^*, X) \) satisfying the relation

(3.7) \[ \|A_{0,\infty}^{-1}\| \leq \alpha_0^{-1} \]
Using the relations (3.1), (3.7) we obtain

\[(3.8) \quad \|A_0^{-1}(t) - A_{0,\infty}^{-1}\| = \|A_{0,\infty}^{-1}(A_{0,\infty} - A_0(t))A_0^{-1}(t)\| \leq \alpha_0^{-2}\|A_{0,\infty} - A_0(t)\|, \quad \forall t \in \mathbb{R}^+\]

and combining with (3.2) we arrive at

\[(3.9) \quad \lim_{t \to \infty} \|A_0^{-1}(t) - A_{0,\infty}^{-1}\| = 0\]

Let us define the operator $\mathcal{A}: L(\mathcal{A}, \mathcal{F})$ by

\[(3.10) \quad \mathcal{A}_0 = \begin{pmatrix} 0, & I, & 0, & \ldots, & 0 \\ 0, & 0, & I, & \ldots, & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ -A_{0,\infty}A_{m,\infty}, & -A_{0,\infty}A_{m-1,\infty}, & \ldots, & -A_{0,\infty}A_{1,\infty} \end{pmatrix}\]

Combining (2.7), (3.2), (3.8), (3.9) we obtain

\[(3.11) \quad \lim_{t \to \infty} \|\mathcal{A}(t) - \mathcal{A}_0\|_{L(\mathcal{A}, \mathcal{F})} = 0.\]

We apply now the results of Theorem 1.2. We must therefore find such a number $\nu_0 > 0$ that

\[(3.12) \quad \text{Re} \lambda < -\nu_0, \quad \forall \lambda \in \sigma(\mathcal{A}_0).\]

It can be verified easily that $\lambda \in \sigma(\mathcal{A}_0)$ if and only if $0 \in \sigma(D(\lambda))$, which means that there does not exist the inverse operator $D(\lambda)^{-1}$. Using the assumption (3.3) we obtain that

\[(3.13) \quad \text{Re} \lambda < 0, \quad \forall \lambda \in \sigma(\mathcal{A}_0).\]

The set $\sigma(\mathcal{A}_0)$ is closed in the complex plane ([6], VIII. 2). Then there must exist such $\nu_0 > 0$ that (3.12) holds. Otherwise there exists such a sequence $\lambda_n \in \sigma(\mathcal{A}_0)$ that $\lim_{n \to \infty} \lambda_n = \lambda_0$, $\text{Re} \lambda_0 = 0$, $\lambda_0 \in \sigma(\mathcal{A}_0)$, which is in contradiction to (3.13).

We can now use Theorem 1.2. Combining (1.9), (1.11), (2.5), (2.6) we obtain

\[(3.14) \quad \|u(t)\| \leq M e^{-\nu t} (\|u_0\| + \int_0^{\infty} e^{\nu t} \|F(t)\| \, dt), \quad \forall t \in \mathbb{R}^+.\]

Using (2.8), (2.9), (2.10), (3.1) we obtain the estimate (3.4) with the constants $M$, $\nu > 0$ depending only on $A_i(t)$, $i = 0, 1, \ldots, m$. 297
It remains to verify the second part of the theorem. Let $f_\infty$ be such a functional from $X^*$ that (3.5) holds. We express a solution $u$ of the problem (1.1), (1.2) in the form

$$u(t) = v(t) + A_{m,\infty}^{-1}f_\infty.$$  

The operator $A_{m,\infty}^{-1}\in L(X^*, X)$ exists, because $D(0) = A_{m,\infty}$. A function $v \in C^{(m)}(R^+, X)$ is a solution of the initial value problem

$$\sum_{i=0}^{m} A_i(t) \frac{d^{m-i}v}{dt^{m-i}} = g(t)$$  

with $v_i \in X$ and

$$g(t) = f(t) - A_{m}(t)A_{m,\infty}^{-1}f_\infty.$$  

Due to the first part of the theorem a function $v$ satisfies

$$\sum_{i=0}^{m-1} \|v^{(i)}(t)\| \leq M e^{-\lambda t} \left( \sum_{i=0}^{m-1} \|v_i\| + \int_{0}^{t} e^{\lambda \tau} \|g(\tau)\| \ast d\tau \right).$$

The relations (3.2), (3.5), (3.17) imply

$$\lim_{t \to \infty} \|g(t)\| = 0.$$  

If

$$\lim_{t \to \infty} \sum_{i=0}^{m-1} \|v^{(i)}(t)\| = 0,$$

then the conclusion of the theorem follows from (3.15). Considering (3.18) we see that it suffices to verify

$$\lim_{t \to \infty} e^{-\lambda t} \int_{0}^{t} e^{\lambda \tau} \|g(\tau)\| \ast d\tau = 0.$$  

If $\int_{0}^{\infty} e^{\lambda \tau} \|g(\tau)\| \ast d\tau < \infty$, then (3.21) follows immediately. If $\lim_{t \to \infty} \int_{0}^{t} e^{\lambda \tau} \|g(\tau)\| \ast d\tau = \infty$, then (3.21) follows from (3.19) after using the L'Hospital rule and the proof is complete.

There arise difficulties with verifying the assumption about the operator $D(\lambda)$ by applying Theorem 3.1. The following corollaries show that under some conditions the polynomial operator $D(\lambda)$ defined in (3.3) satisfies the assumption of Theorem 3.3. We shall be dealing with the problem of the first and the second order.

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Corollary 3.1. \((m = 1)\). Assume that the operators \(A_{i, \infty} \in L(X, X^*)\), \(i = 0, 1\) satisfy the assumptions

\[
\langle A_{0, \infty}x, y \rangle = \langle A_{0, \infty}y, x \rangle, \quad \forall x, y \in X,
\]

\[
0 \leq \langle A_{0, \infty}x, x \rangle, \quad \forall x \in X,
\]

\[
\alpha_1 \|x\|^2 \leq \text{Re} \left\langle A_{1, \infty}x, x \right\rangle, \quad \alpha_1 > 0, \forall x \in X.
\]

Then the operator

\[
D(\lambda) = \lambda A_{0, \infty} + A_{1, \infty}
\]

possesses the inverse operator \(D(\lambda)^{-1}\) for all \(\lambda \in C\) with \(\text{Re} \lambda \geq 0\).

Proof. Using (3.22) we obtain

\[
\text{Re} \left\langle D(\lambda)x, x \right\rangle = \text{Re} \lambda \left\langle A_{0, \infty}x, x \right\rangle + \text{Re} \left\langle A_{1, \infty}x, x \right\rangle.
\]

Considering (3.23), (3.24) we arrive at

\[
\text{Re} \left\langle D(\lambda)x, x \right\rangle \geq \alpha_1 \|x\|^2, \quad \lambda \in C, \text{Re} \lambda \geq 0, \forall x \in X.
\]

The last inequality implies the existence of the inverse operator \(D(\lambda)^{-1}\) for all \(\lambda \in C\) with \(\text{Re} \lambda \geq 0\) and the proof is complete.

Corollary 3.2. \((m = 2)\). Assume that the operators \(A_{i, \infty} \in L(X, X^*)\), \(i = 0, 1, 2\) satisfy the next assumptions

\[
\langle A_{i, \infty}x, y \rangle = \langle A_{i, \infty}y, x \rangle, \quad j = 0, 2, \forall x, y \in X,
\]

\[
0 \leq \langle A_{0, \infty}x, x \rangle, \quad \forall x \in X,
\]

\[
\alpha_1 \|x\|^2 \leq \text{Re} \left\langle A_{1, \infty}x, x \right\rangle, \quad \alpha_1 > 0, \forall x \in X,
\]

\[
\alpha_2 \|x\|^2 \leq \left\langle A_{2, \infty}x, x \right\rangle, \quad \alpha_2 > 0, \forall x \in X.
\]

Then the operator

\[
D(\lambda) = \lambda^2 A_{0, \infty} + \lambda A_{1, \infty} + A_{2, \infty}
\]

possesses the inverse operator \(D(\lambda)^{-1}\) for all \(\lambda \in C\) with \(\text{Re} \lambda \geq 0\).

Proof. Assume first that \(\lambda = 0\). Then \(D(\lambda) = D(0) = A_{2, \infty}\). There exists due to (3.31) the inverse operator \(A_{2, \infty}^{-1} = D(0)^{-1}\).

Let \(\lambda \neq 0\), \(\text{Re} \lambda \geq 0\). Consider the operator \(T(\lambda) = \lambda^{-1}D(\lambda)\). \(T(\lambda)\) can be expressed in the form

\[
T(\lambda) = \lambda A_{0, \infty} + A_{1, \infty} + \frac{\lambda}{|\lambda|^2} A_{2, \infty}, \quad \lambda \neq 0
\]

With the help of (3.28) we obtain
(3.34) \[ \text{Re} \langle T(\lambda)x, x \rangle = \text{Re} \lambda \langle A_0, x, x \rangle + \text{Re} \langle A_1, x, x \rangle + \]
\[ + \frac{\text{Re} \lambda}{|\lambda|^2} \langle A_2, x, x \rangle, \quad \lambda \neq 0, x \in X \]

Using (3.29), (3.30) we obtain the inequality

(3.35) \[ \text{Re} \langle T(\lambda)x, x \rangle \geq \alpha_1 \|x\|^2, \quad \forall x \in X, \lambda \in C, \; \text{Re} \lambda \geq 0, \lambda \neq 0, \]

which implies the existence of the operator \( T(\lambda)^{-1} \). Then, however, there exists the inverse operator \( D(\lambda)^{-1} = \lambda^{-1} T(\lambda)^{-1} \) for all \( \lambda \neq 0 \) with \( \text{Re} \lambda \geq 0 \) and the proof is complete.

**Remark 3.1.** The previous results can be applied to the case of the real Hilbert space \( X \), too. We can extend the space \( X \) onto the complex Hilbert space \( \hat{X} = \{ \hat{x} = (x_1, x_2) \in X \times X \} \) with the scalar product [\( \langle \hat{x}, \hat{y} \rangle = (x_1, y_1) + (x_2, y_2) + i((x_2, y_1) - (x_1, y_2)) \)]. The operator \( A \in L(X, X^*) \) can be extended onto the operator \( \hat{A} \in L(\hat{X}, \hat{X}^*) \) by \( \langle \hat{A} \hat{x}, \hat{y} \rangle = \langle Ax_1, y_1 \rangle + \langle Ax_2, y_2 \rangle + i(\langle Ax_1, y_2 \rangle - \langle Ax_2, y_1 \rangle) \).

4. **Bending of viscoelastic plates with aging**

The previous theory can be applied to the initial boundary value problem, which expresses a bending of a viscoelastic plate made of aging material with a short memory ([5], IV.). We suppose, that the central surface of the plate is the bounded region \( \Omega \subset E_2 \) with the Lipschitz boundary \( \partial \Omega \) (def. [4]). We assume that \( \partial \Omega = \Gamma_1 \cup \Gamma_2, \Gamma_1 \cap \Gamma_2 = \emptyset \). A plate is clamped on \( \Gamma_1 \) and simply supported on \( \Gamma_2 \). The case \( \Gamma_1 = \partial \Omega \), or \( \Gamma_2 = \partial \Omega \) is always possible. The bending \( u(x_1, x_2, t) \) of the plate is a solution of the initial boundary value problem

\[ \sum_{r=0}^{m} K^{(r)}_{ijkl}(t) \frac{d^{m-r}u_{,ijkl}}{dt^{m-r}}(x_1, x_2, t) = f(x_1, x_2, t), \quad (x_1, x_2, t) \in \Omega \times R^+ \]

\[ \frac{d^r u}{dt^r} \bigg|_{t=0} = u_r, \quad r = 0, 1, ..., m - 1 \]

\[ u = 0 \text{ on } \partial \Omega \times R^+ \]

\[ \frac{\partial u}{\partial n} = 0 \text{ on } \Gamma_1 \times R^+ \]

\[ M(t)u = \sum_{r=0}^{m} K^{(r)}_{ijkl}(t) \frac{d^{m-r}u_{,ijkl}}{dt^{m-r}}(n, x_k) \cos (n, x_1) \cos (n, x_1) = 0 \]

\[ \text{on } \Gamma_2 \times R^+. \]
We denote by \( n \) the exterior normal to \( \partial \Omega \). The above problem with constant coefficients is investigated in [2]. We use the notation \( u \) is a weak solution of the problem (4.1)—(4.5).

\[
D^r u = \frac{\partial^{r+1} u}{\partial x_1^{i_1} \cdots \partial x_d^{i_d}}, \quad i = (i_1, i_2), \quad |i| = i_1 + i_2.
\]

We denote by \( W(Q) \) the space of all functions from \( H^2(Q) \) which satisfy the essential (or geometrical) boundary conditions (4.3), (4.4) in the sense of traces (def. [4]). It can be verified with the help of the Fridrichs and Poincaré inequalities ([4]), that \( W(Q) \) is a Hilbert space with the scalar product and the norm

\[
(u, v)_2 = \sum_{|i| \leq 2} \int_Q D'u D'v \, d\Omega
\]

\[
||u|| = \left( \sum_{|i| \leq 2} \int_Q (D'u)^2 \, d\Omega \right)^{1/2},
\]

which is equivalent to the original norm in the space \( H^2(\Omega) \). Let us denote by \( W(\Omega)^* \) the space dual to \( W(\Omega) \). We define now a weak solution of the problem (4.1)—(4.5).

**Definition 4.1.** Let \( f \in C(R^+, W(\Omega)^*), \quad u_i \in W(\Omega), \quad i = 0, 1, \ldots, m - 1, \quad K_{ijkl}(.) \in C(R^+), \quad r = 0, 1, \ldots, m ; i, j, k, l \in \{1, 2\}. A function \( u \in C^{(m)}(R^+, W(\Omega)) \), which is for each \( h \in W(\Omega) \) a solution of the initial value problem

\[
\sum_{r=0}^{m} \int_{\Omega} K_{ijkl}^{(r)} \frac{d^{m-r}}{dt^{m-r}} u_{ijkl}(t) h_{ijkl} \, d\Omega = \langle f(t), h \rangle
\]

\[
\left. \frac{d^r u}{dt^r} \right|_{t=0} = u_r, \quad r = 0, 1, \ldots, m - 1,
\]

is a weak solution of the problem (4.1)—(4.5).
If we define the operators $A_r(t)$ by

\[
\langle A_r(t)u, h \rangle = \int_\Omega K^{(r)}_{ijkl}(t)u_{,ij}h_{,kl} \, d\Omega ,
\]

(4.13)

$u, h \in W(\Omega), \quad t \in \mathbb{R}^+, \quad r = 0, 1, \ldots, m$,

then the operators $A_r(t)$ (extended to $A_r(t)$ according to Remark 3.1) satisfy all the assumptions of Theorem 2.1 with $X = W(\Omega), \quad X^* = W(\Omega)^*$ and hence there exists a unique weak solution of the problem (4.1)—(4.5).

If $\lim_{t \to \infty} K^{(r)}_{ijkl}(t) = K^{(r)}_{ijkl}, \quad r = 0, 1, \ldots, m \quad \lim_{t \to \infty} \|f(t) - f_\infty\|^* = 0, \quad f_\infty \in W(\Omega)^*$, then the assumptions of Corollaries 3.1, 3.2 are fulfilled and hence a weak solution $u$ of (4.1)—(4.5) satisfies in the cases $m = 1, 2$ the relation

\[
\lim_{t \to \infty} \|u(t) - u_\infty\| = 0,
\]

where $u_\infty \in W(\Omega)$ is a weak solution of the corresponding elastic problem, i.e.

\[
\int_\Omega K^{m,\infty}_{ijkl}u_{,ij}h_{,kl} \, d\Omega = \langle f_\infty, h \rangle, \quad \forall h \in W(\Omega).
\]

(4.15)

This result corresponds with the physical experience.

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Received September 22, 1977

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АСИМПТОТИЧЕСКОЕ ПОВЕДЕНИЕ РЕШЕНИЯ ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ В ПРОСТРАНСТВЕ ГИЛЬБЕРТА

Игорь Бок

Резюме

В этой работе изучается начальная задача (1.1), (1.2) в пространстве Гильберта $X$ с операторными функциями $A_t(.) \in C(R^+, L(X, X^*))$. Если оператор $A_t$ коэрцитивный для любого $t \in R^+$, то для любой функции $f \in C(R^+, X^*)$ и для любых элементов $u \in X$ существует единственное решение задачи (1.1), (1.2). Если выполнены некоторые предположения и если

$$\lim_{t \to \infty} \|A_t(t) - A_{t-} \| = \lim_{t \to \infty} \|f(t) - f_\infty \| = 0,$$

то

$$\lim_{t \to \infty} \|u(t) - A_{t-}^{-1} f_\infty \| = 0.$$

Полученные результаты используются для решения начально краевых задач, решения которых определяют изгибы вязкоупругих плит со свойствами зависящими от времени.