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Peter Horák

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A NOTE ON REMOVING A POINT OF A STRONG DIGRAPH

PETER HORÁK

Throughout the paper we use the notation and terminology of Harary [3]. In particular, the set of points and the set of lines in a given digraph D will be denoted by $V(D)$ and $E(D)$ respectively. If there is a walk from a point u to a point v, then v is said to be reachable from u. A digraph is strong if every two points are mutually reachable and is unilateral if for every pair u, v of points either v is reachable from u or u is reachable from v.

The effect that the removal of a point from a strong digraph has on its connectivity class was studied by Ross and Harary [6], Harary, Norman and Cartwright [4], Manvel, Stockmeyer and Welsh [5], Geller [2], Fink [1].

From [2] it immediately follows that any strong digraph D has a point v such that $D-v$ is unilateral. Further, in [5] it is stated that for any point v of a strong digraph D there is a point $u(v)\neq v$ such that v can reach every point in $D-u(v)$. We combine these two statements in the following theorem.

Theorem. *Let D be a strong digraph with at least two points. Then for every point v of D there exists a point $u(v)\neq v$ such that $D-u(v)$ is unilateral and v can reach every point in $D-u(v)$.*

Proof. We prove our theorem by the induction on $|V(D)|$. The first step of the induction is straightforward. Now, let D be a strong digraph with $p>2$ points and let v be a point of D. If D is hamiltonian, then the initial point of the line of the hamiltonian cycle whose terminal point is v has the required properties. Otherwise D contains a cycle $C=v, v_1, \ldots, v_k, v$, where $k<p-1$. Let D_1 be the digraph obtained from D by the contraction of C to the point v (i.e., $V(D_1)=V(D)-W$, where $W=\{v_1, v_2, \ldots, v_k\}$ and a line x belongs to $E(D_1)$ iff either $x\in E(D)$ and $x=u_1u_2$ where $u_1\notin W$, $u_2\notin W$ or $x=vz$ $(x=zv)$ in the case when there is an i, $1\leq i\leq k$ such that $v_iz\in E(D)$ $(zv_i\in E(D))$). As $2\leq|V(D_1)|<p$, by the induction hypothesis there exists a point $u(v)\neq v$, $u(v)\in V(D_1)$ such that the digraph $D_1-u(v)$ is unilateral and v can reach every point in $D_1-u(v)$. From the construction of D_1 it is clear that the point $u(v)$ has the same properties in D.

Corollary. *Let D be a strong digraph with at least two points. Then there exists* *a line $x(v)$ such that $D - x(v)$ is unilateral and v can reach every point in $D - x(v)$ for every point v of D.*

Proof. Let $u(v)$ be a point assigned to v by the Theorem. Then it is sufficient to define $x(v)$ as the line directed from $u(v)$.

Obviously, the theorem and corollary dual to the preceding ones (i.e. that can be obtained by replacing the phrase *"can reach"* by *"can be reached"*) hold.

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Katedra matematiky a deskriptívnej geometrie
Stavebnej fakulty SVŠT
Radlinského 11
813 68 Bratislava

ЗАМЕЧАНИЕ ОБ УДАЛЕНИИ ВЕРШИНЫ ИЗ СИЛЬНОГО ОРГРАФА

Peter Horák

Резюме

В статье доказана следующая теорема. Пусть D сильный орграф. Тогда для каждой вершины v из D существует вершина $u(v) \neq v$ так, что орграф $D - u(v)$ односторонний и каждая вершина орграфа $D - u(v)$ достижима из вершины v.