Ján Mikunda; Jozef Rovder

On nonoscillatory solutions of a class of nonlinear differential equations

Mathematica Slovaca, Vol. 36 (1986), No. 1, 29--38

Persistent URL: http://dml.cz/dmlcz/136412

Math. Slovaca 36, 1986, No. 1, 29—38

ON NONOSCILLATORY SOLUTIONS OF A CLASS OF NONLINEAR DIFFERENTIAL EQUATIONS

JÁN MIKUNDA — JOZEF ROVDER

1. Introduction

The present paper will deal with the diferential equation

$$L_n y \pm (-1)^n f(t, y, y', \dots, y^{(m)}) = 0 . \qquad \text{(E)}$$

where $m \in \{0, 1, \dots, n-1\}$ and $L_n y$ is the quasi-desrivative of y of order n.

Throughout the paper we suppose that the function $f(t, u_0, u_1, \dots, u_m)$ is continuous on a region

$$D: a \leq t < \infty, \ -\infty < u_i < \infty, \ i = 0, 1, \dots, m$$

and for every point $(c_0, c_1, \dots, c_m) \neq (0, 0, \dots, 0)$ the function $f(t, c_0, \dots, c_m)$ is not equal to zero in any sub-interval of the interval $[a, \infty)$.

Further we suppose that in the quasi-derivates $L_i y$, defined by $L_0 y = a_0(t) y$, $L_i y = a_i(t)\,(L_{i-1} y)'$, $i = 1, 2, \dots, n$, the functions $a_i(t)$, $i = 0, 1, \dots, n$ are positive and continuous functions on $[a, \infty)$ and

$$\int_a^\infty \frac{1}{a_i(t)}\, dt = \infty \qquad (1)$$

for $i = 1, \dots, n-1$.

A function $u(t)$ is called a solution of (E) iff $u(t)$ has continuous quasi-derivatives $L_i u(t)$, $i = 0, 1, \dots, n$, continuous derivatives of order m on the interval $[a, \infty)$ and it satisfies (E).

A solution $u(t)$ of (E) is called nonoscillatory iff there exists a number $c \geq a$ such that $u(t) \neq 0$ on $[c, \infty)$. The aim of this paper is to extend the results of [1], [2] and [3] for differential equations with quasi-derivatives. It is proved that every nonoscillatory solution of (E) (if there exists one) belongs to one set defined before. The existence of a nonoscillatory solution of (E) was studied in [4], [5].

2. Preliminary results

If the sign +, resp. −, holds in (E), then the equation (E) will be signed by (E^+), resp. (E^-).

For $k=0, 1, \dots, n-1$ let us define the function $\omega^k(t)$ as follows:

$$\omega^0(t)=1$$

$$\omega^k(t)=\int_a^t \frac{ds_1}{a_1(s_1)}\int_a^{s_1}\frac{ds_2}{a_2(s_2)}\cdots\int_a^{s_{k-1}}\frac{ds_k}{a_k(s_k)} \quad \text{for } k=1, \dots, n-1$$

and $\omega_{i,k}(t)$:

$$\omega_{0,k}=1 \quad \text{for } k=1, \dots, n$$

$$\omega_{i,k}(t)=\int_a^t \frac{1}{a_{n+i-k}(s)}\,\omega_{i-1,k}(s)\,ds \quad \text{for } k=1, \dots, n$$

and $i=1, 2, \dots, k-1$.

Let us define the following sets on nonoscillatory solutions of (E). Let S_0 be the set of a nonoscillatory solution $y(t)$ of (E) such that $L_0y(t)$ be bounded, let S_k, $k=1, 2, \dots, n-1$, be the set of nonoscillatory solutions $y(t)$ of (E) with the properties

$$\lim_{t\to\infty}\frac{|L_0y(t)|}{\omega^{k-1}(t)}>0 \quad \text{and} \quad \lim_{t\to\infty}\frac{L_0y(t)}{\omega^k(t)}=0\,,$$

and let S_n be the set of nonoscillatory solutions $y(t)$ of (E) such that

$$\lim_{t\to\infty}\frac{|L_0y(t)|}{\omega^{n-1}(t)}>0\,.$$

Lemma 1. [Švec [5]]. *Let (1) be valid. Then*

$$\lim_{t\to\infty}\omega^i(t)=\infty \quad \textit{as } t\to\infty \quad \textit{for } i=1, 2, \dots, n-1$$

$$\lim_{t\to\infty}\frac{\omega^j(t)}{\omega^i(t)}=\infty \quad \textit{as } t\to\infty \quad \textit{for } 0\leqslant i<j\leqslant n-1\,.$$

Lemma 2. *Suppose that $y(t)\geqq 0$ on $[b\,;\infty)$, $L_ny(t)$ exists on $[b\,;\infty)$ and*

$$\lim_{t\to\infty}\frac{L_0y(t)}{\omega^r(t)}=0$$

for an integer r, $1\leqslant r\leqslant n-1$. Suppose that $L_ny(t)\not\equiv 0$ on any subinterval of $[b\,;\infty)$.

If $L_n y(t) \leqslant 0$ *on* $[b\,;\infty)$, *then*

$$(-1)^{k+1} L_{n-k} y(t) > 0 \quad on \quad [b\,;\infty)$$

for $k = 1, 2, \ldots, n - r$, *and also for* $k = n - r + 1$ *if* $n - r$ *is even.*

If $L_n y(t) \geqslant 0$ *on* $[b\,;\infty)$, *then*

$$(-1)^{k} L_{n-k} y(t) > 0 \quad on \quad [b\,;\infty)$$

for $k = 1, 2, \ldots, n - r$, *and also for* $k = n - r + 1$ *if* $n - r$ *is odd.*

Proof. Suppose $L_n y(t) \leqslant 0$ *on* $[b\,;\infty)$. We need to prove $L_{n-1} y(t) > 0$ on $[b\,;\infty)$. If $L_{n-1} y(\alpha) \leqslant 0$ for some $\alpha \geqslant b$, then $L_{n-1} y(t)$ is negative and decreasing on $[\alpha\,;\infty)$. So there exist a negative constant K and a number $\beta > \alpha$ such that $L_{n-1} y(t) < K$ on $[\beta\,;\infty)$.

Integrating the last inequality $(n-1)$ times over (β, t) we get

$$L_0 y(t) < K\omega^{n-1}(t) + K_1 \omega^{n-2}(t) + \ldots + K_{n-1}\,\omega^0(t)\,.$$

From the Lemma 1 it follows that $\lim\limits_{t\to\infty} L_0 y(t) = -\infty$, which contradicts the assumption $y(t) \geqslant 0$. Therefore $L_{n-1} y(t) > 0$ on $[b\,;\infty)$. Now we are to prove that $L_{n-2} y(t) < 0$. If $L_{n-2} y(\alpha) \geqslant 0$ for some $\alpha \geqslant b$; then $L_{n-2} y(t)$ is positive and increasing on $[\alpha\,;\infty)$ and so there exist a positive number M and a number β_1 such that $L_{n-2} y(t) > M$ on $[\beta_1\,;\infty)$. From this inequality and from Lemma 1 we obtain

$$\lim_{t\to\infty} \frac{L_0 y(t)}{\omega^{n-2}(t)} > M > 0\,.$$

On the other hand

$$\lim_{t\to\infty} \frac{L_0 y(t)}{\omega^{n-2}(t)} = \lim_{t\to\infty} \frac{L_0 y(t)}{\omega^{r}(t)} \cdot \frac{\omega^{r}(t)}{\omega^{n-2}(t)} = 0$$

for $r \leqslant n-2$, which is a contradiction. Repeating the above arguments we complete the proof.

Lemma 3. *Let* $L_n y(t)$ *exist on* $[b\,;\infty)$ *and* $L_n y(t) \not\equiv 0$ *on any subinterval of* $[b\,;\infty)$. *Let* $L_0 y(t)$ *be bounded on* $[b\,;\infty)$.

If $L_n y(t) \leqslant 0$ *on* $[b\,;\infty)$, *then there exists a number* $c \geqslant b$ *such that*

$$(-1)^{k+1} L_{n-k} y(t) > 0 \quad on \quad [c\,;\infty)$$

for $k = 1, 2, \ldots, n-1$.

If $L_n y(t) \geqslant 0$ *on* $[b\,;\infty)$, *then*

$$(-1)^{k} L_{n-k} y(t) > 0 \quad on \quad [c\,;\infty)$$

for $k = 1, 2, \ldots, n-1$.

Proof. Let $L_n y(t) \leqslant 0$ on $[b\,;\,\infty)$ and a non-identically zero on any subinterval of $[b\,;\,\infty)$. Then there exists a number c such that $L_k y(t)$ is onesigned on $[c\,;\,\infty)$ for all $k=0, 1, \ldots, n-1$. Now we prove that $L_k y(t) \cdot L'_k y(t) < 0$ on $[c\,;\,\infty)$ for $k=1, \ldots, n-1$. From the definition $L_k y(t)$ it follows that

$$L_{k-1}y(t) = L_{k-1}y(c) + \int_c^t \frac{1}{a_{k-1}(s)} L_k y(s)\, ds\,. \tag{2}$$

Suppose that for some $k \geqslant 1$ $L_k y(t) \cdot L'_k y(t) < 0$ fails on $[c\,;\,\infty)$, i.e. $L_k y(t) \cdot L'_k y(t) > 0$ on $[c\,;\,\infty)$. Then $L_k y(t)$ is either positive and increasing or negative and decreasing. From (2) we get that $L_{k-1}y(t)$ is unbounded and has the same sign as $L_k y(t)$. Repeating this procedure we get that $L_0 y(t)$ is unbounded, which is a contradiction. Therefore $L_k y(t) \cdot L'_k y(t) < 0$ on $[c\,;\,\infty)$ for $k=1, \ldots, n-1$. From the last condition we have that $L_{n-1}y(t) > 0$, $L_{n\ 2}y(t) < 0$, ... i.e. $(-1)^{k+1} L_{n-k}y(t) > 0$ for $k=1, \ldots, n-1$. If $L_n y(t) \geqslant 0$, then the proof is similar.

Lemma 4. *Let $y(t)$ be a solution of* (E), *then*

$$L_{n-k}y(t) = L_{n-k}y(c) + \sum_{i=1}^{k-1} (-1)^{i+1} L_{n+i-k}y(t)\omega_{i,k}(t) -$$
$$- \sum_{i=1}^{k-1} (-1)^{i+1} L_{n+i-k}y(c) \cdot \omega_{i,k}(c) \pm$$
$$\pm (-1)^n (-1)^{k+1} \int_c^t \frac{1}{a_n(s)} \omega_{k-1,k}(s) \cdot f(s, y(s), \ldots, y^{(m)}(s))\, ds$$

holds for $t \geqslant c \geqslant a$ and $1 \leqslant k \leqslant n$ $\left(\text{if } k=1 \text{ we put } \sum_{i=1}^{0} = 0\right)$

Proof. Let $y(t)$ be a solution of (E). Integrating

$$[L_{n-k}y(t)]' = \frac{1}{a_{n-k+1}(t)} L_{n-k+1}y(t)$$

over $[c, t]$ we get

$$L_{n-k}y(t) = L_{n-k}y(c) + \int_c^t \frac{1}{a_{n-k+1}(s)} L_{n-k+1}y(s)\, ds\,.$$

Calculating the integral by parts we have

$$L_{n-k}y(t) = L_{n-k}y(c) + [\omega_{1,k}(s) L_{n-k+1}y(s)]_c^t - \int_c^t \omega_{1,k}(s) \cdot$$
$$\cdot \frac{1}{a_{n-k+2}(s)} L_{n-k+2}y(s)\, ds\,.$$

Repeating this procedure i times we get

$$L_{n-k}y(t)=L_{n-k}y(c)+\sum_{j=1}^{i}(-1)^{j+1}[\omega_{j,k}(s)\,L_{n-k+j}(s)]_c^t+$$

$$+(-1)^i\int_c^t \omega_{i,k}(s)\,\frac{1}{a_{n-k+i+1}(s)}\,L_{n-k+i+1}y(s)\,ds\,.$$

Finally for $i=k-1$ there holds

$$L_{n-k}y(t)=L_{n-k}y(c)+\sum_{j=1}^{k-1}(-1)^{j+1}[\omega_{j,k}(s)L_{n-k+j}y(s)]_c^t+$$

$$+(-1)^{k+1}\int_c^t \frac{1}{a_n(s)}\,\omega_{k-1,k}(s)L_ny(s)\,ds=$$

$$L_{n-k}y(c)+\sum_{j=1}^{k-1}(-1)^{j+1}[\omega_{j,k}(s)L_{n-k+j}y(s)]_c^t\pm$$

$$\pm(-1)^n(-1)^{k+1}\int_c^t \frac{1}{a_n(s)}\,\omega_{k-1,k}(s)f(s,\,y(s),\,\ldots,\,y^{(m)}(s))\,ds\,.$$

3. Results.

Theorem 1. *Let the function $f(t,\,u_0,\,u_1,\,\ldots,\,u_m)$ have the following properties*
(H$_1$) $u_0f(t,\,u_0,\,u_1,\,\ldots,\,u_m)\geqslant 0$

(H$_2$) *If* $\alpha(t)\in C^m[a\,;\,\infty)$ *and* $\lim_{t\to\infty} L_0\alpha(t)=K\neq 0,$ *then*

$$\{\operatorname{sgn}\alpha(t)\}\int^\infty \omega_{n-1,n}(s)\,\frac{1}{a_n(s)}\,f(s,\,\alpha(s),\,\alpha'(s),\,\ldots,\,\alpha^{(m)}(s))\,ds=\infty.$$

Then (i) $S_0=\emptyset$ *for equation* (E$^+$), *i.e. if $L_0y(t)$ is bounded, then $y(t)$ is oscillatory.*

(ii) *If $y(t)$ is a solution of* (E$^-$) *and $y(t)\in S_0$, then $\lim_{t\to\infty} L_0y(t)=0$*

Proof. (i). From Lemma 4 it follows that every solution of (E$^+$) satisfies the equation

$$L_0y(t)=L_0y(c)+\sum_{i=1}^{n-1}(-1)^{i+1}\omega_{i,n}(t)L_iy(t)-\sum_{i=1}^{n-1}(-1)^{i+1}\omega_{i,n}(c)$$

$$L_iy(c)-(-1)^n(-1)^{n+1}\int_c^t \omega_{n-1,n}(s)\,\frac{1}{a_n(s)}\,f(s,\,y(s),\,y'(s),\,\ldots,\,y^{(m)}(s))\,ds$$

Suppose $S_0\neq\emptyset$, i.e. there exists a nonoscillatory solution $y(t)$ such that $L_0y(t)$ is bounded.

Let $y(t)<0$, n be even. Then $L_n y(t)=-f(t, y(t), \ldots, y^{(m)}(t))\geqslant 0$ by hypothesis (H_1). By Lemma 3 there hold

$$\sum_{i=1}^{n-1}(-1)^{i+1}L_i y(t)<0 \text{ on } [c, \infty) \text{ for } c>a \tag{3}$$

and so

$$L_0 y(t)\leqslant L+\int_c^t \frac{1}{a_n(s)}\,\omega_{n-1,n}(s)f(s, y(s), \ldots, y^{(m)}(s))\,ds\,,$$

where $L=L_0 y(c)-\sum_{i=1}^{n-1}(-1)^{i+1}\omega_{i,n}(c)L_i y(c)$.

Since $L_1 y(t)<0$, then $L_0 y(t)$ is decreasing and so there exists $\lim_{t\to\infty} L_0 y(t)=K<0$ Hence, by hypothesis (H_2), the righthand side of (3) diverges to $-\infty$, which contradicts the boundedness of $L_0 y(t)$.

(ii) Let $y(t)$ be a solution of (E^-), $y(t)\in S_0$ and $\lim_{t\to\infty} L_0 y(t)=K\neq 0$. If $y(t)<0$, then, by Lemmas 3 and 4, $y(t)$ satisfies the inequality

$$L_0 y(t)\geqslant L-\int_c^t \frac{1}{a_n(s)}\,\omega_{n-1,n}(s)f(s, y(s), \ldots, y^{(m)}(s))\,ds\,.$$

Now we have a contradiction, because $L_0 y(t)$ is bounded while the right-hand side diverges to ∞ for $t\to\infty$.

Let $S=S_0\cup S_2\cup\ldots\cup S_n$ if n is even and let $S=S_0\cup S_2\cup\ldots\cup S_{n-1}$ if n is odd for equation (E^+).

For equation (E^-) denote $S=S_1\cup S_3\cup\ldots\cup S_n$ if n is odd and $S=S_1\cup S_3\cup\ldots\cup S_{n-1}$ if n is even.

Theorem 2. *Suppose that the differential equation (E) satisfies the following hypotheses:*

(h_1) $u_0 f(t, u_0, u_1, \ldots, u_m)\geqq 0$

(h_2) *Let* $r\in\{1, 2, \ldots, n\}$. *If* $\alpha(t)\in C^m[a, \infty)$, $L_{r-1}\alpha(t)\in C[a, \infty)$ *and* $\lim_{t\to\infty} L_{r-1}\alpha(t)\neq 0$, *then*

$$\operatorname{sgn}\{\alpha(t)\}\int^{\infty}\frac{1}{a_n(s)}\,\omega_{n-r,n-r+1}(s)f[s, \alpha(s), \ldots, \alpha^{(m)}(s)]\,ds=\infty\,.$$

Then $S_r=\emptyset$ *in the equation* (E^+) *if* r *is even and* $S_r=\emptyset$ *in the equation* (E^-) *if* r *is odd.*

Proof. Let us consider the equation (E^+) and n is even. Suppose on the contrary, $S_r \neq \emptyset$ for some $r \in \{2, 4, \ldots, n\}$. Let $y(t) \in S_r$, $y(t) > 0$. Then by l'Hospital's rule we obtain

$$\lim_{t\to\infty} \frac{L_0 y(t)}{\omega^{r-1}(t)} = \lim_{t\to\infty} L_{r-1} y(t) > 0, \quad \lim_{t\to\infty} \frac{L_0 y(t)}{\omega^r(t)} = \lim_{t\to\infty} L_r y(t) = 0 .$$

Since $n-r$ is even then from Lemma 2 there yields

$$(-1)^{k+1} L_{n-k} y(t) > 0, \text{ for } k = 1, 2, \ldots, n-r, n-r+1 , \tag{4}$$

and sgn $L_{r-1}y(t) = \operatorname{sgn} y(t) > 0$.

If we put $k = n-r+1$ into $L_{n-k}y(t)$ given by Lemma 4, we get

$$L_{r-1}y(t) = L_{r-1}y(c) + \sum_{j=1}^{n-r} (-1)^{j+1} \omega_{j,\, n-r+1}(t) \cdot L_{r+j-1} y(t) -$$

$$- \sum_{j=1}^{n-r} (-1)^{j+1} \omega_{j,\, n-r+1}(c) L_{r+j-1}(c) - \int_c^t \frac{1}{a_n(s)}\, \omega_{n-r,\, n-r+1}(s) f[s, y(s) \ldots, y^{(m)} s)]\, ds . \tag{5}$$

From (4) it follows that the sums in (5) are negative and so

$$L_{r-1} y(t) < L - \int_c^t \frac{1}{a_n(s)}\, \omega_{n-r,\, n-r+1}(s) f[s, y(s), \ldots, y^{(m)}(s)]\, ds , \tag{6}$$

where L is a constant. Since $\lim_{t\to\infty} L_{r-1}y(t) > 0$, then, by ($h_2$), the right-hand side of (6) diverges to $-\infty$, while the left-hand side of (6) is positive, which is a contradiction.

If $r = n$, then the contradiction follows immediately from the equality

$$L_{n-1} y(t) = L_{n-1} y(c) - \int_c^t \frac{1}{a_n(s)} f[s, y(s), \ldots, y^{(m)}(s)]\, ds ,$$

since $L_{n-1}y(t) > 0$ and the right-hand side diverges to $-\infty$. In a similar way we can prove all the other cases.

If in (h_2) we put $r = 1$, then (h_2) implies (H_2), and so the following theorem holds.

Theorem 3. *If* (h_1) *and* (h_2) *hold for every* $r \in \{1, 2, \ldots, n\}$, *then* $S = \emptyset$.

Theorem 4. *Suppose the following assumptions are valid:*

(a_1) *There exists a continuous function* $p(t) \geqq 0$ *on* $[a, \infty)$ *such that* sgn $\{u_0\} f(t, u_0, \ldots, u_m) \geqq p(t)|u_0|$.

(a_2) $\displaystyle\int^{\infty} \frac{1}{a_0(s) a_n(s)}\, \omega_{n-1,\, n}(s)\, ds = \infty$.

(a_3) $\omega^{k-1}(t) \cdot \omega_{n-k,\, n-k+1}(t) \geqq \omega_{n-1,\, n}(t)$ *for* $k = 1, 2, \ldots, n$.

Then $S = \emptyset$.

Proof. Let $y(t) \in S_r$ and (a_1) holds. Then

$$\operatorname{sgn}\{y(t)\} a_0(t) f[t, y(t), \ldots, y^{(m)}(t)] \geq p(t)\, a_0(t)\, |y(t)| - p(t)\, |L_0 y(t)|\,,$$

$$\operatorname{sgn}\{y(t)\} \frac{1}{a_n(t)} f[t, y(t), \ldots, y^{(m)}(t)] \geq K \frac{1}{a_n(t) a_0(t)} \omega^{r-1}(t) p(t)\,,$$

$$\operatorname{sgn}\{y(t)\} \frac{1}{a_n(t)} \omega_{n-r, n-r+1}(t) f[t, y(t), \ldots, y^{(m)}(t)] \geq$$

$$\frac{K}{a_n(t) a_0(t)} \omega^{r-1}(t) \cdot \omega_{n-r, n-r+1}(t) p(t) \geq \frac{K}{a_n(t) a_0(t)} \omega_{n-1, n}(t) p(t)\,. \qquad (6')$$

Thus the assumptions (h_1) and (h_2) of Theorem 2 hold for each $k \in \{1, 2, \ldots, n\}$, therefore $S = \emptyset$.

From the definition of S_k it is evident that $S_i \cap S_j = \emptyset$, $i \neq j$, $i, j = 0, 1, \ldots, n$ except for $S_0 \cap S_1$ which consists of solutions $y(x)$ such that $\lim_{t \to \infty} L_0 y(t) \neq 0$. However, if (H_1), (H_2) are satisfied, then by Theorem 1 every nonoscillatory solution of (E) has $L_0 y(t)$ unbounded or approaches zero, i.e. $S_0 \cap S_1$ is empty too.

Let $S' = S_1 \cup S_3 \cup \ldots \cup S_{n-1}$ if n is even and $S' = S_1 \cup S_3 \ldots \cup S_n$ if n is odd for equation (E^+). For equation (E^-) let $S' = S_0 \cup S_2 \cup \ldots \cup S_{n-1}$ if n is odd and $S' = S_0 \cup S_2 \cup \ldots \cup S_n$ if n is even.

Theorem 5. *Let* (h_1) *and* (h_2) *hold for every* $r \in \{1, 2, \ldots, n\}$. *Then every nonoscillatory solution of* (E) *belongs to* S'.

Proof. First of all we see that

$$\lim_{t \to \infty} \frac{L_0 y(t)}{\omega^k(t)}, \qquad k = 0, 1, \ldots, n-1$$

exists for every nonoscillatory solution $y(t)$ of (E), because

$$\lim_{t \to \infty} \frac{L_0 y(t)}{\omega^k(t)} = \lim_{t \to \infty} L_k y(t), \quad \text{which exists.}$$

If a nonoscillatory solution $y(t)$ has $L_0 y(t)$ bounded, then it belongs to S. Let now $L_0 y(t)$ be unbounded. If

$$\lim_{t \to \infty} \frac{|L_0 y(t)|}{\omega^{n-1}(t)} > 0\,,$$

then $y(t)$ belongs to S_n. Otherwise, there exists a largest integer $p < n$ such that

$$\lim_{t \to \infty} \frac{|L_0 y(t)|}{\omega^{p-1}(t)} > 0 \quad \text{and} \quad \lim_{t \to \infty} \frac{L_0 y(t)}{\omega^p(t)} = 0\,.$$

Hence $y(t) \in S_p$. This shows that any nonoscillatory solution of (E) belongs to some S_k, $0 \leqslant k \leqslant n$. Since $S = \emptyset$, then every nonosillatory solution of (E) belongs to S'.

Corollary. *Let* $yg(y)>0$, $p(t)>0$, $a_0=1$, (a_2), (a_3) *be valid. Then every bounded solution of the equation*

$$L_n y + p(t)g(y) = 0 \tag{7}$$

is oscillatory if n is even and every bounded solution of (7) is either oscillatory or nonoscillatory with the property $\lim_{t\to\infty} y(t)=0$ *if n is odd.*

If we put $a_i = 1$ for all $i = 0, 1, \ldots, n$, then $\omega_{n-1,n}(t) = t^{n-1}$ and then the paper gegeralizes the results in [1, 2, 3]. Theorem 3 is the same as Theorem 8 in [5]. (We can see from the proof of Theorem 3 that instead of (a_2) and (a_3) it is sufficient to suppose that the right-hand side of (6') diverges for all r, which is the assumption in Theorem 8 [5]).

Finally we note that (a_3) holds for the equation (E) of the second, the third and the fourth order

$$a_4(a_1(a_2(a_1(a_0y)')')')' + f(t, y, \ldots, y^{(m)}) = 0.$$

Indeed for $n=4$, e.g. we get,

$$\omega_{n-1,n} = \omega_{3,4} = \int_{t_0}^{t} \frac{1}{a_1(s)} \left(\int_{t_0}^{s} \frac{1}{a_2(\tau)} \left(\int_{t_0}^{\tau} \frac{1}{a_1(\xi)} \mathrm{d}\xi \right) \mathrm{d}\tau \right) \mathrm{d}s$$

$$\omega_{n-k,n-k+1} \cdot \omega^{k-1} = \omega_{n-1,n} \quad \text{for} \quad k = 1, 4 \,,$$

$$\omega_{n-k,n-k+1} \cdot \omega^{k-1} = \int_{t_0}^{t} \frac{1}{a_1(s)} \left(\int_{t_0}^{s} \frac{1}{a_2(\tau)} \mathrm{d}\tau \right) \mathrm{d}s \cdot \int_{t_0}^{t} \frac{1}{a_1(s)} \mathrm{d}s, \qquad k = 2, 3 \,.$$

However

$$\int_{t_0}^{t} \frac{1}{a_1(s)} \left(\int_{t_0}^{s} \frac{1}{a_2(\tau)} \left(\int_{t_0}^{\tau} \frac{1}{a_1(\xi)} \mathrm{d}\xi \right) \mathrm{d}\tau \right) \mathrm{d}s \leqslant$$

$$\leqslant \int_{t_0}^{t} \frac{1}{a_1(s)} \left(\int_{t_0}^{s} \frac{1}{a_2(\tau)} \left(\int_{t_0}^{t} \frac{1}{a_1(\xi)} \mathrm{d}\xi \right) \mathrm{d}\tau \right) \mathrm{d}s =$$

$$= \int_{t_0}^{t} \frac{1}{a_1(s)} \left(\int_{t_0}^{s} \frac{1}{a_2(\tau)} \mathrm{d}\tau \right) \mathrm{d}s \cdot \int_{t_0}^{t} \frac{1}{a_1(s)} \mathrm{d}s \,,$$

therefore $\omega_{n-k,n-k+1} \cdot \omega^{k-1} \geqq \omega_{n-1,n}$ for $k = 2, 3$ as well.

REFERENCES

[1] KIM W. J.: Nonoscillatory solutions of class of n-th order linear differential equations. J. Differential Equations 27, 1978, 19—27.

[2] LOVELADY D. L.: On the oscillatory behaviour of bounded solutions of higher order differential equations. J. Differential Equations 19, 1975, 167—175.
[3] ROVDER J.: Nonoscillatory solutions of n-th order nonlinear differential equation. Čas. pro pěst. mat. 107, 1982, 159—166.
[4] ŠVEC M.: Monotone solution of some differential equations. Colloquium Mathematicum XVIII, 1967, 7—21.
[5] ŠVEC M.: Behaviour of nonoscillatory solutions of some nonlinear differential equations. Acta Mathematica Universitatis Comenianae XXXIX, 1980, 115—130.

Received October 25, 1983

Katedra matematiky a deskriptívnej geom.
Strojnícka fakulta SVŠT
Gottwaldovo nám. 17
812 31 Bratislava

О НЕКОЛЕБАТЕЛЬНЫХ РЕШЕНИЯХ ОДНОГО КЛАССА ЛИНЕЙНЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ

J. Mikunda, J. Rovder

Резюме

В статье изучается асимптотическое поведение одного класса нелинейных дифференциальных уравнений n-го порядка с квази-производными.