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ON THE OSCILLATION OF A CLASS OF NONLINEAR DIFFERENTIAL SYSTEMS WITH DEVIATING ARGUMENTS

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1. Introduction

Much attention has been paid recently to the oscillatory properties of nonlinear functional differential equations with deviating arguments. However, most of the published papers dealt with scalar differential equations; comparatively little is known about the properties of systems of differential equations.

Fundamental results concerning the oscillatory properties of two-dimensional systems of differential equations have been obtained by Varech, Gritsai, Ševelo, Kitamura, Kusano. The oscillatory properties of n-dimensional systems were studied by Foltýnska, Werbowski and Marušiak.

The aim of the present paper is to extend certain results from [4, 7, 8] to a differential equation system

$$(p_i(t)\varphi_i(x_i'(t)))' = f_i(t, x_1(t), \dots, x_n(t), x_1(\tau_1(t), \dots, x_n(\tau_n(t))) \quad i = 1, \dots, n \tag{A}$$

under the assumption that the following conditions hold:

(a) $p_i \in C([a; \infty), R)$, $p_i(t) > 0$ and $\int^{\infty} \frac{ds}{p_i(s)} = \infty$, $i = 1, \dots, n$;

(b) $\varphi_i \in C(R, R)$ and $\varphi_i(u) . u > 0$ for $u \neq 0$, $|\varphi_i(u)| \leqslant a_i|u|$, $i = 1, \dots, n$; $a_i > 0$, *const.*

(c) $f_i \in C([a; \infty) \times R^{2n}, R)$, $i = 1, \dots, n$ and

$$f_i(t, u_1, \dots, u_n, v_1, \dots, v_n)v_{i+1} \begin{cases} > 0 \text{ if } i = 1, \dots, n-1 \\ < 0 \text{ if } i = n(v_{n+1} = v_1) \end{cases} \text{for } v_i . u_i > 0;$$

(d) $\tau_i \in C([a; \infty), R)$ and $\lim_{t \in \infty} \tau_i(t) = \infty$, $i = 1, \dots, n$.

The term "solution $\mathbf{x}(t) = (x_1(t), \ldots, x_n(t))$ of (A)" will be understood in the sequel to refer to a solution of (A) which exists on an interval $[T_x{:}\infty) \subset [a; \infty)$ and satisfies the condition

$$\sup\left\{\sum_{i=1}^{n} |x_i(t)|; t \geqslant T\right\} > 0 \text{ for every } T \geqslant T_x.$$

A solution $\mathbf{x}(t)$ of (A) is said to be (weakly) oscillatory if each (at least one) of its components has a sequence of zeros tending to ∞.

A solution $\mathbf{x}(t)$ of (A) is said to be (weakly) nonoscillatory if each (at least one) of its components has a constant sign for sufficiently large values of t.

2. Oscillatory theorems

Lemma 1. *If $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is a weakly nonoscillatory solution of* (A), *then $\mathbf{x}$ is nonoscillatory.*

Proof. Suppose that $x_i(t)$ is a nonoscillatory component of $\mathbf{x}(t) = (x_1(t), x_2(t), \ldots, x_n(t))$ and $x_i(t) \neq 0$ for $t \geqslant T \geqslant a$.

1) Let $1 < i \leqslant n$. Owing to (c), (d) we obtain from (A)

$$(p_{i-1}(t)\varphi_{i-1}(x'_{i-1}((t)))' \neq 0 \text{ for } t \geqslant T_1,$$

with t_1 such that $\tau_i(t) \geqslant T$ for $t \geqslant t_1$. From (a) and (b) we see that $x_{i-1}(t)$ is monotonic and therefore there exists $t_2 \geqslant t_1$ such that $x_{i-1}(t) \neq 0$ for $t \geqslant t_2$. This shows that $x_{i-1}(t)$ is a nonoscillatory component of $\mathbf{x}$. Analogously it can be shown that the components $x_{i-2}(t), \ldots, x_1(t)$ are nonoscillatory.

2) Let $i = 1$. From the nth equation of (A) we see that

$$(p_n(t)\varphi_n(x'_n(t)))' \neq 0 \text{ for } t \geqslant T_1 \geqslant T$$

where T_1 is such that $\tau_1(t) \geqslant T$ for $t \geqslant T_1$. The function is monotonic and from (a) and (b) it is evident that there exists $t_3 \geqslant T_1$ such that $x_n(t) \neq 0$ for $t \geqslant T_3$. Using the same method as that we used in 1) starting with $i = n$ we prove that all the components are nonoscillatory.

Now let us consider the system (A) assuming that

$$f_i(t, u_1, \ldots, u_n, v_1, \ldots, v_n)\operatorname{sgn} v_{i+1} \geqslant a_i(t)q_i(v_{i+1})\operatorname{sgn} v_{i+1} \geqslant 0 \quad i = 1, \ldots, n-1 \quad (1)$$
$$f_n(t, u_1, \ldots, u_n, v_1, \ldots, v_n)\operatorname{sng} v_1 \leqslant a_n(t)q_n(v_1)\operatorname{sng} \mathrm{v}_1 \leqslant 0$$

where $a_i \in \mathrm{C}([a; \infty), R)$, $a_i(t) \geqslant 0$, $i = 1, \ldots, n$,
$a_i \in \mathrm{C}(\mathrm{R}; \mathrm{R})$ and $q_i(v) \,.\, v > 0$, $i = 1, \ldots, n-1$, $q_n(v) \,.\, v < 0$, $v \neq 0$.

Lemma 2. *Let the conditions* (1) *and*

$$\lim_{|v|\to\infty} \inf |q_i(v)| \neq 0\,,\ i = 1,\dots,n-1 \tag{2}$$

hold. If

$$\int^{\infty} a_i(t)\,\mathrm{d}t = \infty \text{ for } i = 1,\dots,n-1\,, \tag{3}$$

then for a nonoscillatory solution $\boldsymbol{x} = (x_1, x_2, \dots, x_n)$ *of* (A) *we have*

1) $x_1(t)\,.\,x_i'(t) > 0$ *for* $t \geqslant t_0 \geqslant a$, $i = 1,\dots,n$;
2) *there exists* $k \in \{1,\dots,n\}$ and $t_0 \geqslant a$ *such that for* $t \geqslant t_0$ $x_1(t)x_i(t) > 0$, $i = 1,\dots,k$, $x_1(t)x_i(t) < 0$, $i = k+1,\dots,n$;
3) *there exists a finite limit* $\lim_{t\to\infty} p_k(t)\varphi_k(x_k'(t))) = c_k$;
4) $\lim_{t\to\infty} x_i(t) = \lim_{t\to\infty} p_i(t)\varphi_i(x_i'(t)) = 0$, $i = k+1,\dots,n$, $k < n$;
5) $\lim_{t\to\infty} x_i(t) = +\infty\,(-\infty)$, $i = 1,\dots,k$ \
   $\lim_{t\to\infty} p_i(t)\varphi_i(x_i'(t)) = +\infty\,(-\infty)$, $i = 1,\dots,k-1$ \
   if $c_k \neq 0$, $k > 1$.

Proof. Let $\boldsymbol{x}(t) = (x_1(t), \dots, x_n(t)$ be a nonoscillatory solution of (A) on $[a;\infty)$. Without loss of generality we may suppose that $x_1(t) > 0)$ for $t \geqslant t_0 \geqslant a$ (the proof is analogous if $x_1(t) < 0$). Owing to assumption (d) there exists $t_1 \geqslant t_0$ such that $x_1(\tau_1(1)) > 0$ for $t \geqslant t_1$. The last equation of (A) leads to the inequality

$$p_n(t)\varphi_n(x_n'(t)) \leqslant p_n(t_1)\varphi_n(x_n'(t_1)),\ t \geqslant t_1\,. \tag{4}$$

We shall show that there exists $t_2 \geqslant t_1$ such that $x_n'(t) > 0$ for $t \geqslant t_2$. For suppose that this were not true. This implies the existence of $T \geqslant t_2$ such that $x_n'(T) < 0$ and $x_n'(t) < 0$ for $t \geqslant T_1$. From (4)

$$\alpha_n x_n(t) \leqslant \alpha_n x_n(T) + p_n(T)\varphi_n(x_n'(T)) \int_T^t \frac{\mathrm{d}s}{p_n(s)} \to -\infty \text{ for } t\to\infty\,,$$

and therefore $x_n(t) \to -\infty$ for $t \to \infty$. By condition (2) for $i = n-1$ there must exist a constant $K > 0$ and $T_2 \geqslant T_1$ such that

$$q_{n-1}(x_n(\tau_n(t))) \leqslant -K < 0 \text{ for } t \geqslant T_2\,.$$

Using this relation and integrating the (n − 1)th equation of (A), we see that

$$p_{n-1}(t)\varphi_{n-1}(x'_{n-1}(t)) \leqslant p_{n-1}(T_2)\varphi_{n-1}(x'_{n-1}(T_2)) - K\int_{T_2}^{t} a_{n-1}(s)\,ds \to -\infty \quad (5)$$
$$\text{for } t \to \infty,$$

and therefore there eixsts $T_3 \geqslant T_2$ such that $x'_{n-1}(t) < 0$ for $t \geqslant T_3$. From (5), integrating and taking into consideration (b), we get

$$\alpha_{n-1}x_{n-1}(t) \leqslant \alpha_{n-1}x_{n-1}(T_3) + p_{n-1}(T_3)\varphi_{n-1}(x'_{n-1}(T_3))\int_{T_3}^{t}\frac{ds}{p_{n-1}(s)} \to -\infty$$
$$\text{for } t \to \infty,$$

and therefore $x_{n-1}(t) \to -\infty$ for $t \to \infty$. Analogously we show that $x_i(t) \to -\infty$, $x'_i(t) < 0$ for $t \to \infty$, $i = n-2, \ldots, 1$, which contradicts the assumption that $x_1(t) > 0$ for $t \geqslant t_0$. Therefore $x'_n(t) > 0$ for $t \geqslant t_2$. Two cases may now obtain for $x_n(t)$:

i) there exists $t_3 \geqslant t_2$ such that $x_n(t) > 0$, $x_n(\tau_n(t)) > 0$ for $t \geqslant t_3$;
ii) $x_n(t) <$ for $t \geqslant t_2$.

Suppose that i) obtains. This means that $x_n(t)$ is a positive increasing function which either has an upper bound or is unbounded as $t \to \infty$. In the first case there exist constant $c > 0$ and $t_4 \geqslant t_3$ such that $0 < c \leqslant x_n(\tau_n(t))$ for $t \geqslant t_4$ and owing to the continuity of q_n, this means that

$$0 < m \leqslant q_{n-1}(x_n(\tau_n(t))) \leqslant M,\ m, M - \text{const.},\ t \geqslant t_4. \quad (6)$$

In the second case because of the condition (2) there exist a constant $K > 0$ and $t_5 \geqslant t_4$ such that

$$q_{n-1}(x_n(\tau_n(t)) \geqslant K > 0 \text{ for } t \geqslant t_5. \quad (7)$$

Integrating the (n − 1)st equation of (A) and using (6) and (7), we have

$$p_{n-1}(t)\varphi_{n-1}(x'_{n-1}(t)) \geqslant p_{n-1}(t_5)\varphi_{n-1}(x'_{n-1}(t_5)) + L\int_{t_5}^{t} a_{n-1}(s)\,ds \to \infty$$
$$\text{for } t \to \infty$$

where L is a suitable positive constant. From this inequality we see that $x'_{n-1}(t) > 0$ for $t \geqslant t_6 \geqslant t_5$ and by suitably transforming and integrating we see that $x_{n-1}(t) > 0$ for $t \geqslant t_7 \geqslant t_6$ as well. Analogously it can be shown that $x_i(t) > 0$, $x'_i(t) > 0$ for $i = n-2, \ldots, 1$ and a sufficiently large t. This proves that 1) holds for $i = 1, \ldots, n$ and 2) hold for $k = n$.

Suppose now that ii) obtains. From (n − 1)st equation of (A),

$$p_{n-1}(t)\varphi_{n-1}(x'_{n-1}(t)) \leqslant p_{n-1}(t_2)\varphi_{n-1}(x'_{n-1}(t_2)),\ t \geqslant t_2. \quad (8)$$

We shall show that $x'_{n-1}(t) > 0$ for $t \geqslant t_3 \geqslant t_2$. We suppose that this is not true and that there exists $t_4 \geqslant t_3$ such that $x'_{n-1}(t_4) < 0$. Then by (7) $x'_{n-1}(t) < 0$ for $t \geqslant t_4$ and

$$\alpha_{n-1}x_{n-1}(t) \leqslant \alpha_{n-1}x_{n-1}(t_4) + p_{n-1}(t_4)\varphi_{n-1}(x'_{n-1}(t_4))\int_{t_4}^{t}\frac{ds}{p_{n-1}(s)} \to -\infty$$
$$\text{for } t \to \infty\,,$$

so that $x_{n-1}(t) \to -\infty$ for $t \to \infty$. Repeating the procedure used in the first part of our proof we arrive at contradiction with the assumption that $x_1(t) > 0$ on $[t_0\colon \infty)$. Thus $x'_{n-1}(t) > 0$ for $t \geqslant t_3$ and two cases may obtain for $x_{n-1}(t)$:

i$_1$) there exists $t_4 \geqslant t_3$ such that $x_{n-1}(t) > 0$, $x_{n-1}(\tau_{n-1}(t)) > 0$ for $t \geqslant t_4$;

ii$_1$) $x_{n-1}(t) < 0$ for $t \geqslant t_3$.

For i$_1$) we use the same method as for i) to prove that $x_i(t) > 0$, $x'_i(t) > 0$ for $i = 1, \ldots, n-1$ and t sufficiently large, which is exactly what statements 1) and 2) of the Lemma state for $k = n-1$.

For ii$_1$) we prove analogously as for ii) that $x'_{n-2}(t) > 0$ for $t \geqslant t_4 \geqslant t_3$ and that the following two possibilities exist for x_{n-2}:

i$_2$) there exists $t_5 \geqslant t_4$ such that $x_{n-2}(t) > 0$, $x_{n-2}(\tau_{n-2}(t)) > 0$ for $t \geqslant t_5$;

ii$_2$) $x_{n-2}(t) < 0$ for $t \geqslant t_4$.

The method used in i$_1$), ii$_1$) is now used repeatedly to prove statements 1) and 2) of the Lemma for $k = n-2, \ldots, 1$.

By hypothesis, $x_1(t) > 0$, $x_1(\tau_1(t)) > 0$ for $t \geqslant t_0$ and therefore the function $p_n(t)\varphi_n(x'_n(t))$ is positive and decreasing and thus has a finite limit. Statement 3) holds for $k = n$.

If k has the property 2) then $p_k(t)\varphi_k(x'_k(t))$ is a positive decreasing function and has a finite limit. If

$$\lim_{t\to\infty} p_k(t)\varphi_k(x'_k(t))) = c_k > 0\,,$$

then there exists $T \geqslant t_0$ sufficiently large and such that

$$\alpha_k p_k(t)x'_k(t) \geqslant p_k(t)\varphi_k(x'_k(t))) \geqslant \frac{1}{2}c_k\,,$$

whence we see by integrating that $\lim_{t\to\infty} x_k(t) = \infty$. Using (3), (2) and (a) it is easy to prove from the first k $-$ 1 equations of (A) that $\lim_{t\to\infty} x_i(t) = \infty$ for $i = 1, \ldots, k$ and $\lim_{t\to\infty} p_i(t)\varphi_i(x'_i(t)) = \infty$ for $i = 1, \ldots, k-1$. This proves statement 5) of the Lemma.

Statement 4) will be proved by contradiction. Assume that there exists $j \in \{k+1, \ldots, n\}$ such that $\lim_{t \to \infty} p_j(t)\varphi_j(x_j'(t)) = c_j > 0$. Using the preceding part of our proof this leads to $\lim_{t \to \infty} p_k(t)\varphi_k(x_k'(t)) = \infty$ which contradicts 3). Analogously assume the existence of $j \in \{k+1, \ldots, n\}$ such that $\lim_{t \to \infty} x_j(t) \neq 0$. Since $x_j(t)$ is a negative increasing function there exist constants c_j, d_j and T sufficiently large such that

$$c_j \leqslant x_j(\tau_j(t)) \leqslant d_j < 0\,, \quad t \geqslant T$$

and the continuity of q_{j-1} implies that there exist constants m, M such that

$$m \leqslant q_{j-1}(x_j(\tau_j(t)) \leqslant M < 0 \text{ for } t \geqslant T.$$

From the $(j-1)$ equation of (A) we have

$$p_{j-1}(t)\varphi_{j-1}(x_{j-1}'(t)) \leqslant p_{j-1}(T)\varphi_{j-1}(x_{j-1}'(T)) + M\int_T^t a_{j-1}(s)\,\mathrm{d}s \to -\infty \quad \text{for } t \to \infty\,,$$

which again yields a contradiction to 3). This completes the proof of the Lemma.

Theorem 1. *Suppose that, in addition to the assumptions of Lemma 2,*

$$\int^{\infty} a_n(t)\,\mathrm{d}t = \infty \tag{9}$$

and

$$\lim_{|v| \to \infty} |q_n(v)| \neq 0\,, \tag{10}$$

then every solution of (A) *is oscillatory.*

Proof. Suppose that (A) has a weakly nonoscillatory solution $(x_1(t), \ldots, x_n(t))$. By Lemma 1 this solution is nonoscillatory. Suppose that $x_1(t) > 0$, $x_1(\tau_1(t)) > 0$ for $t \geqslant t_1 \geqslant a$. By Lemma 2, $x_1(t)$ is a positive increasing function and there exists $\lim_{t \to \infty} x_1(t) = d_1$ such that either $d_1 < \infty$ or $d_1 = \infty$. In both cases, owing to (10) and the continuity of q_n, there exist a constant $L > 0$ and T sufficiently large so that

$$q_n(x_1(\tau_1(t))) \leqslant -L \text{ for } t \geqslant T.$$

By Lemma 2, $p_n(t)\varphi_n(x_n'(t))$ is a positive decreasing function. Using these properties, we see after integrating the last equation of (A) that

$$p_n(t)\varphi_n(x_n'(t)) - p_n(T)\varphi_n(x_n'(T)) \leqslant -L\int_T^t a_n(s)\,\mathrm{d}s,$$

which contradicts (9).

Remark 1. Theorem 1 is a generalization of Theorem 2 of [8]. If $\tau_i(t) = t$ for $i = 1, 2, \ldots, n$, we obtain the results formulated in Theorem 1 of [7] under weaker assumptions about f_i.

The following example shows that the assumption (10) of Theorem 1 is indispensable.

Example 1. The system

$$(t^{\frac{1}{2}}x_1'(t))' = \frac{3}{2}t^{-\frac{1}{4}}(x_2(t^{\frac{1}{4}}))^3$$

$$(t^{\frac{1}{2}}x_2'(t))' = -\frac{1}{18}t^{-\frac{5}{3}}(1+t)\frac{x_1(t^{\frac{1}{3}})}{1+(x_1(t^{\frac{1}{3}}))^2}$$

satisfies all conditions of Theorem 1 except (10), but the system has a nonoscillatory solution $(x_1(t), x_2(t)) = (t^{\frac{3}{2}}, t^{\frac{1}{3}})$ for $t > 0$.

Remark 2. The assumptions of Theorem 1 are rather strong in the sense that the deviating arguments $\tau_i(t)$ have no influence on the oscillatory properties of solutions of (A).

Theorem 2. *Suppose that, in addition to (1) and (3),*

$$\liminf_{|v|\to\infty}\frac{q_i(v)}{v} \neq 0,\ \liminf_{|v|\to 0}\frac{q_i(v)}{v} \neq 0 \textit{ for } i = 1, \ldots, n-1 \tag{11}$$

holds. If

$$|q_n(v)| \leqslant |q_n(u)| \textit{ for } |v| \leqslant |u| \tag{12}$$

and

$$\infty = \int_T^\infty a_k(v_k)\int_{\tau_{k+1}(v_k)}^\infty \frac{1}{p_{k+1}(u_{k+1})}\int_{u_{k+1}}^\infty a_{k+1}(v_{k+1})\int_{\tau_{k+2}(v_{k+1})}^\infty \frac{1}{p_{k+2}(u_{k+2})}\int_{u_{k+2}}^\infty \ldots$$

$$\ldots\int_{u_n}^\infty a_n(v)|q_n\Bigl(c\int_T^{\tau_1(v)}\frac{1}{p_1(u_1)}\int_T^{u_1}a_1(v_1)\int_T^{\tau_2(v_1)}\frac{1}{p_2(u_2)}\int_T^{u_2}\ldots$$

$$\ldots\int_T^{u_{k-1}} a_{k-1}(v_{k-1})\,\mathrm{d}v_{k-1}\ldots\mathrm{d}u_1\Bigr)|\,\mathrm{d}v\ldots\mathrm{d}v_k$$

for every $c \neq 0$ and $k = 1, \ldots, n$; then every solution of (A) *is oscillatory.*

Proof. Suppose that (A) has a weakly nonoscillatory solution. By Lemma 1 it is nonoscillatory; assume that $x_1(t) > 0$ for $t \geqslant t_0 \geqslant a$. The first part of (11) implies the validity of (2) and therefore by Lemma 2 there exist $k \in \in \{1, \ldots, n\}$ and $T_0 \geqslant t_0$ such that for $t \geqslant T_0$ and $i = 1, \ldots, k$ all $x_i(t)$ are positive and increasing and $\lim_{t \to \infty} x_i(t) = \infty$. Owing to (11) there exist positive constants K_i and $T \geqslant T_0$ such that

$$q_i(x_{i+1}(\tau_{i+1}(t))) \geqslant K_i x_{i+1}(\tau_{i+1}(t)) \text{ for } t \geqslant T,\ i = 1, \ldots, k-1. \tag{14}$$

By transforming the first (k − 1) equations of (A) as follows

$$\alpha_i p_i x_i'(t) \geqslant p_i(t)\varphi_i(x_i'(t)) \geqslant p_i(T)\varphi_i(x_i'(T)) + \int_T^t a_i(s) q_i(x_{i+1}(\tau_{i+1}(s)))\,\mathrm{d}s \geqslant$$

$$\geqslant \int_T^t a_i(s) q_i(x_{i+1}(\tau_{i+1}(s)))\,\mathrm{d}s > 0,\ t \geqslant T,\ i = 1, \ldots, k-1$$

and integrating we obtain

$$x_i(t) \geqslant \frac{K_i}{\alpha_i}\int_T^t \frac{1}{p_i(u)}\int_T^u a_i(s) x_{i+1}(\tau_{i+1}(s))\,\mathrm{d}s\,\mathrm{d}u,\ i = 1, \ldots, k-2 \tag{15}$$

$$x_{k-1}(t) \geqslant \frac{d_k K_{k-1}}{\alpha_{k-1}}\int_T^t \frac{1}{p_{k-1}(u)}\int_T^u a_{k-1}(s)\,\mathrm{d}s\,\mathrm{d}u. \tag{16}$$

Since the kth component of the solution is an increasing function there exists a constant $d_k > 0$ such that $x_k(\tau_k(t)) \geqslant d_k$ for $t \geqslant T$. After a transformation of (15), (16) we have

$$\begin{aligned} x_1(t) \geqslant c \int_T^t \frac{1}{p_1(u_1)}\int_T^{u_1} a_1(v_1)\int_T^{\tau_1(v_1)} \frac{1}{p_2(u_2)}\int_T^{u_2} a_2(v_2)\int_T^{\tau_2(v_2)} \ldots \\ \ldots \int_T^{\tau_{k-2}(v_{k-2})} \frac{1}{p_{k-1}(u_{k-1})}\int_T^{u_{k-1}} a_{k-1}(v_{k-1})\,\mathrm{d}v_{k-1}\,\mathrm{d}u_{k-1} \ldots \mathrm{d}v_2\,\mathrm{d}u_2\,\mathrm{d}v_1\,\mathrm{d}u_1, \end{aligned} \tag{17}$$

where $c = d_k \prod_{i=1}^{k-1} \frac{K_i}{\alpha_i}$.

By Lemma 2, $\lim_{t \to \infty} x_i(t) = 0$, $\lim_{t \to \infty} p_i(t))\varphi_i(x_i'(t) = 0$ for $i = k+1, \ldots, n$; therefore the (k + 1)st to the nth equation of (A) yield

$$|x_i(\tau_i(t))| \geqslant \frac{1}{\alpha_i}\int_{\tau_i(t)}^\infty \frac{1}{p_i(u)}\int_u^\infty a_i(v)|q_i(x_{i+1}(\tau_{i+1}(v)))|\,\mathrm{d}v\,\mathrm{d}u. \tag{18}$$

Further, owing to (11) there exist constants $M_i > 0$ and $T_1 \geqslant T$ such that

$$|q_i(x_{i+1}(\tau_{i+1}(t)))| \geqslant M_i|x_{i+1}(\tau_{i+1}(t)))|,\ t \geqslant T_1,\ i = k+1,\dots,n-1 .$$

Using this property, we can transform (18) to obtain

$$|x_{k+1}(t)| \geqslant D\int_t^\infty \frac{1}{p_{k+1}(u_{k+1})}\int_{u_{k+1}}^\infty a_{k+1}(v_{k+1})\int_{\tau_{k+2}(v_{k+1})}^\infty \frac{1}{p_{k+2}(u_{k+2})}\int_{u_{k+2}}^\infty \dots$$

$$\dots\int_{\tau_n(v_{n-1})}^\infty \frac{1}{p_n(u_n)}\int_{u_n}^\infty a_n(v)|q_n(x_1(\tau_1(v)))|\,\mathrm{d}v\,\mathrm{d}u_n\dots\mathrm{d}v_{k+1}\,\mathrm{d}u_{k+1}, \tag{19}$$

$$t \geqslant T_1,\ D = \frac{1}{\alpha_n}\prod_{i=k+1}^{n-1}\frac{M_i}{\alpha_i}.$$

Now by Lemma 2 there exists a finite limit $\lim\limits_{t\to\infty} p_k(t))\varphi_k(x_k'(t) = L$. Integrating the kth equation of (A) we have after some manipulations

$$|L - p_k(T_1)\varphi_k(x_k'(T_1))| \geqslant M_k\int_{T_1}^\infty a_k(s)|x_{k+1}(\tau_{k+1}(s))|\,\mathrm{d}s\,. \tag{20}$$

Using the fact that $|q_n(v)|$ is nondecreasing we substitute (17) into (19). The resulting expression is then substitued into (20) and this yields a contradiction to (13).

Corollary 1. If in addition to the assumptions of Theorem 2 with the exception of the second condition in (11)

$$\frac{q_i(v)}{v} \geqslant \frac{q_i(u)}{u} \text{ for } |u| \leqslant |v|,\ i = 1,\dots,n-1\,,$$

holds, then every solution of (A) is oscillatory.

Example 2. The system

$$\left(\left(\frac{3}{13}t^{-\frac{5}{6}} + \frac{1}{3}t^{-\frac{2}{3}}\right)x_1'(t)\right)' = 2t^{-\frac{1}{2}}((x_2(\tau_2(t)))^{\frac{5}{3}} + x_2(\tau_2(t)))$$

$$(t^{-\frac{1}{2}}x_2'(t))' = -\frac{1}{2}t^{-3}(x_1(\tau_1(t)))^3,\ t \geqslant 0$$

with the deviating arguments $\tau_1(t) = t^{\frac{1}{12}}$, $\tau_2(t) = t^2$ has an nonoscillatory solution $(x_1(t),\ x_2(t)) = (t^4, t^{\frac{1}{2}})$ for $t \geqslant 0$ (since for $k = 1$ the asumption (13) does not hold), but for the deviating arguments $\tau_1(t) = t^2$, $\tau_2(t) = t^{\frac{1}{4}}$ every solution of the system is oscillatory.

Example 3. For the system

$$(t^{-3}x_1'(t))' = t^{-\frac{7}{2}}\left(\frac{15}{4} + t^2\right)\left(t + \frac{\pi}{2}\right)^{\frac{1}{2}} x_2\left(t + \frac{\pi}{2}\right)$$

$$(t^{-3}x_2'(t))' = -t^{-\frac{3}{2}}\left(t^2 - \frac{1}{4}\right)\left(t + \frac{\pi}{2}\right)^{-\frac{3}{2}} x_1\left(t + \frac{\pi}{2}\right)$$

all the conditions of Theorem 2 are satisfied and therefore every one of its solutions is oscillatory on $[\pi; \infty)$.

$(x_1(t), x_2(t)) = (t^{\frac{3}{2}} \sin t, t^{-\frac{1}{2}} \cos t)$ is one such solution.

We shall now study the behaviour of (A) under the following assumptions:

$$f_i(t, u_1, \dots, u_n, v_1, \dots, v_n) \operatorname{sgn} v_{i+1} \geqslant a_i(t) g_i(u_{i+1}) \operatorname{sgn} u_{i+1} \geqslant 0, \quad i = 1, \dots, n-1$$

$$f_n(t, u_1, \dots, u_n, v_1, \dots, v_n) \operatorname{sgn} v_1 \leqslant g_n(t, v_1) \operatorname{sgn} v_1 \leqslant 0, \tag{21}$$

where

$$a_i \in C([a; \infty), R), a_i(t) \geqslant 0, i = 1, \dots, n-1;$$

$$g_i \in C(R; R), g_i(v)v > 0 \text{ for } v \neq 0, i = 1, \dots, n-1;$$

$$g_n \in C([a; \infty) \times R; R), g_n(t, v) . v < 0 \text{ for } v \neq 0.$$

Let $i_k \in \{1, 2, \dots, 2n-1\}$, $1 \leqslant k \leqslant 2n-1$ and $t, s \in [a; \infty)$. Define

$$I_0(t, s) = J_0(t, s) = 1$$

$$I_k(t, s; y_{i_k}, \dots, y_{i_1}) = \int_s^t y_{i_k}(x) I_{k-1}(x, s; y_{i_{k-1}}, \dots, y_{i_1}) \, dx,$$

$$J_k(t, s; y_{i_k}, \dots, y_{i_1}) = \int_s^t y_{i_1}(x) J_{k-1}(t, x; y_{i_k}, \dots, y_{i_2}) \, dx.$$

Further let us introduce the following notation

$$R_k(t, T) = I_{2n-1}\left(t, T; \frac{1}{p_1}, a_1, \frac{1}{p_2}, a_2, \dots, \frac{1}{p_{k-1}}, a_{k-1}, \frac{1}{p_n}, a_{n-1}, \dots, a_k, \frac{1}{p_k}\right)$$

$$1 \leqslant k \leqslant n.$$

$R_k(t, a) = R_k(t)$

Lemma 3. *Suppose that, in addition to* (21),

$$\lim_{|u| \to \infty} \inf |g_i(u)| \neq \textit{ for } i = 1, \dots, n-1\,, \tag{22}$$

and

$$\int^{\infty} a_i(t)\,\mathrm{d}t = \infty \textit{ for } i = 1, \dots, n-1\,. \tag{23}$$

Then for any nonoscillatory solution $\boldsymbol{x} = (x_1, \dots, x_n)$ *the statements* 1) *to* 5) *of Lemma* 2 *hold.*

The proof of the Lemma is analogous to that of Lemma 2.

Lemma 4. *Suppose that, in addition to* (21) *and* (23),

$$\frac{g_i(u)}{u} \leqslant \frac{g_i(v)}{v} \textit{ for } |u| \leqslant |v|,\ i = 1, \dots, n-1\,. \tag{24}$$

Then for any nonoscillatory solution $\boldsymbol{x} = (x_1, \dots, x_n)$ *of* (A) *and* $a \leqslant s < t$ *we have*

$$\begin{aligned} |p_1(t)\varphi_1(x_1'(t)) &\geqslant \prod_{i=1}^{j} \frac{|g_i(x_{i+1}(s))|}{a_{i+1}|x_{i+1}(s)|} \int_s^t |\varphi_{j+1}(x_{j+1}'(u))| \times \\ &\times \mathrm{J}_{2j-1}\left(t, u; a_1, \frac{1}{p_2}, \frac{1}{p_3}, \dots, \frac{1}{p_j} a_j\right) \mathrm{d}u,\ 1 \leqslant j \leqslant k-1\,; \end{aligned} \tag{25}$$

and

$$\begin{aligned} |p_k(s)\varphi_k(x_k'(s)) &\geqslant \prod_{i=k}^{j} \frac{|g_i(x_{i+1}(t))|}{a_{i+1}|x_{i+1}(t)|} \int_s^t |\varphi_{j+1}(x_{j+1}'(u))| \times \\ &\times \mathrm{I}_{2(j-k)+1}\left(u, s; a_j, \frac{1}{p_j}, a_{j-1}, \dots, \frac{1}{p_{k+1}}, a_k\right) \mathrm{d}u,\ k \leqslant j \leqslant n-1\,, \end{aligned} \tag{26}$$

where $k = 1, \dots, n-1$ *is determined according to Lemma 3.*

Proof. Let $\boldsymbol{x} = (x_1, \dots, x_n)$ be a nonoscillatory solution of (A) defined on $[a; \infty)$ and suppose that $x_1(t) > 0$, $x_1(\tau_1(t)) > 0$ for $t \geqslant t_0 \geqslant a$. The condition (24) implies the validity of (22). Thus by Lemma 3 there exist $T \geqslant t_0$ and $k \in \{1, \dots, n\}$ such that $x_i'(t) > 0$ for $i = 1, \dots, n$, $x_j(t) > 0$ for $j = 1, \dots, k$, $x_j(t) < 0$ for $j = k+1, \dots, n$ and $t \geqslant T$.

To prove (25), we shall use the monotonicity of the first k components of the solution, the relations (21) and (24), the first (k − 1) equations of (A) and integration by parts.

Suppose that $T \leqslant s < t$. Then

$$p_1(t)\varphi_1(x_1'(t)) = p_1(s)\varphi_1(x_1'(s)) + \int_s^t (p_1(z)\varphi_1(x_1'(z)))'\,\mathrm{d}z \geqslant$$

$$\geqslant \int_s^t a_1(z)g_1(x_2(z))\,\mathrm{d}z \geqslant \frac{g_1(x_2(s))}{x_2(s)}\int_s^t a_1(z)x_2(z)\,\mathrm{d}z =$$

$$= g_1(x_2(s))\,.\,\mathrm{J}_1(t,s;a_1) + \frac{g_1(x_2(s))}{x_2(s)}\int_s^t x_2'(z)\mathrm{J}_1(t,z;a_1)\,\mathrm{d}z \geqslant$$

$$\geqslant \frac{g_1(x_2(s))}{\alpha_2 x_2(s)}\int_s^t \varphi_2(x_2'(z))\mathrm{J}_1(t,z;a_1)\,\mathrm{d}z\,,$$

which is (25) for $j = 1$. Integrating the last integral we have

$$p_1(t)\varphi_1(x_1'(t)) \geqslant \frac{g_1(x_2(s))}{\alpha_2 x_2(s)}p_2(s)\varphi_2(x_2'(s))\mathrm{J}_2\left(t,s;a_1,\frac{1}{p_2}\right) +$$

$$+ \frac{g_1(x_2(s))}{\alpha_2 x_2(s)}\int_s^t a_2(z)g_2(x_3(z))\mathrm{J}_2\left(t,z;a_1,\frac{1}{p_2}\right)\mathrm{d}z \geqslant$$

$$\geqslant \frac{g_1(x_2(s))}{\alpha_2 x_2(s)}\cdot\frac{g_2(x_3(s))}{x_3(s)}\int_s^t a_2(z)x_3(z)\mathrm{J}_2\left(t,z;a_1,\frac{1}{p_2}\right)\mathrm{d}z\,.$$

By the above transformations and (2j-2) integrations we obtain (25).

To prove (26), we use the last $(n-k+1)$ equations of (A), the relations (21) and (24) and the properties of the last $(n-k+1)$ components of the solution as well as the fact that they are negative increasing functions.

For $T \leqslant s < t$ we have

$$p_k(s)\varphi_k(x_k'(s)) = p_k(t)\varphi_k(x_k'(t)) - \int_s^t (p_k(u)\varphi_k(x_k'(u)))'\,\mathrm{d}u \geqslant$$

$$\geqslant -\int_s^t a_k(u)g_k(x_{k+1}(u))\,\mathrm{d}u \geqslant -\frac{g_k(x_{k+1}(t))}{x_{k+1}(t)}\int_s^t a_k(u)x_{k+1}(u)\,\mathrm{d}u =$$

$$= -g_k(x_{k+1}(t))\mathrm{I}_1(t,s;a_k) + \frac{g_k(x_{k+1}(t))}{\alpha_{k+1}x_{k+1}(t)}\int_s^t \varphi_{k+1}(x_{k+1}'(u))\mathrm{I}_1(u,s;a_k)\,\mathrm{d}u \geqslant$$

$$\geqslant \frac{g_k(x_{k+1}(t))}{\alpha_{k+1}x_{k+1}(t)}\int_s^t p_{k+1}(u)\varphi_{k+1}(x_{k+1}'(u))\frac{1}{p_{k+1}(u)}\mathrm{I}_1(u,s;a_k)\,\mathrm{d}u\,,$$

which is (26) for $j = k$. Again integrating by parts and using the above properties (n-1-k) times we obtain (26).

For $x_1(t) < 0$ the proof is analogous.

Theorem 3. *Suppose that, in addition to the assumptions of Lemma 4, the following conditions hold*:

1) $\liminf_{|u|\to 0} \frac{g_i(u)}{u} \neq 0$ *for* $i = 1, \dots, n-1$;

2) $\frac{|g_n(t, u)|}{|u|^\beta} \leqslant \frac{|g_n(t, v)|}{|v|^\beta}$ *for* $|u| \leqslant |v|$, $\beta > 1$;

3) *There exists a function* $h(t)$ *continuous and differentiable on* $[a; \infty)$, *such that* $0 < h(t) \leqslant \tau_1(t)$, $h'(t) \geqslant 0$, $\lim_{t\to\infty} h(t) = \infty$.

If

$$\int^\infty \mathrm{R}_k(h(t))|g_n(t, c)|\, \mathrm{d}t = \infty \textit{ for all } c \neq 0 \textit{ and } k = 1, \dots, n, \tag{27}$$

then all solutions of (A) *are oscillatory.*

Proof. Suppose that (A) has a weakly nonoscillatory solution $\boldsymbol{x} = (x_1, \dots, x_n)$. By Lemma 1 this solution is nonoscillatory and without loss of generality we may assume that $x_1(t) > 0$, $x_1(h(t)) > 0$ for $t \geqslant t_0 \geqslant a$. By Lemma 3, $\lim_{t\to\infty} x_i(t) = 0$ for $i = k+1, \dots, n$ and by the assumption 1) there exist constants $\delta_i > 0$ and $T \geqslant t_0$ such that

$$\frac{g_i(x_{i+1}(t))}{x_{i+1}(t)} \geqslant \delta_i,\ i = k+1, \dots, n-1, t \geqslant T.$$

Since $p_n(t)\varphi_n(x_n'(t))$ is decreasing, we have the following relation from (26) for $j = n-1$:

$$p_k(s)\varphi_k(x_k'(s)) \geqslant p_n(t)\varphi_n(x_n'(t)) \prod_{i=k}^{n-1} \frac{\delta_i}{\alpha_{i+1}} \mathrm{I}_{2(n-k)}\left(t, s; \frac{1}{p_n}, a_{n-1}, \dots, \frac{1}{p_{k+1}}, a_k\right), \quad T \leqslant s < t. \tag{28}$$

Substituting (28) into (25) for $s = T, j = k-1$ we have

$$p_1(t)\varphi_1(x_1'(t)) \geqslant a p_n(t)\varphi_n(x_n'(t)) \int_T^t \mathrm{I}_{2(n-k)}\left(t, u; \frac{1}{p_n}, a_{n-1}, \dots, \frac{1}{p_{k+1}}, a_k\right) \times$$
$$\times \frac{1}{p_k(u)} \mathrm{J}_{2k-3}\left(t, u; a_1, \frac{1}{p_2}, \dots, \frac{1}{p_{k-1}}, a_{k-1}\right) \mathrm{d}u,$$

where $a = \prod_{i=1}^{k-1} \frac{q_i(x_{i+1}(T))}{\alpha_{i+1} x_{i+1}(T)} \prod_{i=k}^{n-1} \frac{\delta_i}{\alpha_{i+1}}$, and therefore

$$x_1'(t) \geqslant \frac{\alpha}{\alpha_1} p_n(t)\varphi_n(x_n'(t)) \frac{1}{p_1(t)} \times$$
$$\times\, \mathrm{I}_{2n-2}\left(t, T; a_1, \frac{1}{p_2}, \ldots, a_{k-1}, \frac{1}{p_n}, a_{n-1}, \ldots, a_k, \frac{1}{p_k}\right). \tag{29}$$

Taking $t_1 \geqslant T$ such that $h(t) \geqslant T$ for $t \geqslant t_1$, calculate the following derivative using the nth equation of (A), the relation (29) and assumption 2) of the theorem:

$$[\mathrm{R}_k(h(t), T)p_n(t)\varphi_n(x_n'(t))x_1^{-\beta}(h(t))]' \leqslant$$
$$\leqslant [\mathrm{R}_k(h(t), T)]'\, h'(t)p_n(t)\varphi_n(x_n'(t))x_1^{-\beta}(h(t)) +$$
$$+ \mathrm{R}_k(h(t), T)x_1^{-\beta}(h(t))g_n(t, x_1(\tau_1(t))) \leqslant$$
$$\leqslant \frac{\alpha_1}{\alpha} x_1'(h(t))h'(t)x_1^{-\beta}(h(t)) + \mathrm{R}_k(h(t), T)g_n(t, K)\,.\,K^{-\beta},$$

where $K = x_1(T)$.

Integrating the last inequality yields after necessary manipulations

$$-K^{-\beta}\int_{t_1}^{t} \mathrm{R}_k(h(s), T)g_n(s, K)\,\mathrm{d}s \leqslant \alpha_1 \frac{x_1^{1-\beta}(h(t_1))}{\alpha(\beta - 1)} +$$
$$+ \mathrm{R}_k(h(t_1), T)p_n(t_1)\varphi_n(x_n'(t_1))x_1^{-\beta}(h(t_1))\,.$$

The right-hand part of this inequality is a finite positive number. Therefore the integral is convergent, which is a contradiction to (27).

Example 4. The system

$$(t^{-2}x_1'(t))' = 4t^{-\frac{1}{2}}x_2(\tau_2(t))$$
$$(t^{-3}x_2'(t))' = -\frac{7}{4}(t^{-\frac{49}{2}} + t^{-\frac{33}{2}})\frac{x_1^5(\tau_1(t))}{1 + x_1(\tau_1(t))}$$

with $\tau_1(t) = \tau_2(t) = t$ has a nonoscillatory solution $(x_1(t),\ x_2(t)) = (t^4, t^{\frac{1}{2}})$ for $t \geqslant 0$. For $\tau_1(t) = t^4$, $\tau_2(t) = t^{\frac{1}{2}}$ every solution is oscillatory.

The following theorem presents a sufficient condition for the oscillation of all solutions of (A) if $0 < \beta < 1$ in condition 2) of Theorem 3.

Let

$$\tau_*(t) = \min(\tau_1(t), t)$$

$$\mathrm{P}_0^1(t, T) = 1$$

$$\mathrm{P}^1_{2j}(t,T) = \mathrm{I}_{2j}\left(t, T; \frac{1}{p_1}, a_1, \frac{1}{p_2}, a_2, \ldots, \frac{1}{p_j}, a_j\right)$$

$$\mathrm{P}^1_{2j+1}(t,T) = \mathrm{I}_{2j+1}\left(t, T; \frac{1}{p_1}, a_1, \frac{1}{p_2}, a_2, \ldots, a_j, \frac{1}{p_{1+j}}\right)$$

$$\mathrm{P}^1_k(t,a) = \mathrm{P}^1_k(t),\ 0 \leqslant k \leqslant 2n-2\,.$$

Theorem 4. *If in addition to the assumptions of Lemma 4*

1) $\liminf\limits_{|u|\to 0} \dfrac{g_i(u)}{u} \neq$ *for* $i = 1, \ldots, n-1$;

2) $\dfrac{|g_n(t,u)|}{|u|^\beta} \leqslant \dfrac{|g_n(t,v)|}{|v|^\beta}$ *for* $|u| \leqslant |v|$, $0 < \beta < 1$

and

$$\int^\infty \left(\frac{\mathrm{R}_k(\tau_*(t))}{\mathrm{P}^1_{2k-2}(\tau_1(t))}\right)^\beta |g_n(t, c\mathrm{P}^1_{2k-2}(\tau_1(t)))|\,\mathrm{d}t = \infty \text{ for all } c \neq 0,\ k = 1, \ldots, n\,. \quad (30)$$

Then every solution of (A) *is oscillatory.*

Proof. The proof will be indirect. We start by repeating the proof of Theorem 3 up to and including the inequality (29). Integrating this inequality from T to $t \geqslant T$ we have

$$x_1(t) \geqslant \frac{a}{a_1} p_n(t)\varphi_n(x_n'(t))\mathrm{R}_k(t,T)\,. \quad (31)$$

By Lemma 3 $x_1(t)$ is increasing and $p_n(t)\varphi_n(x_n'(t))$ decreasing. Using this, it is possible to transform (31) as follows:

$$\begin{aligned}(p_n(t)\varphi_n(x_n'(t)))^{-\beta} &\geqslant (p_n(\tau_*(t))\varphi_n(x_n'(\tau_*(t))))^{-\beta} \geqslant \\ \geqslant \left(\frac{a}{a_1}\right)^\beta \mathrm{R}^\beta_k(\tau_*(t),T)x_1^{-\beta}(\tau_*(t)) &\geqslant \left(\frac{a}{a_1}\right)^\beta \mathrm{R}^\beta_k(\tau_*(t),T)x_1^{-\beta}(\tau_1(t))\,,\end{aligned} \quad (32)$$

where $t \geqslant t_1 \geqslant T$ such that $\tau_*(\mathrm{t}) \geqslant T$ for $t \geqslant t_1$.

Starting with (25) for $j = k-2$, $s = T$, integrating by parts and using the (k − 1)th equation of (A) and the monotonicity of x_k leads to

$$p_1(t)\varphi_1(x_1'(t)) \geqslant g_{k-1}(x_k(T)) \times$$

$$\times \prod_{i=1}^{k-2} \frac{g_i(x_{i+1}(T))}{a_{i+1}(x_{i+1}(T))} \mathrm{J}_{2k-3}\left(t, T; a_1, \frac{1}{p_2}, a_2, \ldots, \frac{1}{p_{k-1}}, a_{k-1}\right).$$

Integrating the last inequality from T to $t \geqslant T$ we have

$$x_1(t) \geqslant c\mathrm{P}^1_{2k-2}(t, T), \text{ where } c = \frac{g_{k-1}(x_k(T))}{\alpha_1} \prod_{i=1}^{k-2} \frac{g_i(x_{i+1}(T))}{\alpha_{i+1} x_{i+1}(T)}. \tag{33}$$

Using the nth equation of (A), the relations (33) and (32) and condition 2) we see that

$$[(p_n(t)\varphi_n(x_n'(t)))^{1-\beta}]' = (1-\beta)(p_n(t)\varphi_n(x_n'(t)))^{-\beta}(p_n(t)\varphi_n(x_n'(t)))' \leqslant$$

$$\leqslant (1-\beta)\left(\frac{\alpha}{\alpha_1}\right)^\beta \mathrm{R}^\beta_k(\tau_1(t(\tau_*(t), T)x_1^{-\beta}(t))g_n(t, x_1(\tau_1(t))) \leqslant$$

$$\leqslant (1-\beta)\left(\frac{\alpha}{\alpha_1}\right)^\beta \mathrm{R}^\beta_k(\tau_*(t), T)(\mathrm{P}^1_{2k-2}(\tau_1(t), T))^{-\beta}|g_n(t, c\mathrm{P}^1_{2k-2}(\tau_1(t)))|.$$

Integrating the last inequality yields a contradicition to (30). This completes the proof.

Remark 4. For the case when (A) is equivalent to a differential equation with deviating arguments of order 2n the theorem yields a result proved in [5].

Example 5. If for some $k \in \{1, \dots, n\}$ the assumption (30) is not satisfied, then there may exist nonoscillatory solutions of the system. The system

$$\left(\frac{1}{t} x_1'(t)\right)' = 3 \, . \, t^{-\frac{2}{3}} x_2(t^{\frac{1}{3}})$$

$$\left(\frac{1}{t^2} x_1'(t)\right)' = -\frac{2}{t^{23}} x_1(t^7)$$

does not satisfy (30) for $k = 2$ and has a nonoscillatory solution $(x_1(t), x_2(t)) = (t^3, t^2)$ for $t \geqslant 0$.

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О КОЛЕБЛЕМОСТИ РЕШЕНИЙ НЕЛИНЕЙНЫХ СИСТЕМ С ОТКЛОНЯЮЩИМСЯ АРГУМЕНТОМ

Božena Mihalíková

Резюме

В статье приведены достаточные условия колеблемости решений сыстемы (A).