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EXISTENCE AND UNIQUENESS OF SOLUTIONS OF QUASILINEAR HYPERBOLIC SYSTEMS OF PARTIAL DIFFERENTIAL-FUNCTIONAL EQUATIONS

JAN TURO

1. Introduction

In the present paper we take into consideration the following Schauder canonic form of quasilinear hyperbolic systems of differential-functional equations

$$\sum_{j=1}^{n} A_{ij}(x, y, z(x, y), (V^{(1)}z)(x, y))[\partial z_j(x, y)/\partial x +$$
$$+\sum_{k=1}^{m} \varrho_{ik}(x, y, z(x, y), (V^{(2)}z)(x, y))\partial z_j(x, y)/\partial y_k] =$$
$$= f_i(x, y, z(x, y), (V^{(3)}z)(x, y)), \quad (x, y) \in D_a = I_a \times R^m, \ i = 1, \dots, n,$$

where $I_a = [0, a]$, $a \geqslant 0$, $y = [y_1, \dots, y_m] \in R^m$, $m \geqslant 1$, $z(x, y) = [z_1(x, y), \dots, \dots, z_n(x, y)]$, and $(V^{(k)}z)(x, y) = [(V_1^{(k)}z)(x, y), \dots, (V_l^{(k)}z)(x, y)]$, $k = 1, 2, 3$, are operatortors of the Volterra type.

For matrices $B = [b_{ij}]$, $C = [c_{ij}]$, $i, j = 1, \dots, n$, we define $B * C = d$, $d = [d_1, \dots, d_n]^T$ where $d_i = \sum_{j=1}^{n} b_{ij}c_{ji}$, $i = 1, \dots, n$, and T means transposition of a vector or matrix.

We can write such systems in the matrix form

$$A(x, y, z(x, y), (V^{(1)}z)(x, y))\partial z(x, y)/\partial x + A(x, y, z(x, y), (V^{(1)}z)(x, y)) *$$
$$* [\varrho(x, y, z(x, y), (V^{(2)}z)(x, y))\partial z(x, y)/\partial y]^T = \tag{1}$$
$$= f(x, y, z(x, y), (V^{(3)}z)(x, y))$$

where $A = [A_{ij}]$, $i, j = 1, \dots, n$, $\partial z/\partial x = [\partial z_1/\partial x, \dots, \partial x, \dots, \partial z_m/\partial x]^T$, $\varrho = [\varrho_{ik}]$, $i = 1, \dots, n$, $k = 1, \dots, m$, $\partial z/\partial y = [\partial z_j/\partial y_i]$, $i = 1, \dots, m$, $j = 1, \dots, n$, and $f = [f_1, \dots, f_n]^T$.

In this paper we consider the existence and uniqueness of a local generalized solution (in the sense "almost everywhere") of the Cauchy problem obtained by adding to systems (1) the following initial condition

$$z(0, y) = \varphi(y), \quad y \in R^m \tag{2}$$

where $\varphi = [\varphi_1, \ldots, \varphi_n]$ is a given function.

Quasilinear hyperbolic systems in the "second canonic" form which have been considered by L. Cesari [5], P. Bassanini [1—3], and M. Cinquini-Cibrario [7], are the special cases (A, ϱ and f do not depend on the last variable) of systems (1).

Systems of differential equations with a retarded argument [10—11], and a few kinds of integrodifferential systems (cf. for instance [4]) can be obtained from systems (1) by specializing the operators $V^{(k)}$ (see Section 4).

System (1) is a generalization of the systems considered in [12] where the matrix function A does not depend on the last variable.

Classical solutions (belonging to C^1) of nonlinear and quasilinear hyperbolic systems with a retarded argument were discussed by Z. Kamont [8—9].

The method used in the present paper is based on the Banach fixed point theorem and it is close to that used in [5].

2. Bicharacteristics

Let $|y|_m = \max_{1 \leqslant k \leqslant m} |y_k|$ and $|z|_n = \max_{1 \leqslant i \leqslant n} |z_i|$, denote the norms of y in R^m and z in R^n, respectively. We denote by $|(x, y)|_{m+1} = \max(|x|, |y|_m)$ the norm of (x, y) in R^{m+1}. If $D = [d_{ij}]$, $i = 1, \ldots, n$, $j = 1, \ldots, m$, is a $n \times m$ matrix, then $D_i = [d_{i1}, \ldots, d_{im}]$. If D an $n \times n$ matrix, then $\|D\| = \max_{1 \leqslant i \leqslant n} \sum_{j=1}^{n} |d_{ij}|$. We shall use the symbol $\bar{\Omega}$ to denote the interval $[-\Omega, \Omega]^n \subset R^n$, $\Omega > 0$.

Let us denote by $\mathcal{J}$ the class of all continuous functions $\varphi\colon R^m \to R^n$, such that, for all $y, \bar{y} \in R^m$, we have

$$|\varphi(y)|_n \leqslant \omega, \quad |\varphi(y) - \varphi(\bar{y})|_n \leqslant \Lambda |y - \bar{y}|_m,$$

where ω, $0 \leqslant \omega < \Omega$, and $\Lambda \geqslant 0$ are given constants.

We denote by K the set of all continuous functions $z\colon D_a \to R^n$, such that $|z(x, y)|_n \leqslant \Omega$, $(x, y) \in D_a$.

For every $\varphi \in \mathcal{J}$ let us consider the set K_φ of all functions $z \in K$ satisfying the following conditions:

(i) $z(0, y) = \varphi(y)$, $y \in R^m$;

(ii) there are constants $P, Q \geqslant 0$, such that, for all $(x, y), (\bar{x}, \bar{y}) \in D_a$, we have

$$|z(x, y) - z(\bar{x}, \bar{y})|_n \leqslant P|x - \bar{x}| + Q|y - \bar{y}|_m,$$

where the constants P and Q will be defined by (4). Here, K_φ is the closed (convex) subset of the Banach space $(\mathscr{C}(D_a) \cap \mathscr{L}_\infty(D_a))^n$ with norm

$$\|z\|_a = \sup_{(x, y) \in D_a} |z(x, y)|_n .$$

We shall denote by K_1 the set of all functions $z\colon D_a \to R^n$ satisfying the following conditions:

(i) $z(\cdot, y)\colon I_a \to R^n$ is measurable for every $y \in R^m$;

(ii) $z(x, \cdot)\colon R^m \to R^n$ is continuous for a.e. $x \in I_a$;

(iii) $|z(x, y)|_n \leqslant \Omega$, $(x, y) \in D_a$.

Assumption H_1. Suppose that

1° $V^{(1)}_j\colon K_\varphi \to K$, $V_j^{(k)}\colon K_\varphi \to K_1$, $k = 2, 3, j = 1, \ldots, l$;

2° there are constants $p_j^{(k)}, q_j^{(k)}, k = 1, 2, 3, j = 1, \ldots, l$, such that, for all $z \in K_\varphi$, we have

$$[\![(V_j^{(1)}z)(\cdot)]\!] \leqslant p_j^{(1)}[\![z(\cdot)]\!] + q_j^{(1)}, \quad [\![(V_j^{(k)}z)(x, \cdot)]\!] \leqslant p_j^{(k)}[\![z(x, \cdot)]\!] + q_j^{(k)},$$
$$k = 2, 3, \quad j = 1, \ldots, l, \quad \text{a.e.} \quad x \in I_{a_0}$$

where

$$[\![z(\cdot)]\!] = \sup_{(x, y), (\bar{x}, \bar{y}) \in D_a} \frac{|z(x, y) - z(\bar{x}, \bar{y})|_n}{|(x, y) - (\bar{x}, \bar{y})|_{m+1}}, \quad [\![z(x, \cdot)]\!] = \sup_{y, \bar{y} \in R^m} \frac{|z(x, y) - z(x, \bar{y})|_n}{|y - \bar{y}|_m}$$

$x \in I_{a_0}$, and a_0 is a given positive constant;

3° there are constants $M_j^{(k)} \geqslant 0$, $k = 1, 2, 3, j = 1, \ldots, l$, such that, for all $z, \bar{z} \in K_\varphi$, $y \in R^m$, and a.e. $x \in I_{a_0}$, we have

$$|(V_j^{(k)}z)(x, y) - (V_j^{(k)}\bar{z})(x, y)|_n \leqslant M_j^{(k)} \|z - \bar{z}\|_x, \qquad k = 1, 2, 3, j = 1, \ldots, l,$$

where $\|z\|_x = \sup_{D_x} |z(x, y)|_n$, $D_x = I_x \times R^m$.

Remark 1. It follows from 3° of H_1 that $V_j^{(k)}$, $k = 1, 2, 3, j = 1, \ldots, l$, satisfy the following Volterra condition: if $z, \bar{z} \in K_\varphi$ and $z(t, y) = \bar{z}(t, y)$ for $t \in I_x$, $y \in R^m$, then $(V_j^{(k)}z)(x, y) = (V_j^{(k)}\bar{z})(x, y)$, $k = 1, 2, 3, j = 1, \ldots, l$.

Assumption H_2. Suppose that

1° the matrix function $\varrho(\cdot, y, z, U) = [\varrho_{ik}(\cdot, y, z, U)]\colon I_{a_0} \to R^{nm}$, $i = 1, \ldots, n$, $k = 1, \ldots, m$, is measurable for every $(y, z, U) \in D = R^m \times \bar{\Omega} \times \bar{\Omega}^l$, where $U = [u_1, \ldots, u_l]$;

2° $\varrho(x, \cdot)\colon D \to R^{nm}$ is continuous for a.e. $x \in I_{a_0}$;

3° there is a function $l\colon I_{a_0} \to R_+ = [0, +\infty)$, $l \in \mathscr{L}_1[0, a_0]$, and a constant $b > 0$, such that, for all $(y, z, U), (\bar{y}, \bar{z}, \bar{U}) \in D$, $i = 1, \ldots, n$, a.e. $x \in I_{a_0}$, we have

$$|\varrho_i(x, y, z, U)|_m \leqslant b,$$

$$|\varrho_i(x, y, z, U) - \varrho_i(x, \bar{y}, \bar{z}, \bar{U})|_m \leq l(x)\left[|y - \bar{y}|_m + |z - \bar{z}|_n + \sum_{j=1}^{l} |u_j - \bar{u}_j|_n\right],$$

where $\bar{U} = [\bar{u}_1, \ldots, \bar{u}_l]$.

We shall use $\tilde{K}_0$ to denote the set of all continuous vector functions $h: \Delta_a = I_a \times I_a \times R^m \to R^m$ satisfying the following conditions

$$h(x, x, y) = 0, \quad (x, y) \in D_a,$$

$$|h(\xi, x, y) - h(\bar{\xi}, x, y)|_m \leq b|\xi - \bar{\xi}|,$$

$$|h(\xi, x, y) - h(\xi, x, \bar{y})|_m \leq s|y - \bar{y}|_m,$$

for all (ξ, x, y), $(\bar{\xi}, x, y)$, $(\xi, x, \bar{y}) \in \Delta_a$, and some constant s, $0 < s < 1$.

Let us consider the set K_0 defined by

$$K_0 = \{g: g(\xi, x, y) = y + h(\xi, x, y), (\xi, x, y) \in \Delta_a, h \in \tilde{K}_0\}.$$

Consequently, for all (ξ, x, y), $(\xi, x, \bar{y}) \in \Delta_a$, and $g \in K_0$, we have

$$|g(\xi, x, y) - g(\xi, x, \bar{y})|_m \leq (1 + s)|y - \bar{y}|_m.$$

Note that $\tilde{K}_0$ is a closed (convex) subset of the Banach space $(\mathscr{C}(\Delta_a) \cap \mathscr{L}_\infty(\Delta_a))^m$ with norm

$$\|h\|_a = \sup_{(\xi, x, y) \in \Delta_a} |h(\xi, x, y)|_m.$$

For further properties of h and g we refer to [5—6].

Let us define the following constants

$$p = \sum_{j=1}^{l} (p_j^{(1)} P + q_j^{(1)}), \quad Q_{(k)} = \sum_{j=1}^{l} (p_j^{(k)} Q + q_j^{(k)}),$$

$$\bar{P} = 1 + P + p, \quad \bar{Q}_{(k)} = 1 + Q + Q_{(k)}, \qquad k = 1, 2, 3.$$

Lemma 1. *If Assumptions H_1 and H_2 are satisfied and a, $0 < a \leq a_0$, is sufficiently small such that $L_a(1 + s)\bar{Q}_{(2)} \leq s$, then for every fixed $z \in K_\varphi$, and for each i, $i = 1, \ldots, n$, the transformation $T_z^i: \tilde{K}_0 \to \tilde{K}_0$ defined by*

$$(T_z^i h)(\xi, x, y) = -\int_\xi^x \varrho_i(t, g(t, x, y), z(t, g(t, x, y)), (V^{(2)}z)(t, g(t, x, y)))\, dt,$$

$(\xi, x, y) \in \Delta_a$, $i = 1, \ldots, n$, has a unique fixed point $h_i[z] \in \tilde{K}_0$. Furthermore, for all z, $\bar{z} \in K_\varphi$, we have

$$\|g_i[z] - g_i[\bar{z}]\|_a = \|h_i[\bar{z}]\|_a \leq L_a[1 + M^{(2)}) \exp(L_a Q_{(2)}) \|z - \bar{z}\|_a,$$

where $g_i[z](\xi, x, y) = h_i[z](\xi, x, y) + y$. *It means that* $z \to h_i[z]$ ($z \to g_i[z]$) *is a continuous map of* K_φ *into* $\tilde{K}_0$ ($K_\varphi \to K_0$), $i = 1, \ldots, n$.

Proof. Note that, for every $h \in \tilde{K}_0$, and i, $i = 1, \ldots, n$, the function $T_z^i h$ is obviously continuous, and that

$$(T_z^i h)(x, x, y) = 0, \quad (x, y) \in D_a,$$

$$|(T_z^i h)(\xi, x, y) - (T_z^i h)(\bar{\xi}, x, y)|_m \leqslant b|\xi - \bar{\xi}|,$$

$$|(T_z^i h)(\xi, x, y) - (T_z^i h)(\xi, x, \bar{y})|_m \leqslant \left| \int_\xi^x |l(t)|\, \bar{Q}_{(2)} |g(t, x, y) - g(t, x, \bar{y})|_m \, \mathrm{d}t \right| \leqslant$$
$$\leqslant L_a(1 + s)\bar{Q}_{(2)} |y - \bar{y}|_m \leqslant s|y - \bar{y}|_m, \quad i = 1, \ldots, n.$$

Hence we conclude that $T_z^i h$ belongs to $\tilde{K}_0$.

In order to prove that T_z^i is a contraction we introduce norm

$$\|h\|_0 = \sup_{\Delta_a} \exp\left[-\lambda \left| \int_\xi^x l(t)\, \mathrm{d}t \right|\right] |h(\xi, x, y)|_m \tag{3}$$

with $\lambda > \bar{Q}_{(2)}$.

Now, we have

$$\|T_z^i h - T_z^i \bar{h}\|_0 \leqslant \sup_{\Delta_a} \exp\left[-\lambda \left| \int_\xi^x l(t)\, \mathrm{d}t \right|\right] \left| \int_\xi^x l(t) \exp\left[\lambda \left| \int_t^x l(s)\, \mathrm{d}s \right|\right] \mathrm{d}t \right| \cdot$$
$$\cdot \bar{Q}_{(2)} \|h - \bar{h}\|_0 \leqslant \frac{\bar{Q}_{(2)}}{\lambda} \|h - \bar{h}\|_0, \quad i = 1, \ldots, n.$$

Hence and by the Banach fixed point theorem it follows that, for every $z \in K_\varphi$ and i, $i = 1, \ldots, n$, the transformation T_z^i has a unique fixed point $h_i[z] \in \tilde{K}_0$.

Let us prove that $z \to h_i[z]$ is a continuous map. Indeed, for any two $z, \bar{z} \in K_\varphi$ and corresponding h_i, $\bar{h}_i$, or fixed points $h_i = T_z^i h_i$, $\bar{h}_i = T_{\bar{z}}^i \bar{h}_i$, and for $\xi \geqslant x$, we have

$$|h_i(\xi, x, y) - \bar{h}_i(\xi, x, y)|_m \leqslant \int_x^\xi l(t)\, \bar{Q}_{(2)} |h_i(t, x, y) - \bar{h}_i(t, x, y)|_m \, \mathrm{d}t +$$
$$+ L_a(1 + M^{(2)}) \|z - \bar{z}\|_a, \quad i = 1, \ldots, n.$$

Hence and by Gronwall's inequality we have

$$|h_i(\xi, x, y) - \bar{h}_i(\xi, x, y)|_m \leqslant L_a(1 + M^{(2)}) \exp(L_a \bar{Q}_{(2)}) \|z - \bar{z}\|_a.$$

By the definition of norm $\|h\|_a$ we get

$$\|h_i - \bar{h}_i\|_a \leqslant L_a(1 + M^{(2)}) \exp(L_a Q_{(2)}) \|z - \bar{z}\|_a, \quad i = 1, \ldots, n.$$

If $\xi < x$, by introducing a new variable η, where $\xi = 2x - \eta$, we obtain the same estimate as above. This ends the proof.

Remark 2. By introducing norm (3) in $\tilde{K}_0$ we can improve the estimate of the slab width a (by which the existence and the uniqueness are proved) (cf. [5, 1, 10]).

Remark 3. Note that, for each i, $i = 1, \ldots, n$, the function $h_i[z]$ of the variables (ξ, x, y) is absolutly continuous in x for every (ξ, y). Indeed, for $h_i \in \tilde{K}_0$ and any two (ξ, x, y), $(\xi, \bar{x}, y) \in \Delta_a$, for $\xi \geqslant x$, we have

$$|h_i(\xi, x, y) - h_i(\xi, \bar{x}, y)|_m \leqslant b|x - \bar{x}| + \int_x^\xi l(t)\, \bar{Q}_{(2)} |h_i(t, x, y) - h_i(t, \bar{x}, y)|_m \,\mathrm{d}t,$$

$i = 1, \ldots, n$. Hence and by Gronwall's inequality we have

$$|h_i(\xi, x, y) - h_i(\xi, \bar{x}, y)|_m \leqslant b \exp(L_a \bar{Q}_{(2)})\, |x - \bar{x}|.$$

For $\xi < x$, similarly as in the proof of Lemma 1, we get by change of the variable the same estimate.

3. Lemmas and the main result

Assumption H_3. Suppose that

1° $A = [A_{ij}]: I_{a_0} \times D \to R^{n^2}$, $i, j = 1, \ldots, n$, is continuous;

2° $\det A(x, y, z, U) \geqslant \varkappa > 0$ in $I_{a_0} \times D$, for some constant $\varkappa$;

3° there are constants $H > 0$, $C \geqslant 0$, such that, for all (x, y, z, U), $(\bar{x}, \bar{y}, \bar{z}, \bar{U}) \in I_{a_0} \times D$, we have

$$\|A(x, y, z, U)\| \leqslant H,$$

$$\|A(x, y, z, U) - A(\bar{x}, \bar{y}, \bar{z}, \bar{U})\| \leqslant$$

$$\leqslant C\left[|x - \bar{x}| + |y - \bar{y}|_m + |z - \bar{z}|_n + \sum_{j=1}^{l} |u_j - \bar{u}_j|_n\right];$$

Since $\det A(x, y, z, U) \geqslant \varkappa > 0$ in $I_{a_0} \times D$, the relations of H_3 yield analogous relations for the inverse matrix A^{-1}. Thus, there are constants H' and C', such that, for all (x, y, z, U), $(\bar{x}, \bar{y}, \bar{z}, \bar{U}) \in I_{a_0} \times D$, we have

$$\|A^{-1}(x, y, z, U)\| \leqslant H',$$

$$\|A^{-1}(x, y, z, U) - A^{-1}(\bar{x}, \bar{y}, \bar{z}, \bar{U})\| \leqslant$$

$$\leqslant C'\left[|x - \bar{x}| + |y - \bar{y}|_m + |z - \bar{z}|_n + \sum_{j=1}^{l} |u_j - \bar{u}_j|_n\right].$$

Assumption H_4. Suppose that

1° $f(\cdot, y, z, U): I_{a_0} \to R^n$ is measurable for every $(y, z, U) \in D$;

2° $f(x, \cdot): D \to R^n$ is continuous for a.e. $x \in I_{a_0}$;

3° there is a constant $N > 0$ and a function $l_1: I_{a_0} \to R_+$, $L_1 \in \mathscr{L}_1[0, a_0]$ such that, for all (y, z, U), $(\bar{y}, \bar{z}, \bar{U}) \in D$, a.e. in I_{a_0}, we have

$$|f(x, y, z, U)|_n \leqslant N,$$

$$|f(x, y, z, U) - f(x, \bar{y}, \bar{z}, \bar{U})|_n \leqslant l_1(x) \left[|y - \bar{y}|_m + |z - \bar{z}|_n + \sum_{j=1}^{l} |u_j - \bar{u}_j|_n \right];$$

4° the vector function $\varphi: R^m \to R^n$ belongs to $\mathscr{J}$.

For every fixed $z \in K_\varphi$ and corresponding $g_i = g_i[z] \in K_0$, $i = 1, \ldots, n$ we consider now the transformation F defined by

$$(Fz)(x, y) = \varphi(y) + A^{-1}(x, y, z(x, y), (V^{(1)}z)(x, y)) \cdot$$
$$\cdot [\Delta^1(x, y) + \Delta^2(x, y) + \Delta^3(x, y)]$$

where $\Delta^k = [\Delta^k_1, \ldots, \Delta^k_n]^T$, $k = 1, 2, 3$,

$$\Delta^1(x, y) = \int_0^x f(t, g(t, x, y), z(t, g(t, x, y)), (V^{(3)}z)(t, g(t, x, y)))\, \mathrm{d}t,$$

$$\Delta^2(x, y) = A(0, g(0, x, y), z(0, g(0, x, y)), (V^{(1)}z)(0, g(0, x, y))) *$$
$$* [\varphi(g(0, x, y)) - \varphi(g(x, x, y))],$$

$$\Delta^3(x, y) = \int_0^x \frac{\mathrm{d}}{\mathrm{d}t} [A(t, g(t, x, y), z(t, g(t, x, y)), (V^{(1)}z)(t, g(t, x, y)))] *$$
$$* [z(t, g(t, x, y)) - \varphi(g(x, x, y))]\, \mathrm{d}t,$$

and

$$f(t, g(t, x, y), z(t, g(t, x, y)), (V^{(3)}z)(t, g(t, x, y))) =$$
$$= [f_1(t, g_1(t, x, y), z(t, g_1(t, x, y)), (V^{(3)}z)(t, g_1(t, x, y))), \ldots$$
$$\ldots, f_n(t, g_n(t, x, y), z(t, g_n(t, x, y)), (V^{(3)}z)(t, g_n(t, x, y)))]^T,$$

$$A(t, g(t, x, y), z(t, g(t, x, y)), (V^{(1)}z)(t, g(t, x, y))) =$$
$$= [A_{ij}(t, g_i(t, x, y), z(t, g_i(t, x, y)), (V^{(1)}z)(t, g_i(t, x, y)))], \quad i, j = 1, \ldots, n,$$

$$\varphi(g(0, x, y)) = [\varphi_i(g_j(0, x, y))], \quad z(t, g(t, x, y)) = [z_i(t, g_j(t, x, y))],$$
$$i, j = 1, \ldots, n.$$

Lemma 2. *Let Assumptions H_1—H_4 hold. Then for sufficiently small a, $0 < a \leqslant a_0$, the transformation F maps K_φ into itself.*

Proof. Let us denote by

$$z(t, g_i(t, x, y)) - \varphi(g_i(x, x, y)) =$$
$$= [z_1(t, g_i(t, x, y)) - \varphi_1(t, g_i(x, x, y)), \ldots, z_n(t, g_i(t, x, y)) - \varphi_n(g_i(x, x, y))]^T$$

the vector of the ith column of the matrix $z(t, g(t, x, y)) - \varphi(g(x, x, y))$.

By applying the Chain Rule Differentiation Lemma (4.ii) of [6] we have (cf. [5])

$$\int_0^x \left\| \frac{\mathrm{d}}{\mathrm{d}t} A(t, g(t, x, y), z(t, g(t, x, y)), (V^{(1)}z)(t, g(t, x, y))) \right\| \mathrm{d}t \leqslant$$
$$\leqslant aC(\bar{P} + b\bar{Q}_{(1)}),$$
$$\left| \frac{\mathrm{d}}{\mathrm{d}t} z(t, g_i(t, x, y)) \right|_n \leqslant P + Qb,$$
$$|z(t, g_i(t, x, y)) - \varphi(g_i(x, x, y))|_n \leqslant a(P + Qb), \quad (t, x, y) \in \Delta_a, \ i = 1, \ldots, n,$$

and hence

$$|\Delta^1(x, y)|_n \leqslant Na,$$
$$|\Delta^2(x, y)|_n \leqslant \|A(0, g(0, x, y), z(0, g(0, x, y)), (V^{(1)}z)(0, g(0, x, y)))\| \cdot$$
$$\cdot \max_{1 \leqslant i \leqslant n} |\varphi(g_i(0, x, y)) - \varphi(y)|_n \leqslant H\Lambda ba,$$
$$|\Delta^3(x, y)|_n \leqslant \int_0^x \left\| \frac{\mathrm{d}}{\mathrm{d}t} A(t, g(t, x, y), z(t, g(t, x, y)), (V^{(1)}z)(t, g(t, x, y))) \right\| \mathrm{d}t \cdot$$
$$\cdot \max_{1 \leqslant i \leqslant n} |z(t, g_i(t, x, y)) - \varphi(g_i(x, x, y))|_n \leqslant C(\bar{P} + b\bar{Q}_{(1)})(P + Qb)a^2.$$

Thus

$$|(Fz)(x, y)|_n \leqslant \omega + h'Sa \leqslant \omega + (\Omega - \omega) = \Omega,$$

provided a is assumed sufficiently small in order that $H'Sa \leqslant \Omega - \omega$, where $S = N + H\Lambda b + C(\bar{P} + b\bar{Q}_{(1)})(P + Qb)a$.

For any two points $(x, y), (\bar{x}, \bar{y}) \in D_a$, we see that the difference $(Fz)(x, y) - (Fz)(\bar{x}, \bar{y})$ can be written as the sum of the terms

$$(Fz)(x, y) - (Fz)(\bar{x}, \bar{y}) = \varphi(y) - \varphi(\bar{y}) + \delta_0 + \delta_1 + \delta_2 + \delta_3,$$

where

$$\delta_0 = [A^{-1}(x, y, z(x, y), (V^{(1)}z)(x, y)) - A^{-1}(\bar{x}, \bar{y}, z(\bar{x}, \bar{y}), (V^{(1)}z)(\bar{x}, \bar{y}))] \cdot$$
$$\cdot [\Delta^1(x, y) + \Delta^2(x, y) + \Delta^3(x, y)],$$
$$\delta_k = A^{-1}(\bar{x}, \bar{y}, z(\bar{x}, \bar{y}), (V^{(1)}z)(\bar{x}, \bar{y}))[\Delta^k(x, y) - \Delta^k(\bar{x}, \bar{y})], \quad k = 1, 2, 3,$$

and estimate below one by one:

$$|\delta_0|_n \leqslant aC'\bar{P}S|x - \bar{x}| + aC'\bar{Q}_{(1)}S|y - \bar{y}|_m,$$
$$|\delta_1|_n \leqslant H'[L_{1a}\bar{Q}_{(3)}b \exp(L_a\bar{Q}_{(2)}) + N]|x - \bar{x}| + H'L_{1a}\bar{Q}_{(3)}(1 + s)|y - \bar{y}|_m,$$
$$|\delta_2|_n \leqslant H'\Lambda b[C\bar{Q}_{(3)}ba + H] \exp(L_a\bar{Q}_{(2)})|x - \bar{x}| +$$

$$+ H'\Lambda[C\bar{Q}_{(1)}(1+s)ba + H(2+s)]\,|y-\bar{y}|_m,$$

$$|\delta_3|_n \leqslant H'Ca\{2\bar{Q}_{(1)}b(P+Qb)\exp(L_a\bar{Q}_{(2)}) +$$
$$+ (\bar{P} + b\bar{Q}_{(1)})[P + Qb(1+\exp(L_a\bar{Q}_{(2)})]\}\,|x-\bar{x}| +$$
$$+ H'Ca[2\bar{Q}_{(1)}(1+s)(P+Qb) + (\bar{P}+b\bar{Q}_{(1)})(\Lambda + Q(1+s))]\,|y-\bar{y}|_m.$$

Combining the previous estimates we have

$$|(Fz)(x,y) - (Fz)(\bar{x},\bar{y})|_n \leqslant [H'N + HH'\Lambda b\exp(L_a\bar{Q}_{(2)}) +$$
$$+ \sigma_1 L_{1a} + \sigma_2 a]\,|x-\bar{x}| + [\Lambda + HH'\Lambda(2+s) + \bar{\sigma}_1 L_{1a} + \bar{\sigma}_2 a]\,|y-\bar{y}|_m,$$

where

$$\sigma_1 = H'\bar{Q}_{(3)}b\exp(L_a\bar{Q}_{(2)}),$$
$$\sigma_2 = C'S + H'Cb\exp(L_a\bar{Q}_{(2)})[\bar{Q}_{(3)}b\Lambda + 2\bar{Q}_{(1)}(P+Qb)] +$$
$$H'C(\bar{P}+b\bar{Q}_{(1)})[P + Qb(1+\exp(L_a\bar{Q}_{(2)}))],$$
$$\bar{\sigma}_1 = H'\bar{Q}_{(3)}(1+s),$$
$$\bar{\sigma}_2 = C'\bar{Q}_{(1)}S + H'C\bar{Q}_{(1)}(1+s)(\Lambda b + 2(P+Qb) +$$
$$+ H'C(\bar{P}+b\bar{Q}_{(1)})(\Lambda + Q(1+s)).$$

Let us choose constants P and Q such that

$$P > H'N + HH'\Lambda b\exp(L_a\bar{Q}_{(2)}), \quad Q > \Lambda(1 + HH'(2+s)). \tag{4}$$

Suppose that a is sufficiently small so that

$$\sigma_1 L_{1a} + \sigma_2 a \leqslant P - (H'N + HH'\Lambda b\exp(L_a\bar{Q}_{(2)})),$$
$$\bar{\sigma}_1 L_{1a} + \bar{\sigma}_2 a \leqslant Q - \Lambda(1 + HH'(2+s)).$$

Then, for all (x, y), $(\bar{x}, \bar{y}) \in D_a$, we have

$$|(Fz)(x,y) - (Fz)(\bar{x},\bar{y})|_n \leqslant P|x-\bar{x}| + Q|y-\bar{y}|_m.$$

This completes the proof.

Lemma 3. *If Assumptions H_1—H_4 are satisfied, then for sufficiently small a, $0 < a \leqslant a_0$, the transformation $F\colon K_\varphi \to K_\varphi$ is a contraction.*

Proof. We first prove the following estimate

$$\|Fz - F\bar{z}\|_a \leqslant [1 + 2HH' + H'C(\bar{P} + b\bar{Q}_{(1)})a]\,\|\varphi - \bar{\varphi}\|_a + \delta\|z - \bar{z}\|_a \tag{5}$$

where

$$\delta = [C'(1+M^{(1)})S + H\Lambda b + H'C(1+M^{(1)})P + 2H'C(P+Qb) +$$
$$+ C(\bar{P}+b\bar{Q}_{(1)})]a + H'L_{1a} + \{H'(\bar{Q}_{(3)} + M^{(3)}) + H'[C(\bar{Q}_{(1)} + M^{(1)})ba +$$
$$+ 2H]\Lambda + 2H'C(P+Qb)(\bar{Q}_{(1)} + M^{(1)})a + C(\bar{P}+b\bar{Q}_{(1)})Qa\}.$$

$$\cdot L_a(1 + M^{(2)}) \exp(L_a \bar{Q}_{(2)}),$$

and $\|\varphi\|_a = \sup_{y \in R^m} |\varphi(y)|_n$.

Let φ, $\bar{\varphi}$ be any two elements of $\mathscr{J}$, z, $\bar{z}$ any two elements K_φ and $K_{\bar{\varphi}}$, respectively, and let $g = g[z]$, $\bar{g} = g[\bar{z}]$ be the corresponding elements in K_0. Then we can derive

$$(Fz)(x, y) - (F\bar{z})(x, y) = \varphi(y) - \bar{\varphi}(y) + \varepsilon_0 + \varepsilon_1 + \varepsilon_2 + \varepsilon_3,$$

where

$$\varepsilon_0 = [A^{-1}(x, y, z(x, y), (V^{(1)}z)(x, y)) - A^{-1}(x, y, \bar{z}(x, y), (V^{(1)}\bar{z})(x, y))] \cdot$$
$$\cdot [\Delta^1(x, y) + \Delta^2(x, y) + \Delta^3(x, y)],$$
$$\varepsilon_k = A^{-1}(x, y, \bar{z}(x, y), (V^{(1)}\bar{z})(x, y)) [\Delta^k(x, y) - \bar{\Delta}^k(x, y)], \quad k = 1, 2, 3,$$

and

$$|\varepsilon_0|_n \leqslant aC'(1 + M^{(1)}) S \|z - \bar{z}\|_a,$$
$$|\varepsilon_1|_n \leqslant H' L_{1a}[(\bar{Q}_{(3)} + M^{(3)})(1 + M^{(2)}) L_a \exp(L_a \bar{Q}_{(2)}) + 1] \|z - \bar{z}\|_a,$$
$$|\varepsilon_2|_n \leqslant 2HH' \|\varphi - \bar{\varphi}\|_a + H'\{[C(\bar{Q}_{(1)} + M^{(1)}) ba + 2H] \cdot$$
$$\cdot \Lambda(1 + M^{(2)}) L_a \exp(L_a \bar{Q}_{(2)}) + C\Lambda ba\} \|z - \bar{z}\|_a,$$
$$|\varepsilon_3|_n \leqslant H'Ca(\bar{P} + b\bar{Q}_{(1)}) \|\varphi - \bar{\varphi}\|_a + H'\{C(1 + M^{(1)}) Pa +$$
$$+ 2C[(\bar{Q}_{(1)} + M^{(1)})(1 + M^{(2)}) L_a \exp(L_a \bar{Q}_{(2)}) + 1](P + Qb) a +$$
$$+ aC(\bar{P} + b\bar{Q}_{(1)})(1 + M^{(2)}) L_a \exp(L_a \bar{Q}_{(2)})\} \|z - \bar{z}\|_a.$$

Here $\bar{\Delta}^k$, $k = 1, 2, 3$, can be obtained from Δ^k, $k = 1, 2, 3$, by replacing φ, z and g with $\bar{\varphi}$, $\bar{z}$ and $\bar{g}$, respectively.

Thus, combining the estimates above, we get estimate (5).

Now we shall take a sufficiently small so that $\delta \leqslant k < 1$. Then from (5), for fixed $\varphi \in \mathscr{J}$ and for every pair z, $\bar{z} \in K_\varphi$, corresponding g, $\bar{g} \in K_0$, we find

$$\|Fz - F\bar{z}\|_a \leqslant k \|z - \bar{z}\|_a,$$

where $k < 1$. Thus, the transformation F is a contraction.

Theorem. *If Assumptions H_1—H_4 are satisfied then for a sufficiently small, $0 < a \leqslant a_0$, there is a vector function $z: D_a \to R^n$, $z \in K_\varphi$, which satisfies* (1) *a.e. in D_a and* (2) *everywhere in R^m. Furthermore, z is unique in the class K_φ and depends continuously on φ.*

Proof. From Lemmas 2 and 3 and by the Banach fixed point theorem it follows that there exists a unique fixed point $z \in K_\varphi$, $Fz = z$, such that the following integral equations hold:

$$g_i(\xi, x, y) = y - (T_z^i g_i)(\xi, x, y), \qquad (\xi, x, y) \in \Delta_a, \; i = 1, \ldots, n,$$

$$z(x, y) = (Fz)(x, y), \qquad (x, y) \in D_a.$$

We can show similarly as in [5] (see also [11]) that the fixed point $z = z[\varphi]$ is the (unique in the class K_φ) solution of the Cauchy problem (1), (2).

It remains to prove that $z[\varphi]$ depends continuously on φ. Indeed, if $\varphi, \bar{\varphi} \in \mathscr{J}$ and $z = z[\varphi]$, $\bar{z} = z[\bar{\varphi}]$, then from (5) we have

$$\|z - \bar{z}\|_a = \|z[\varphi] - z[\bar{\varphi}]\| \leqslant (1 - \delta)^{-1}[1 + 2HH' + H'C(\bar{P} + b\bar{Q}_{(1)})a] \, \|\varphi - \bar{\varphi}\|_a.$$

The Theorem is thereby proved.

4. Examples. We list below a few particular cases of systems (1) which can be derived from (1) be specializing the operators $V^{(k)}$, $k = 1, 2, 3$.

(i) Let

$$(V_j^{(k)} z)(x, y) = (z \circ \alpha_j^{(k)})(x, y) \tag{6}$$

where $(z \circ \alpha^{(k)})(x, y) = [(z \circ \alpha_1^{(k)})(x, y), \ldots, (z \circ \alpha_1^{(k)})(x, y)]$, $(z \circ \alpha_j^{(k)})(x, y) = z(\alpha_j^{(k)}(x, y))$, $\alpha_j^{(k)}(x, y) = [\alpha_{j0}^{(k)}(x, y), \bar{\alpha}_j^{(k)}(x, y)]$, $\bar{\alpha}_j^{(k)}(x, y) = [\alpha_{j1}^{(k)}(x, y), \ldots, \alpha_{jm}^{(k)}(x, y)]$, $k = 1, 2, 3$, $j = 1, \ldots, l$. Then problem (1), (2) reduces to the Cauchy problem for quasiliniear hyperbolic systems of partial differential equations with a retarded argument (cf. [11])

$$\sum_{j=1}^{n} A_{ij}(x, y, z(x, y), (z \circ \alpha^{(1)})(x, y)) \Big[\partial z_j(x, y)/\partial x +$$

$$+ \sum_{k=1}^{m} \varrho_{ik}(x, y, z(x, y), (z \circ \alpha^{(2)})(x, y)) \, \partial z_j(x, y)/\partial y_k \Big] =$$

$$= f_i(x, y, z(x, y), (z \circ \alpha^{(3)})(x, y)), \qquad i = 1, \ldots, n, \; (x, y) \in D_a,$$

$$z(0, y) = \varphi(y), \qquad y \in R^m.$$

Let us suppose that

1° $\alpha_j^{(1)}: I_{a_0} \times R^m \to I_{a_0} \times R^m$, $j = 1, \ldots, l$, are continuous, $\alpha_{j0}^{(1)}(x, y) \leqslant x$ $(x, y) \in I_{a_0} \times R^m$, $j = 1, \ldots, l$, and there constants $c_j^{(1)} \geqslant 0$, such that, for all (x, y), $(\bar{x}, \bar{y}) \in I_{a_0} \times R^m$, we have

$$|\alpha_j^{(1)}(x, y) - \alpha_j^{(1)}(\bar{x}, \bar{y})|_{m+1} \leqslant c_j^{(1)} |(x, y) - (\bar{x}, \bar{y})|_{m+1};$$

2° $\alpha_j^{(k)}(\cdot, y): I_{a_0} \to I_{a_0} \times R^m$, $j = 1, \ldots, l$, $k = 2, 3$, are measurable for every $y \in R^m$, $\alpha_{j0}^{(k)}(x, y) \leqslant x$, $(x, y) \in I_{a_0} \times R^m$, $k = 2, 3$, $j = 1, \ldots, l$, and there are constants $c_j^{(k)} \geqslant 0$, such that, for all $y, \bar{y} \in R^m$, a.e. $x \in I_{a_0}$, we have

$$|\alpha_j^{(k)}(x, y) - \alpha_j^{(k)}(x, \bar{y})|_{m+1} \leqslant c_j^{(k)}|y - \bar{y}|_m, \qquad i = 2, 3,\ j = 1, \ldots, l.$$

Then Assumption H_1 is satisfied for the operators $V_j^{(k)}$ defined by (6) with $p_j^{(1)} = c_j^{(1)}$, $q_j^{(1)} = 0$, $p_j^{(k)} = c_j^{(k)}$, $q_j^{(k)} = 0$, $k = 2, 3$, and $M_j^{(i)} = 1$, $i = 1, 2, 3$, $j = 1, \ldots, l$.

(ii) As a particular case of (1), (2) we get the initial problem for systems of partial integrodifferential equations if we put

$$(V_j^{(k)}z)(x, y) = \int_{\beta_j^{(k)}(x, y)}^{\gamma_j^{(k)}(x, y)} K_j^{(k)}(s, t, x, y)\, z(s, t)\, \mathrm{d}s\, \mathrm{d}t \tag{7}$$

where $K_j^{(k)}$, $k = 1, 2, 3, j = 1, \ldots, l$, are $n \times n$ matrices.

Let us assume that

1° $\beta_j^{(1)}, \gamma_j^{(1)}: I_{a_0} \times R^m \to I_{a_0} \times R^m$ are continuous, $\beta_{j0}^{(1)}(x, y) \leqslant x$, $\gamma_{j0}^{(1)}(x, y) \leqslant x$, $(x, y) \in I_{a_0} \times R^m$, and there are constants $d_j^{(1)}, \bar{d}_j^{(1)} \geqslant 0$, such that, for all (x, y), $(\bar{x}, \bar{y}) \in I_{a_0} \times R^m$, we have

$$|\beta_j^{(1)}(x, y) - \beta_j^{(1)}(\bar{x}, \bar{y})|_{m+1} \leqslant d_j^{(1)}|(x, y) - (\bar{x}, \bar{y})|_m^{1/m+1},$$
$$|\gamma_j^{(1)}(x, y) - \gamma_j^{(1)}(\bar{x}, \bar{y})|_{m+1} \leqslant \bar{d}_j^{(1)}|(x, y) - (\bar{x}, \bar{y})|_m^{1/m+1}, \quad j = 1, \ldots, l;$$

2° $\beta_j^{(k)}(\cdot, y), \gamma_j^{(k)}(\cdot, \bar{y}): I_{a_0} \to I_{a_0} \times R^m$ are measurable, $\beta_{j0}^{(k)}(x, y) \leqslant x$, $\gamma_{j0}^{(k)}(x, y) \leqslant x$, $(x, y) \in I_{a_0} \times R^m$, $k = 2, 3, j = 1, \ldots, l$, and there are constants $d_j^{(k)}, \bar{d}_j^{(k)} \geqslant 0$, such that, for all $y, \bar{y} \in R^m$, a.e. $x \in I_{a_0}$, we have

$$|\beta_j^{(k)}(x, y) - \beta_j^{(k)}(x, \bar{y})|_{m+1} \leqslant d_j^{(k)}|y - \bar{y}|_m^{1/m+1},$$
$$|\gamma_j^{(k)}(x, y) - \gamma_j^{(k)}(x, \bar{y})|_{m+1} \leqslant \bar{d}_j^{(k)}|y - \bar{y}|_m^{1/m+1}, \qquad k = 2, 3,\ j = 1, \ldots, l;$$

3° there are constants $e_j^{(k)} > 0$, such that, for every $(x, y) \in I_{a_0} \times R^m$, we have

$$\prod_{i=0}^{m} |\gamma_{ji}^{(k)}(x, y) - \beta_{ji}^{(k)}(x, y)| \leqslant e_j^{(k)}, \qquad k = 1, 2, 3,\ j = 1, \ldots, l;$$

4° the matrix functions $K_j^{(1)}(\cdot, x, y): I_{a_0} \times R^m \to R^{n^2}$, $K_j^{(k)}(\cdot, y): I_{a_0} \times R^m \times I_{a_0} \to R^{n^2}$, are measurable for every $(x, y) \in I_{a_0} \times R^m$, $k = 2, 3, j = 1, \ldots, l$, and there are constants $\bar{c}_j^{(k)} > 0$, $r_j^{(1)}, r_j^{(i)} \geqslant 0$, such that, for all (x, y), $(\bar{x}, \bar{y}) \in I_{a_0} \times R^m$, $(s, t, x) \in I_{a_0} \times R^m \times I_{a_0}$, we have

$$\|K_j^{(k)}(s, t, x, y)\| \leqslant \bar{c}_j^{(k)}, \qquad k = 1, 2, 3,\ j = 1, \ldots, l,$$
$$\|K_j^{(1)}(s, t, x, y) - K_j^{(1)}(s, t, \bar{x}, \bar{y})\| \leqslant r_j^{(1)}|(x, y) - (\bar{x}, \bar{y})|_{m+1},$$
$$\|K_j^{(i)}(s, t, x, y) - K_j^{(i)}(s, t, x, \bar{y})\| \leqslant r_j^{(i)}|y - \bar{y}|_m, \qquad i = 2, 3,\ j = 1, \ldots, l.$$

Then Assumption H_1 is satisfied for the operators $V_j^{(k)}$ defined by (7) with $p_j^{(k)} = 0$, $q_j^{(k)} = \Omega\{e_j^{(k)} r_j^{(k)} + \bar{c}_j^{(k)}[(d_j^{(k)})^{m+1} + (\bar{d}_j^{(k)})^{m+1}]\}$, and $M_j^{(k)} = e_j^{(k)} \bar{c}_j^{(k)}$, $k = 1, 2, 3, j = 1, \dots, l$, provided $e_j^{(k)} \bar{c}_j^{(k)} < 1$, $k = 1, 2, 3, j = 1, \dots, l$.

(iii) Let $(V_j^{(k)} z)(x, y) = \int_{-\infty}^{y} K_j^{(k)}(y - t) z(x, t)\, dt$, $k = 1, 2, 3, j = 1, \dots, l$. Then systems (1) are systems of integrodifferential equations of which the particular case ($l = 1$, $A(x, y, z, u) = \bar{A}(x, y, z)$, $\varrho(x, y, z, u) = \bar{\varrho}(x, y, z)$ and $f(x, y, z, u) = \bar{f}(x, y, z) + u$) were considered by P. Bassanini, M. C. Salvatori [4].

(iv) We denote by A_m the set of all elements $\mu = (\mu_0, \mu_1, \dots, \mu_m)$, such that $\mu_i = 0$ or $\mu_i = 1$ for $i = 0, 1, \dots, m$, and $1 \leqslant |\mu| = \mu_0 + \dots + \mu_m$. It is easy to see that the number of elements of A_m is equal to $2^{m+1} - 1$. Let $N_\mu = \{i : \mu_i = 1\}$. For $(s, t) \in D_a$ we define $\mu \cdot (s, t) = (\mu_0 s, \mu_1 t_1, \dots, \mu_m t_m)$ (we shall often write $\mu(s, t)$). Let $1 - \mu = (1 - \mu_0, 1 - \mu_1, \dots, 1 - \mu_m)$ and $(1 - \mu)(s, t) = ((1 - \mu_0) s, (1 - \mu_1) t_1, \dots, (1 - \mu_m) t_m)$. Suppose that

$$\mu \, ds \, dt = \begin{cases} ds \, dt_{i_1} \dots dt_{i_k} & \text{if} \quad 0 \in N_\mu, \quad i_1, \dots, i_k \in N_\mu, \\ dt_{i_0} \, dt_{i_1} \dots dt_{i_k} & \text{if} \quad 0 \bar{\in} N_\mu, \quad i_0, i_1, \dots, i_k \in N_\mu, \quad k = 1, \dots, m, \end{cases}$$

and $\beta_{(\mu)}^{(s)}, \gamma_{(\mu)}^{(s)} : D_a \to R^{|\mu|}$, where

$$\beta_{(\mu)}^{(s)} = (\beta_{(\mu) i_0}^{(s)}, \dots, \beta_{(\mu) i_k}^{(s)}), \quad \gamma_{(\mu)}^{(s)} = (\gamma_{(\mu) i_0}^{(s)}, \dots, \gamma_{(\mu) i_k}^{(s)}),$$

$$0 \leqslant i_0 < i_1 \dots < i_k \leqslant m, \quad i_0, i_1, \dots, i_k \in N_\mu, \quad k = 1, \dots, m, \quad s = 1, 2, 3.$$

We define the operators $V_\mu^{(s)}$ in the following way

$$(V_\mu^{(s)} z)(x, y) = \int_{\beta_{(\mu)}^{(s)}(x, y)}^{\gamma_{(\mu)}^{(s)}(x, y)} z(\mu(s, t) + (1 - \mu)(x, y)) \, \mu \, ds \, dt.$$

Here $\int \mu \, ds \, dt$ is the $|\mu|$-dimensional integral with respect to the variables s, $t_{i_1}, \dots, t_{i_k}$ if $0 \in N_\mu$, $i_1, \dots, i_k \in N_\mu$, and it is the integral with respect to $t_{i_0}, \dots, t_{i_k}$ if $0 \bar{\in} N_\mu$.

Now we consider the Cauchy problem (1), (2) for integrodifferential systems with $V^{(s)} z = (V_{(1, \dots, 1)}^{(s)} z, V_{(0, 1, \dots, 1)}^{(s)} z, V_{(1, 0, 1, \dots, 1)}^{(s)} z, \dots, V_{(1, \dots, 1, 0)}^{(s)} z, V_{(0, 0, 1, \dots, 1)}^{(s)} z, \dots, V_{(1, \dots, 1, 0, 0)}^{(s)} z, \dots, V_{(1, 0, \dots, 0)}^{(s)} z)$, $s = 1, 2, 3$.

We introduce the following assumptions:

1° $\beta_{(\mu)}^{(1)}, \gamma_{(\mu)}^{(1)} : I_{a_0} \times R^m \to R$, $\mu \in A_m$, are continuous, $\beta_{(\mu) 0}^{(1)}(x, y) \leqslant x$, $\gamma_{(\mu) 0}^{(1)}(x, y) \leqslant x$, $(x, y) \in I_{a_0} \times R^m$, and $\beta_{(\mu)}^{(s)}(\cdot, y)$, $\gamma_{(\mu)}^{(s)}(\cdot, y) : I_{a_0} \to R$, $s = 2, 3$, $\mu \in A_m$, are measurable, $\beta_{(\mu) 0}^{(s)}(x, y) \leqslant x$, $\gamma_{(\mu) 0}^{(s)}(x, y) \leqslant x$, $s = 2, 3$, $(x, y) \in I_{a_0} \times R^m$;

2° there are constants $d_{(\mu)}^{(s)}, \bar{d}_{(\mu)}^{(s)} \geqslant 0$, such that, for all $(x, y), (\bar{x}, \bar{y}) \in I_{a_0} \times R^m$, we have

$$|\beta^{(1)}_{(\mu)j}(x, y) - \beta^{(1)}_{(\mu)j}(\bar{x}, \bar{y})| \leqslant d^{(1)}_{(\mu)} |(x, y) - (\bar{x}, \bar{y})|^{1/|\mu|}_{m+1},$$
$$|\gamma^{(1)}_{(\mu)j}(x, y) - \gamma^{(1)}_{(\mu)j}(\bar{x}, \bar{y})| \leqslant \bar{d}^{(1)}_{(\mu)} |(x, y) - (\bar{x}, \bar{y})|^{1/|\mu|}_{m+1},$$
$$|\beta^{(s)}_{(\mu)j}(x, y) - \beta^{(s)}_{(\mu)j}(x, \bar{y})| \leqslant d^{(s)}_{(\mu)} |y - \bar{y}|^{1/|\mu|}_{m},$$
$$|\gamma^{(s)}_{(\mu)j}(x, y) - \gamma^{(s)}_{(\mu)j}(x, \bar{y})| \leqslant \bar{d}^{(s)}_{(\mu)} |y - \bar{y}|^{1/|\mu|}_{m}, \qquad s = 2, 3,\ j = 1, \ldots, m;$$

3° there are constants $e^{(s)}_{(\mu)} > 0$, such that, for every $(x, y) \in I_{a_0} \times R^m$, we have

$$\prod_{j \in N_\mu} |\gamma^{(s)}_{(\mu)j}(x, y) - \beta^{(s)}_{(\mu)j}(x, y)| \leqslant e^{(s)}_{(\mu)}, \qquad s = 1, 2, 3.$$

Then Assumption H_1 is satisfied for the operators $V^{(s)}_\mu$ defined by (8) with $p^{(s)}_\mu = e^{(s)}_{(\mu)}$, $q^{(s)}_\mu = \Omega[(d^{(s)}_{(\mu)})^{|\mu|} + (\bar{d}^{(s)}_{(\mu)})^{|\mu|}]$, and $M^{(s)}_\mu = e^{(s)}_{(\mu)}$, $s = 1, 2, 3$, (here $l = 2^{m+1} - 1$).

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СУЩЕСТВОВАНИЕ И ЕДИНСТВЕННОСТЬ РЕШЕНИЙ КВАЗИЛИНЕЙНЫХ ГИПЕРБОЛИЧЕСКИХ СИСТЕМ ДИФФЕРЕНЦИАЛЬНО-ФУНКЦИОНАЛЬНЫХ УРАВНЕНИЙ С ЧАСТНЫМИ ПРОИЗВОДНЫМИ

Jan Turo

Резюме

В работе доказывается теорема о существовании, единственности и непрерывной зависимости обобщенных решений (в смысле всюд «почти всюду») от начальных данных задачи Коши для квазилинейных гиперболических систем дифференциально-фукциональных уравнений с частными производными первого порядка.