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A NOTE ON RANDOMLY COMPLETE $n$-PARTITE GRAPHS

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ABSTRACT. The paper of A l a v i et al. [1] contains two results concerning randomly complete $n$-partite graphs. We give improvements and a correction of their result.

Let $G$ be a graph containing a subgraph $H$ with no isolated vertices. In [2] the concept of “randomly $H$ graphs” is introduced as follows: The graph $G$ is said to be a randomly $H$ graph if and only if any subgraph of $G$ without isolated vertices, which is isomorphic to a subgraph of $H$, can be extended to a subgraph $H_1$ of $G$ such that $H_1$ is isomorphic to $H$.

The authors [1] characterized graphs $G$ that are randomly complete $n$-partite. Two of their results, namely, Theorem 1 and 2, are improved upon.

The first of them asserts:

**THEOREM 1.** [1]. A graph $G$ is randomly $K_{p,q}$, $q \geq p \geq 2$, if and only if $G$ is isomorphic to a complete bipartite graph $K_{s,t}$, where $s \geq p$ and $t \geq q$, or $G$ is isomorphic to a complete graph $K_r$ where $r \geq p + q$.

However, the graph $K_{3,5}$ is not randomly $K_{3,4}$. Otherwise, take $F$ (Fig. 1.a), which is a subgraph of $K_{3,5}$. The graph $F'$ is isomorphic to a subgraph $F''$ (Fig. 1.b) of $K_{3,4}$. But there is no way to extend $F$ in $K_{3,5}$ to a graph isomorphic to $K_{3,4}$.

The result in Theorem 1 can be improved as follows:

**THEOREM A.** A graph $G$ is randomly $K_{p,q}$, $q \geq p \geq 2$, if and only if $G$ is isomorphic to a complete graph $K_r$, where $r \geq p + q$ or

(i) $p = q = 2$ and $G = K_{s,t}$, where $t \geq s \geq 2$,
(ii) $p = 2$, $q = 3$ and $G = K_{s,t}$, where $s \geq 2$, $t \geq 3$,
(iii) $p = 2$, $q \geq 4$ and $G = K_{s,t}$, where $s = 2$, $t \geq q$,

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(iv) $p = q = r \geq 3$ and $G = K_{r,r}$,
(v) $p = r \geq 3$, $q = r + 1$ and $G = K_{r,r+1}$ or $G = K_{r+1,r+1}$,
(vi) $p = r \geq 3$, $q = r + 2$ and $G = K_{p,q}$.

For the proof see [4, Theorem 2 and Theorem 3] (H-closed is there used for the term randomly H graph).
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The second result asserts:

**THEOREM 2.** [1]. Let \( m \geq 2 \) and \( n \geq 3 \) be integers. The graph \( G \) is randomly complete \( K_{n-1} + \overline{K}_m \) if and only if \( G \) is isomorphic to \( K_r + \overline{K}_s \), where: \( r \geq n - 1 \), \( m \geq s \geq 0 \), and \( r + s \geq m + n - 1 \).

(The operation \( "+" \) is taken in the usual sense of Harary [3]).

The above assertion is true only in the case \( m \leq 2 \). For the case \( m > 2 \) we need the following:

**PROPOSITION.** Let \( m > 2 \), and \( n \geq 3 \) be integers. Let \( G = K_r + \overline{K}_s \), \( H = K_{n-1} + \overline{K}_m \), \( r > n - 1 \), \( m \geq s \geq 2 \), \( r + s \geq m + n - 1 \). The graph \( G \) is not randomly \( H \) graph.

**Proof.** Assume that \( G = K_r + \overline{K}_s \). Let

\[
V_1 = \{v_1, v_2, \ldots, v_n, \ldots, v_r\} = V(K_r), \quad V_2 = \{u_1, u_2, \ldots, u_s\} = V(\overline{K}_s)
\]

be two disjoint sets of vertices of \( G \). Form a graph \( F \) as follows: the edges of \( F \) are

\[
E(F) = \{(u_1, v_1), (u_1, v_2), \ldots, (u_1, v_n), (u_2, v_n)\},
\]

(see Fig. 2.a). Obviously, \( F \) is a subgraph of \( K_r + \overline{K}_s \), which is isomorphic to a subgraph \( F' \) of \( K_{n-1} + \overline{K}_m \), (see Fig. 2.b). But there is no way to extend \( F \) in \( G \) to a subgraph that is isomorphic to \( K_{n-1} + \overline{K}_m \).

And now, from this Proposition and from [5, Theorem 5] it follows:

**THEOREM B.** Let \( m = 2 \) and \( n \geq 3 \). The graph \( G \) is randomly complete \( K_{n-1} + \overline{K}_2 \) if and only if \( G \) is isomorphic to \( K_r + \overline{K}_s \), where \( r \geq n - 1 \), \( 2 \geq s \geq 0 \), \( r + s \geq m + n - 1 \).

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