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Dedicated to Professor Tibor Šalát
on the occasion of his 70th birthday

MAXIMA AND MINIMA OF SIMPLY CONTINUOUS AND QUASICONTINUOUS FUNCTIONS

JÁN BORSÍK

(*Communicated by Ľubica Holá*)

ABSTRACT. Functions which are maxima and minima of simply continuous and quasicontinuous functions are characterized.

In what follows, X denotes a topological space. For a subset A of a topological space denote by $\operatorname{Cl} A$ and $\operatorname{Int} A$ the closure and the interior of A, respectively. The letters $\mathbb{N}$, $\mathbb{Q}$ and $\mathbb{R}$ stand for the set of natural, rational and real numbers, respectively. For $x \in X$ denote by $\mathcal{U}_x$ the family of all neighbourhoods of x. A regular (normal) space is not assumed to be T_1.

A family $\mathcal{F} \subset \mathbb{R}^X$ of real functions is a lattice if and only if $\min(f,g) \in \mathcal{F}$ and $\max(f,g) \in \mathcal{F}$ for $f,g \in \mathcal{F}$. The symbol $L(\mathcal{F})$ stands for the lattice generated by $\mathcal{F}$, i.e., the smallest lattice of functions containing $\mathcal{F}$.

We recall that a function $f\colon X \to \mathbb{R}$ is *quasicontinuous* (*cliquish*) at a point $x \in X$ if for each $\varepsilon > 0$ and each neighbourhood U of x there is a nonempty open set $G \subset U$ such that $|f(y) - f(x)| < \varepsilon$ for each $y \in G$ ($|f(y) - f(z)| < \varepsilon$ for each $y, z \in G$). A function $f\colon X \to \mathbb{R}$ is said to be quasicontinuous (cliquish) if it is quasicontinuous (cliquish) at each point $x \in X$ (see [6]).

A function $f\colon X \to \mathbb{R}$ is *simply continuous* if $f^{-1}(V)$ is a simply open set in X for each open set V in $\mathbb{R}$. A set A is *simply open* if it is the union of an open set and a nowhere dense set (see [1]).

Denote by $\mathcal{Q}$, $\mathcal{S}$ and $\mathcal{K}$ the set of all functions (in $\mathbb{R}^X$) which are quasicontinuous, simply continuous and cliquish, respectively. By $C(f)$ and $Q(f)$ we will denote the set of all continuity points of a function $f\colon X \to \mathbb{R}$ and the set of all

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quasicontinuity points of f, respectively. Furthermore, let $A(f) = X \setminus Q(f)$. It is easy to see that $\mathcal{Q} \subset \mathcal{S}$ and $\mathcal{Q} \subset \mathcal{K}$. If X is a Baire space, then every simply continuous function $f\colon X \to \mathbb{R}$ is cliquish ([7]). Example 1 in [3] shows that the assumption "X is a Baire space" cannot be omitted.

In [5], T. Natkaniec has characterized the maximum of real quasicontinuous functions of one real variable. Properties of $\mathbb{R}$ (the ordering and the completeness) in his proof play the key role. In this paper, we shall give a characterization of the maximum of real quasicontinuous functions defined on a regular second countable topological space.

It is well known that $f+g$, $|f|$ and cf ($c \in \mathbb{R}$) are cliquish functions for cliquish functions $f, g\colon X \to \mathbb{R}$ for an arbitrary topological space X. Hence $\max(f,g)$ and $\min(f,g)$ are cliquish functions for cliquish f and g and $L(\mathcal{K}) = \mathcal{K}$.

By [2], the sum of a simply continuous function $f\colon \mathbb{R} \to \mathbb{R}$ and a continuous function $g\colon \mathbb{R} \to \mathbb{R}$ need not be a simply continuous function. Nevertheless, we have:

THEOREM 1. *Assume that X is a topological space with the following property:*

(∗) *if $(X_n)_n$ is a partition of X such that $\bigcup_{n\in M} X_n$ is simply open for each $M \subset \mathbb{N}$, and G is a nonempty open set in X, then $G \cap \operatorname{Int} X_n \neq \emptyset$ for some $n \in \mathbb{N}$.*

Then both $\min(f,g)$ and $\max(f,g)$ are simply continuous functions whenever $f, g\colon X \to \mathbb{R}$ are simply continuous, i.e., $L(\mathcal{S}) = \mathcal{S}$.

Proof. Suppose that $h = \max(f,g)$ is not simply continuous. Then there is an open set V in $\mathbb{R}$ such that $h^{-1}(V)$ is not simply open, i.e., $h^{-1}(V) \setminus \operatorname{Int} h^{-1}(V)$ is not nowhere dense. Let $A \subset X$ be an open set such that $h^{-1}(V) \setminus \operatorname{Int} h^{-1}(V)$ is dense in A. This yields

(1) $h^{-1}(V)$ is dense in A,

(2) $\operatorname{Int} h^{-1}(V) \cap A = \emptyset$.

Since f is simply continuous, there is an open nonempty set $B \subset A$ such that $B \cap \big(f^{-1}(V) \setminus \operatorname{Int} f^{-1}(V)\big) = \emptyset$. Further, there is an open nonempty set $C \subset B$ such that $C \cap \big(g^{-1}(V) \setminus \operatorname{Int} g^{-1}(V)\big) = \emptyset$. We have four possibilities:

a) $C \cap g^{-1}(V) = \emptyset = C \cap f^{-1}(V)$. Then $g(x) \notin V$, $f(x) \notin V$, and hence $h(x) \notin V$ for each $x \in C$, which is a contradiction with (1).

b) $C \subset g^{-1}(V) \cap f^{-1}(V)$. Then $g(x), f(x) \in V$, and hence $h(x) \in V$ for each $x \in C$, which is a contradiction with (2).

c) $C \cap g^{-1}(V) = \emptyset$ and $C \subset f^{-1}(V)$. Let $(J_n)_n$ be a sequence of all components of V. By (∗), there are an $n \in \mathbb{N}$ and a nonempty open subset $W \subset C$ such that $f(x) \in J_n$ for each $x \in W$. Since $g(W) \cap J_n = \emptyset$, $g(W) \subset (-\infty, \inf J_n] \cup [\sup J_n, \infty)$. It is easy to observe that the sets $g^{-1}\big((-\infty, \inf J_n]\big)$

and $g^{-1}([\sup J_n, \infty))$ are simply open, so there is a nonempty open set $U \subset W$ such that either $U \subset g^{-1}((-\infty, \inf J_n])$ or $U \subset g^{-1}([\sup J_n, \infty))$. This yields $h|_U = f|_U$ and $U \subset h^{-1}(V)$, which is a contradiction with (2), or $h|_U = g|_U$ and $U \cap h^{-1}(V) = \emptyset$, which is a contradiction with (1).

d) $C \cap f^{-1}(V) = \emptyset$ and $C \subset g^{-1}(V)$. The proof is similar as in c). □

Recall that a collection $\mathcal{P}$ of nonempty open sets in X is a *π-base* for X if every nonempty open subset of X contains at least one member of $\mathcal{P}$. A π-base $\mathcal{P}$ is said to be locally countable if each member of $\mathcal{P}$ contains only countably many members of $\mathcal{P}$ (see [4]).

PROPOSITION 1. *If X either is a Baire topological space, or has a locally countable π-base, then it possesses the property $(*)$.*

Proof. If X is a Baire space, then this is a consequence of the Baire Category Theorem. Assume that X has a locally countable π-base $\mathcal{P}$. Suppose that $(*)$ fails. Then there are a nonempty set G and a partition of X of simply open sets $(X_n)_n$ such that $G \cap \operatorname{Int} X_n = \emptyset$ for each $n \in \mathbb{N}$. Let P be a member of $\mathcal{P}$ with $P \subset G$, and let $(U_n)_n$ be a π-base for P. Since $X_n \cap P$ are nowhere dense sets, we can choose by induction a one-to-one sequence of positive integers $(t_n)_n$ such that

$$X_{t_{2n}} \cap U_n \neq \emptyset \neq X_{t_{2n+1}} \cap U_n \qquad \text{for each} \quad n \in \mathbb{N}.$$

Then the set $Y = \bigcup_n X_{t_{2n}}$ is dense in P, and $P \cap \operatorname{Int} Y = \emptyset$. Thus Y is not simply open, which is a contradiction. □

Remark 1. There are topological spaces which neither are Baire nor have locally countable π-base, but still possess the property $(*)$. If fact, let X be uncountable product of $\mathbb{Q}$ ($\mathbb{Q}$ with the Euclidean topology of $\mathbb{R}$) with the Tychonoff topology. Then it is easy to verify that X has the property $(*)$, has no locally countable π-base and it is not Baire.

Remark 2. There are topological spaces which do not satisfy the condition $(*)$. In fact, let X be a space from [3], i.e., $X = \mathbb{N}$ with the topology $\mathcal{T} = \mathcal{D} \cup \{\emptyset\}$, where $\mathcal{D}$ is an ultrafilter on X which contains no finite sets. (By [3; Example 1], there is a real simply continuous function on X which is not cliquish. By a slight modification of the proof of Theorem 1 in [3], we see that every real simply continuous function on a space with the property $(*)$ is cliquish; hence the space X does not satisfy $(*)$.)

PROBLEM 1. Is Theorem 1 true for an arbitrary topological space?

LEMMA 1. *Let $f\colon X \to \mathbb{R}$ (X is an arbitrary topological space), and let the set $A(f)$ be nowhere dense. Then* $\liminf\limits_{u\to x,\, u\in Q(f)} f(u) \leqq f(x) \leqq \limsup\limits_{u\to x,\, u\in Q(f)} f(u)$ *for each* $x \in Q(f)$.

Proof. Let $x \in Q(f)$, and suppose that $\limsup\limits_{u\to x,\, u\in Q(f)} f(u) < c < f(x)$ for some $c \in \mathbb{R}$. Then there is a neighbourhood U of x such that $f(u) < c$ for each $u \in Q(f) \cap U$, $u \neq x$. Since $x \in Q(f)$, there is an open nonempty set $G \subset U$ such that $f(u) > c$ for $u \in G$. Since $A(f)$ is nowhere dense, there is $t \in G \cap Q(f)$. Now, $c < f(t) < c$, which is a contradiction. □

Remark 3. If X is a Baire space, then $\limsup\limits_{u\to x,\, u\in Q(f)} f(u) = \limsup\limits_{u\to x,\, u\in C(f)} f(u)$. This is not true for an arbitrary topological space.

LEMMA 2. *Let X be a regular second countable topological space. Let $f\colon X \to \mathbb{R}$ be a function such that the set $A(f)$ is nowhere dense and $f(x) \leqq \limsup\limits_{u\to x,\, u\in Q(f)} f(u)$ for each $x \in A(f)$. Then there is a sequence $(K_{n,m})_{m\leqq n}$ of nonempty open sets in X such that*

(i) $\operatorname{Cl} K_{n,m} \cap \operatorname{Cl} A(f) = \emptyset$ *for each $n \in \mathbb{N}$ and $m \leqq n$,*
(ii) $\operatorname{Cl} K_{r,s} \cap \operatorname{Cl} K_{i,j} = \emptyset$ *whenever $(r,s) \neq (i,j)$,*
(iii) *for each $x \in X \setminus \operatorname{Cl} A(f)$ there is a neighbourhood V of x such that the set $\{(n,m):\ V \cap \operatorname{Cl} K_{n,m} \neq \emptyset\}$ is finite,*
(iv) *for each $x \in \operatorname{Cl} A(f)$, for each neighbourhood U of x, and for each $m \in \mathbb{N}$ there is $n \geqq m$ such that $\operatorname{Cl} K_{n,m} \subset U$ and $f(x) - \frac{1}{m} < \sup\{f(u):\ u \in K_{n,m}\}$.*

Proof. Let $(B_n)_n$ be a countable base of open sets in X, and let $(G_n)_n$ be a sequence of all sets in $(B_n)_n$ such that $G_n \cap \operatorname{Cl} A(f) \neq \emptyset$ for $n \in \mathbb{N}$. Let $(E_n)_n$ be a sequence of open sets in X such that $\operatorname{Cl} E_{n+1} \subset E_n$ for each $n \in \mathbb{N}$ and $\bigcap\limits_{n=1}^{\infty} E_n = \operatorname{Cl} A(f)$.

We put $H_{1,1} = (G_1 \cap E_1) \setminus \operatorname{Cl} A(f)$ ($\neq \emptyset$) and $\beta_{1,1} = \min\{1, \sup\{f(t):\ t \in H_{1,1}\} - 1\}$. Let $z_{1,1} \in H_{1,1}$ be such that $f(z_{1,1}) > \beta_{1,1}$, and let $K_{1,1}$ be an open set such that $z_{1,1} \in K_{1,1} \subset \operatorname{Cl} K_{1,1} \subset H_{1,1}$.

Suppose that we have constructed sets $K_{r,s}$ for all $r < n$ ($n > 1$) and $s \leqq r$. Put

$$H_{n,1} = (G_n \cap E_n) \setminus \Big(\operatorname{Cl} A(f) \cup \bigcup_{r<n} \bigcup_{s\leqq r} \operatorname{Cl} K_{r,s}\Big).$$

Since $\operatorname{Cl} K_{r,s} \cap \operatorname{Cl} A(f) = \emptyset$ and $E_n \cap G_n \cap \operatorname{Cl} A(f) \neq \emptyset$, the set $H_{n,1}$ is nonempty open. Put

$$\beta_{n,1} = \min\{n, \sup\{f(t):\ t \in H_{n,1}\} - \tfrac{1}{n}\}.$$

Then there is $z_{n,1} \in H_{n,1}$ such that $f(z_{n,1}) > \beta_{n,1}$. Let $K_{n,1}$ be an open set such that $z_{n,1} \in K_{n,1} \subset \operatorname{Cl} K_{n,1} \subset H_{n,1}$.

Now suppose that we have constructed sets $K_{n,p}$ for all $p < m$ $(1 < m \leqq n)$. Since $\operatorname{Cl} K_{n,p} \cap \operatorname{Cl} A(f) = \emptyset$ for $p < m$, the set

$$H_{n,m} = H_{n,1} \setminus \bigcup_{p<m} \operatorname{Cl} K_{n,p}$$

is nonempty open. Put

$$\beta_{n,m} = \min\{n, \sup\{f(t) : \ t \in H_{n,m}\} - \tfrac{1}{n}\}.$$

Let $z_{n,m} \in H_{n,m}$ be such that $f(z_{n,m}) > \beta_{n,m}$, and let $K_{n,m}$ be an open set such that $z_{n,m} \in K_{n,m} \subset \operatorname{Cl} K_{n,m} \subset H_{n,m}$.

We shall verify that $K_{n,m}$ satisfy (i), (ii), (iii), (iv). Conditions (i) and (ii) are obvious.

(iii): Let $x \in X \setminus \operatorname{Cl} A(f)$. Then there is a neighbourhood V of x and $k \in \mathbb{N}$ such that $V \cap E_k = \emptyset$. Therefore, if $V \cap \operatorname{Cl} K_{n,m} \neq \emptyset$, then $n < k$ and the set $\{(n,m) : \ V \cap \operatorname{Cl} K_{n,m} \neq \emptyset\}$ is finite.

(iv): Let $x \in \operatorname{Cl} A(f)$. Let U be a neighbourhood of x and $m \in \mathbb{N}$. Let $n \in \mathbb{N}$ be such that $n > \max\{m, f(x)\}$ and $x \in G_n \subset U$. Then $\operatorname{Cl} K_{n,m} \subset H_{n,m} \subset U$. According to assumptions and Lemma 1, we have $f(x) \leqq \limsup\limits_{u \to x,\, u \in Q(f)} f(u)$. The set

$J_{n,m} = \Big((G_n \cap E_n) \setminus \bigcup\limits_{r<n} \bigcup\limits_{s \leqq r} \operatorname{Cl} K_{r,s}\Big) \setminus \bigcup\limits_{p<m} \operatorname{Cl} K_{n,p}$ is a neighbourhood of x.

Hence, there is $y \in J_{n,m} \cap Q(f)$ such that $f(x) - \frac{1}{n} < f(y)$. Since $y \in Q(f)$ and the set $A(f)$ is nowhere dense, there is $z \in J_{n,m} \cap \operatorname{Int} Q(f) = H_{n,m}$ such that $f(z) > f(x) - \frac{1}{n}$. Now, if $\sup\{f(t) : \ t \in H_{n,m}\} - \frac{1}{n} \leqq n$, then $f(z) \leqq \beta_{n,m} + \frac{1}{n} < f(z_{n,m}) + \frac{1}{n}$ and $f(x) - \frac{1}{m} < f(x) - \frac{1}{n} < f(x) - \frac{1}{2n} < f(z) - \frac{1}{n} < f(z_{n,m}) \leqq \sup\{f(t) : \ t \in K_{n,m}\}$. If $\sup\{f(t) : \ t \in H_{n,m}\} - \frac{1}{n} > n$, then $f(x) - \frac{1}{m} < n < f(z_{n,m}) \leqq \sup\{f(t) : \ t \in K_{n,m}\}$. □

THEOREM 2. *Let X be a regular second countable topological space. Let $f\colon X \to \mathbb{R}$. Then $f = \max(f_0, f_1)$ for some quasicontinuous functions f_0 and f_1 if and only if the set $A(f)$ is nowhere dense and $f(x) \leqq \limsup\limits_{u \to x,\, u \in Q(f)} f(u)$ for each $x \in A(f)$.*

P r o o f. Let $f = \max(f_0, f_1)$, where $f_0, f_1 \in \mathcal{Q}$. Then $A(f)$ is nowhere dense by [5; Lemma 2] and $f(x) \leqq \limsup\limits_{u \to x,\, u \in Q(f)} f(u)$ for each $x \in A(f)$ by [5; Lemma 3].

Now, let $f\colon X \to \mathbb{R}$ be such that $A(f)$ is nowhere dense and $f(x) \leqq \limsup\limits_{u \to x,\, u \in Q(f)} f(u)$ for each $x \in A(f)$. Then, by Lemma 2, there is a sequence of open sets $(K_{n,m})_{m \leqq n}$ satisfying (i)–(iv). Let $\mathbb{Q} = \{q_1, q_2, \dots\}$ be a one-to-one sequence of all rationals. For $i \in \{0,1\}$ we define functions $f_i \colon X \to \mathbb{R}$ as follows:

$$f_i(x) = \begin{cases} \min\{f(x), q_m\} & \text{if there is } m \in \mathbb{N} \text{ and } n \geqq 2m - i \text{ such that} \\ & x \in \operatorname{Cl} K_{n,2m-i} \text{ and} \\ & f(x) \in \bigcap\limits_{U \in \mathcal{U}_x} \operatorname{Cl} f(U \cap K_{n,2m-i}), \\ f(x) & \text{otherwise.} \end{cases}$$

Then evidently $f = \max(f_0, f_1)$. We shall verify that f_i are quasicontinuous. Let $i \in \{0,1\}$. Let $x \in X$, let U be a neighbourhood of x, and let $\varepsilon > 0$. We have four possibilities.

a) Suppose that $x \in X \setminus \Big(\operatorname{Cl} A(f) \cup \bigcup\limits_{m \in \mathbb{N}} \bigcup\limits_{n \geqq 2m-i} \operatorname{Cl} K_{n,2m-i}\Big)$. Then, with respect to (iii), there is a neighbourhood $V \subset U$ of x such that $f_i(t) = f(t)$ for each $t \in V$, and the quasicontinuity of f at x yields $x \in Q(f_i)$.

b) Suppose that $x \in \operatorname{Cl} K_{n,2m-i}$ for some $m \in \mathbb{N}$ and $n \geqq 2m - i$ and $f(x) \notin \bigcap\limits_{U \in \mathcal{U}_x} \operatorname{Cl} f(U \cap K_{n,2m-i})$. Then there are $0 < \delta < \varepsilon$ and $W \subset U$, $W \in \mathcal{U}_x$ such that $\big(f(x)-\delta, f(x)+\delta\big) \cap \operatorname{Cl} f(W \cap K_{n,2m-i}) = \emptyset$. Then, with respect to (iii), there is a neighbourhood $V \subset W$ of x such that $V \cap \operatorname{Cl} K_{r,s} = \emptyset$ for $(r,s) \neq (n, 2m-i)$. Since $x \in Q(f)$, there is an open nonempty set $G \subset V$ such that $|f(t) - f(x)| < \delta$ for each $t \in G$. This implies $G \cap K_{n,2m-i} = \emptyset$ (so $G \cap \operatorname{Cl} K_{n,2m-i} = \emptyset$), and hence $|f_i(t) - f_i(x)| = |f(t) - f(x)| < \delta < \varepsilon$ for each $t \in G$, i.e., $x \in Q(f_i)$.

c) Suppose that $x \in \operatorname{Cl} K_{n,2m-i}$ for some $m \in \mathbb{N}$ and $n \geqq 2m - i$, and $f(x) \in \bigcap\limits_{U \in \mathcal{U}_x} \operatorname{Cl} f(U \cap K_{n,2m-i})$. Then there is $y \in U \cap K_{n,2m-i}$ such that $|f(x) - f(y)| < \frac{\varepsilon}{2}$. Since $y \in Q(f)$, there is an open nonempty $G \subset U \cap K_{n,2m-i}$ such that $|f(t) - f(y)| < \frac{\varepsilon}{2}$ for each $t \in G$. Then clearly $\big|\min\big(f(t), q_m\big) - \min\big(f(x), q_m\big)\big| \leqq |f(t) - f(x)| < \varepsilon$, so $|f_i(t) - f_i(x)| < \varepsilon$ for each $t \in G$, and thus $x \in Q(f_i)$.

d) Suppose that $x \in \operatorname{Cl} A(f)$. Let $m \in \mathbb{N}$ be such that $\frac{1}{m} < \varepsilon$ and $|f(x) - q_m| < \varepsilon$. With respect to (iv), there is $n \geqq 2m - i$ such that $\operatorname{Cl} K_{n,2m-i} \subset U$ and $f(x) - \frac{1}{2m-i} < \sup\{f(u) : u \in K_{n,2m-i}\}$. Choose $y \in K_{n,2m-i}$ with $f(x) - \frac{1}{2m-i} < f(y)$. Since $y \in Q(f)$, there is an open nonempty $G \subset K_{n,2m-i}$ such that $f(t) > f(x) - \frac{1}{2m-i}$ for each $t \in G$. Thus, for $t \in G$ we have $f(x) - \varepsilon < f(x) - \frac{1}{2m-i} < f(t)$, $f(x) - \varepsilon < q_m$, and hence $f(x) - \varepsilon < \min\{f(t), q_m\} = f_i(t)$. Further, we have $q_m < f(x) + \varepsilon$, and hence $f_i(t) < f(x) + \varepsilon$. Therefore, we obtain $|f_i(t) - f_i(x)| < \varepsilon$ and $x \in Q(f_i)$. $\square$

Remark 4. The assumption "X is regular second countable" cannot be replaced with "X is normal second countable". If $X = \mathbb{R}$ with the topology $\mathcal{T}$, where $A \in \mathcal{T}$ if and only if $A = \emptyset$ or $A = (a, \infty)$ $(a \in \mathbb{R})$, then every quasicontinuous function on X is constant, but there are nonconstant functions satisfying assumptions of Theorem 2 (e.g., $f(x) = 0$ for $x \leqq 0$ and $f(x) = 1$ for $x > 0$).

PROBLEM 2. Is Theorem 2 true for an arbitrary metric space X?

THEOREM 2'. *Let X be a regular second countable topological space, and let $f\colon X \to \mathbb{R}$. Then $f = \min(f_0, f_1)$ for some $f_0, f_1 \in \mathcal{Q}$ if and only if the set $A(f)$ is nowhere dense and $f(x) \geqq \liminf_{u \to x, u \in Q(f)} f(u)$ for each $x \in A(f)$.*

T. Natkaniec has shown in [5] that if $X' = X \setminus X_0$ is a regular second countable space without isolated points (where X_0 is the set of all isolated points of X), then $L(\mathcal{Q}) = \mathcal{Q}^*$, where Q^* is the family of all functions for which the set $A(f)$ is nowhere dense.

One can see (similarly as in the proof of our Lemma 2) that his Lemma 2 is true for an arbitrary regular second countable space. Hence, from his proof, we see that $L(\mathcal{Q}) = \mathcal{Q}^*$, if X' is a regular second countable space; especially, if X is a regular second countable topological space.

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REFERENCES

[1] BISWAS, N.: *On some mappings in topological spaces*, Calcutta Math. Soc. **61** (1969), 127–135.

[2] BORSÍK, J.—DOBOŠ, J.: *A note on real cliquish functions*, Real Anal. Exchange **18** (1992-93), 139–145.

[3] DOBOŠ, J.: *Simple continuity and cliquishness*, Časopis Pěst. Mat. **112** (1987), 355–358.

[4] HAWORTH, R. C.—MCCOY, R. A.: *Baire spaces*, Dissertationes Math. (Rozprawy Mat.) **141** (1977).

[5] NATKANIEC, T.: *On the maximum and the minimum of quasi-continuous functions*, Math. Slovaca **42** (1992), 103–110.

[6] NEUBRUNN, T.: *Quasi-continuity*, Real Anal. Exchange **14** (1988-89), 259–306.

[7] NEUBRUNNOVÁ, A.: *On certain generalizations of the notion of continuity*, Mat. Časopis **23** (1973), 374–384.

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