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NORMAL AUTOMETRIZED LATTICE ORDERED ALGEBRAS

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ABSTRACT. Results proved for normal autometrized lattice ordered algebras under the assumption of semiregularity are shown to be valid without this assumption.

Autometrized algebras were introduced by Swamy (cf. [6]) as an attempt to obtain a unified theory of abelian lattice ordered groups and Brouwerian algebras. Swamy and Rao (cf. [7]) studied the concept of an autometrized lattice ordered algebra.

Swamy and Rao (cf. [7]) remarked that the notion of an autometrized algebra is too general and they introduced the notions of a normal autometrized algebra and a semiregular autometrized algebra. This work was continued by Hansen (cf. [1] and [2]) and Rachůnek (cf. [3], [4] and [5]).

In this paper we show that several results which were proved in the above quoted papers under the assumption of semiregularity can be proved without this assumption. We also give a characterization of an ideal of a normal autometrized lattice ordered algebra.

An algebra $A = (A; 0; +; \wedge; \vee; *)$ of type $\langle 0; 2; 2; 2; 2\rangle$ is a normal autometrized lattice ordered algebra (abbreviated, NAℓ-algebra) if the following holds (cf. [6; Definition 1] and [7; Definition 1]):

(i) $(A; 0; +; \leq)$ is an abelian lattice ordered monoid, i.e.

- (a) $(A; 0; +)$ is an abelian monoid,
- (b) $(A; \wedge; \vee)$ is a lattice (the induced order is denoted by $\leq$),
- (c) $x + (y \wedge z) = (x + y) \wedge (x + z)$ for all $x, y, z \in A$,
- (d) $x + (y \vee z) = (x + y) \vee (x + z)$ for all $x, y, z \in A$,

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(ii) $*$ is a metric operation, i.e.

(a) $x * y \geq 0$ and $x * y = 0$ if and only if $x = y$ for all $x, y \in A$,

(b) $x * y = y * x$ for all $x, y \in A$,

(c) $x * y \leq (x * z) + (z * y)$ for all $x, y, z \in A$,

(iii) $x * 0 \geq x$ for each $x \in A$,

(iv) $(x + y) * (x' + y') \leq (x * x') + (y * y')$ for all $x, y, x', y' \in A$,

(v) $(x * y) * (x' * y') \leq (x * x') + (y * y')$ for all $x, y, x', y' \in A$,

(vi) $x, y \in A$ and $x \leq y$ imply there exists $z \in A$ such that $x + z = y$.

An NAℓ-algebra A is semiregular if the following holds (cf. [7; Definition 5]):

(vii) $x \geq 0$ implies $x * 0 = x$ for each $x \in A$.

In what follows A stands for an NAℓ-algebra.

A subset $I \subseteq A$ is an ideal of A if the following holds (cf. [7; Definition 2]):

(i) $0 \in I$,

(ii) $x, y \in I$ implies $(x + y) \in I$,

(iii) $x \in A$, $y \in I$ and $x * 0 \leq y * 0$ imply $x \in I$.

The set of all ideals of A ordered by set inclusion is a complete algebraic lattice $\mathbf{I}(A)$ (cf. [7; Theorem 1]). In this lattice, $I \wedge J = I \cap J$ (cf. [7; Lemma 1]) and $I \vee J = \{z \in A \mid z * 0 \leq x + y$ for some $x \in I$ and $y \in J\}$ (cf. [7; Corollary 1]). An ideal generated by a set $B \subseteq A$ is denoted by $I(B)$ and an ideal generated by a singleton $\{x\} \subseteq A$ is denoted by $I(x)$. Furthermore, $I(x) = \{y \in A \mid y * 0 \leq n(x * 0)$ for some natural $n\}$ (cf. [7; Lemma 2]).

A finitely meet-irreducible ideal $I \in \mathbf{I}(A)$ is a prime ideal (cf. [3]). The set of all prime ideals of A ordered by set inclusion is denoted by $\mathbf{I}_P(A)$.

Elements $x, y \in A$ are orthogonal, $x \perp y$, if $(x * 0) \wedge (y * 0) = 0$ (cf. [4]). If $B \subseteq A$, then $B^{\perp} = \{x \in A \mid x \perp y$ for all $y \in B\}$ is the polar of the set B. The polar of a singleton $\{x\} \subseteq A$ is denoted by $x^{\perp}$. A set $C \subseteq A$ is a polar if there exists the set $B \subseteq A$ such that $C = B^{\perp}$. The set of all polars in A ordered by set inclusion is denoted by $\mathbf{P}(A)$.

The set of all (additively) invertible elements of A endowed by $+$, $\wedge$ and $\vee$ is denoted by $\mathrm{In}(A)$ and the set of all (additively) idempotent elements of A endowed by $+$, $\wedge$ and $\vee$ is denoted by $\mathrm{Id}(A)$.

1. THEOREM. $\mathrm{In}(A)$ *is an abelian lattice ordered group.*

Proof. Clear. □

2. THEOREM. $x \in \mathrm{In}(A)$, $y \in A$ *and* $y \leq x$ *imply* $y \in \mathrm{In}(A)$.

Proof. Since $y + (-x) \leq 0$ therefore there exists $z \in A$ such that $y + (-x) + z = 0$. Hence $y \in \mathrm{In}(A)$. □

3. Theorem. *$\mathrm{Id}(A)$ is an abelian lattice ordered monoid. Moreover, in $\mathrm{Id}(A)$, the following holds:*

(i) $x \geq 0$,
(ii) $x + y = x \vee y$.

Proof. Assume that $x, y \in \mathrm{Id}(A)$. Since $(x \wedge 0) + (x \wedge 0) = (x + x) \wedge (x + 0) \wedge (0 + x) \wedge (0 + 0) = x \wedge 0$ therefore $(x \wedge 0) \in \mathrm{Id}(A)$. Since $x \wedge 0 \leq 0$ therefore Theorem 2 implies $(x \wedge 0) \in \mathrm{In}(A)$ and thus $x \wedge 0 = 0$. Hence $x \geq 0$.

Clearly, $0 \in \mathrm{Id}(A)$. Since $(x+y)+(x+y) = (x+x)+(y+y) = x+y$ therefore $(x+y) \in \mathrm{Id}(A)$. Since $(x \wedge y)+(x \wedge y) = (x+x) \wedge (x+y) \wedge (y+x) \wedge (y+y) = x \wedge y$ therefore $(x \wedge y) \in \mathrm{Id}(A)$. Since $x \leq x \vee y$ and $y \leq x \vee y$ therefore there exist $x_1 \in A$ and $y_1 \in A$ such that $x + x_1 = x \vee y$ and $y + y_1 = x \vee y$. Then $(x \vee y) + (x \vee y) = \big[x + (x \vee y)\big] \vee \big[y + (x \vee y)\big] = (x + x + x_1) \vee (y + y + y_1) = (x + x_1) \vee (y + y_1) = x \vee y \vee x \vee y = x \vee y$ and therefore $(x \vee y) \in \mathrm{Id}(A)$. Hence $\mathrm{Id}(A)$ is an abelian lattice ordered monoid.

Finally, $x + y \leq (x \vee y) + (x \vee y) = x \vee y \leq x + y$. Hence $x + y = x \vee y$. □

4. Lemma. *For $x, y \in A$ and $z \in \mathrm{In}(A)$, the following holds:*

(i) $x * 0 \geq x \vee 0$,
(ii) $x * y = (x + z) * (y + z)$.

Proof.

(i) Clear.

(ii) It follows from $x * y = \big[x + z + (-z)\big] * \big[y + z + (-z)\big] \leq \big[(x + z) * (y + z)\big] + \big[(-z) * (-z)\big] = (x + z) * (y + z) \leq (x * y) + (z * z) = x * y$. □

In view of (ii) of Lemma 4 we observe that any mapping $f\colon A \to A$, $f(x) = x + y$, where $y \in \mathrm{In}(A)$ is a fixed element, is an isometry of A, i.e. a surjective and distance preserving mapping.

5. Lemma. *For $x \in A$ and $I \in \mathbf{I}(A)$, the following holds:*

(i) $x \in I$ *if and only if* $(x * 0) \in I$,
(ii) $I(x) = I(x * 0)$,
(iii) $x \in \mathrm{In}(A)$ *implies* $I(x) = I(-x) = I(x * 0)$.

Proof.

(i) In view of [7; Lemma 5], we obtain $(x * 0) * 0 = x * 0$, which yields the assertion.

(ii) It follows from (i).

(iii) In view of (ii) of Lemma 4, we obtain $x * 0 = \big[x + (-x)\big] * \big[0 + (-x)\big] = 0 * (-x) = (-x) * 0$ and (ii) yields $I(x) = I(x * 0) = I\big((-x) * 0\big) = I(-x)$. □

6. THEOREM. (*cf.* [2; Proposition 3]) *A subset $I \subseteq A$ is an ideal of A if and only if the following holds:*

(i) *I is a sub-NAℓ-algebra of A,*
(ii) *I is a convex subset of A,*
(iii) *$(x * 0) \in I$ implies $x \in I$ for each $x \in A$.*

Proof. Assume that $I \in \mathbf{I}(A)$. In view of [7; Theorem 4], there exist a normal autometrized algebra B (cf. [6; Definition 1] and [7; Definition 1]) and a homomorphism $f\colon A \to B$ (cf. [7; Definition 4]) such that $I = \ker(f) = \{x \in A \mid f(x) = 0\}$. If $x, y \in I$, then $f(x+y) = f(x) + f(y) = 0 + 0 = 0$ and $f(x*y) = f(x) * f(y) = 0 * 0 = 0$, i.e. $(x+y) \in I$ and $(x*y) \in I$. Since $x \le x \vee y \le (x*0)+(y*0)$ therefore $0 = f(x) \le f(x \vee y) \le f\big((x*0)+(y*0)\big) = 0$, i.e. $(x \vee y) \in I$. Since in any abelian lattice ordered monoid the identity $x + y = (x \wedge y) + (x \vee y)$ holds therefore $f(x \wedge y) = f(x \wedge y) + f(x \vee y) = f\big((x\wedge y)+(x\vee y)\big) = f(x+y) = 0$, i.e. $(x \wedge y) \in I$. If $x \le y$, then there exists $z \in A$ such that $x + z = y$ and thus $f(z) = f(x) + f(z) = f(x+z) = f(y) = 0$, i.e. $z \in I$. Hence I is a NAℓ-algebra.

If $x \le z \le y$ and $z \in A$, then $0 = f(x) \le f(z) \le f(y) = 0$, i.e. $z \in I$. Hence I is a convex subset.

If $(x*0) \in I$, then $f(x)*0 = f(x)*f(0) = f(x*0) = 0$, i.e. $f(x) = 0$. Hence $x \in I$.

Conversely, assume that $I \subseteq A$ satisfies the conditions (i), (ii) and (iii) and $x, y \in I$. Obviously, $0 \in I$ and $(x+y) \in I$. If $z \in A$ and $z * 0 \le x * 0$, then in view of (ii), we observe $(z*0) \in I$, and (iii) implies $z \in I$. Hence I is an ideal. □

7. LEMMA. (*cf.* [3; Propositions 2, 3]) *For $x, y \in A$, the following holds:*

(i) *$I(x) \cap I(y) = I\big((x*0) \wedge (y*0)\big)$,*
(ii) *$I(x) \vee I(y) = I\big((x*0) \vee (y*0)\big) = I\big((x*0) + (y*0)\big)$,*
(iii) *$x \ge 0$ and $y \ge 0$ imply $I(x) \vee I(y) = I(x \vee y) = I(x+y)$.*

Proof.

(i) In view of (ii) of Lemma 5, (ii) of Theorem 6 and $0 \le (x*0) \wedge (y*0) \le (x*0), (y*0)$, we obtain $I\big((x*0)\wedge(y*0)\big) \subseteq I(x*0) \cap I(y*0) = I(x) \cap I(y)$. Conversely, if $z \in I(x) \cap I(y)$, then there exist natural numbers n and m such that $z*0 \le n(x*0)$ and $z*0 \le m(y*0)$. In view of [1; Lemma 1.2], we obtain $z*0 \le [n(x*0)] \wedge [m(y*0)] \le nm[(x*0)\wedge(y*0)]$, i.e. $z \in I\big((x*0)\wedge(y*0)\big)$. Hence $I(x) \cap I(y) \subseteq I\big((x*0)\wedge(y*0)\big)$.

(ii) In view of (ii) of Lemma 5, (ii) of Theorem 6 and $0 \le (x*0), (y*0) < (x*0) \vee (y*0) \le (x*0)+(y*0)$ we obtain $I(x) \vee I(y) = I(x*0) \vee I(y*0) \subseteq I\big((x*0) \vee (y*0)\big) \subseteq I\big((x*0)+(y*0)\big)$. Conversely, if $z \in I\big((x*0)+(y*0)\big)$, then there exist a natural number n such that $z*0 \le n\big[(x*0)+(y*0)\big] = n(x*0)+n(y*0)$, i.e. $z \in I(x) \vee I(y)$. Hence $I\big((x*0)+(y*0)\big) \subseteq I(x) \vee I(y)$.

(iii) In view of (ii) of Theorem 6 and $0 \le x, y \le x \vee y \le x+y$ we obtain $I(x) \vee I(y) = I(x \vee y) \subseteq I(x+y)$. Conversely, if $z \in I(x+y)$, then there exist a natural number n such that $z*0 \le n\big[(x+y)*0\big] = n\big[(x+y)*(0+0)\big] \le n\big[(x*0) + (y*0)\big] = n(x*0)+n(y*0)$, i.e. $z \in I(x) \vee I(y)$. Hence $I(x+y) \subseteq I(x) \vee I(y)$. □

8. THEOREM. (*cf.* [7; Lemma 6, Theorem 6]) *The following holds:*

(i) $\mathbf{I}(A)$ *is an algebraic lattice,*
(ii) $\mathbf{I}(A)$ *is a complete lattice,*
(iii) $\mathbf{I}(A)$ *is a distributive lattice,*
(iv) $\mathbf{I}(A)$ *is a Brouwerian lattice,*
(v) $\mathbf{I}(A)$ *is a pseudocomplemented lattice.*

Proof.

(i), (ii) Cf. [7; Theorem 1].

(iii) Assume that $I, J, K \in \mathbf{I}(A)$ and $u \in I \cap (J \vee K)$. There exist $x \in I$, $y \in J$ and $z \in K$ such that $u*0 \le x \le x*0$ and $u*0 \le y+z \le (y*0)+(z*0)$. In view of [1; Lemma 1.2], we obtain $u*0 \le (x*0) \wedge \big[(y*0)+(z*0)\big] \le \big[(x*0) \wedge (y*0)\big] + \big[(x*0) \wedge (z*0)\big]$, and (i) of Lemma 7 yields $u \in I\big((x*0) \wedge (y*0)\big) \vee I\big((x*0) \wedge (z*0)\big) = \big[I(x) \cap I(y)\big] \vee \big[I(x) \cap I(z)\big] \subseteq (I \cap J) \vee (I \cap K)$. Hence $I \cap (J \vee K) \subseteq (I \cap J) \vee (I \cap K)$. The rest is clear.

(iv) It follows from (i) and (ii).

(v) It follows from (iv). □

9. THEOREM. (*cf.* [3; Theorem 4]) *For $I \in \mathbf{I}(A)$, the following are equivalent:*

(i) $I \in \mathbf{I}_P(A)$,
(ii) $J \cap K \subseteq I$ *implies* $J \subseteq I$ *or* $K \subseteq I$ *for all* $J, K \in \mathbf{I}(A)$,
(iii) $(x*0) \wedge (y*0) \in I$ *implies* $x \in I$ *or* $y \in I$ *for all* $x, y \in A$.

Proof.

(i) $\Longrightarrow$ (ii) Assume that $J \cap K \subseteq I$. In view of (iii) of Theorem 8, we obtain $I = I \vee (J \cap K) = (I \vee J) \cap (I \vee K)$ and therefore $I = I \vee J$ or $I = I \vee K$. Hence $J \subseteq I$ or $K \subseteq I$.

(ii) $\Longrightarrow$ (iii) Assume that $(x*0) \wedge (y*0) \in I$. In view of (i) of Lemma 7, we obtain $I(x) \cap I(y) = I\big((x*0) \wedge (y*0)\big) \subseteq I$ and therefore $I(x) \subseteq I$ or $I(y) \subseteq I$. Hence $x \in I$ or $y \in I$.

(iii) $\Longrightarrow$ (i) Assume that $I = J \cap K$, $I \ne J$ and $y \in K$. There exists $x \in J \backslash I$. In view of (i) of Lemma 7, we obtain $(x*0) \wedge (y*0) \in I\big((x*0) \wedge (y*0)\big) = I(x) \cap I(y) \subseteq J \cap K = I$. Since $x \notin I$ therefore $y \in I$. Hence $K = I$. □

10. THEOREM. (*cf.* [3; Theorem 8]) *If $\{I_\lambda\}_{\lambda\in\Lambda}$ is a totally ordered system of prime ideals of A, then $I = \bigcap\{I_\lambda\}_{\lambda\in\Lambda}$ is a prime ideal of A.*

Proof. Assume that $x, y \in A$, $(x*0)\wedge(y*0) \in I$ and $x \notin I$. There exists $\lambda_0 \in \Lambda$ such that $x \notin I_\lambda$ for each $\lambda \in \Lambda$, $\lambda \geq \lambda_0$. In view of (iii) of Theorem 9, we observe that $y \in I_\lambda$ for each $\lambda \in \Lambda$, $\lambda \geq \lambda_0$, i.e. $y \in I$. Hence $I \in \mathbf{I}_P(A)$. □

11. COROLLARY. (*cf.* [3; Corollary 9]) *Each prime ideal contains a minimal prime ideal.*

12. THEOREM. *$I \in \mathbf{I}_P(A)$ and $x \in A \setminus I$ imply $x^\perp \subseteq I$.*

Proof. Assume that $y \in x^\perp$. Then $(x*0)\wedge(y*0) = 0$ and in view of (i) of Lemma 7, we obtain $I(x)\cap I(y) = I\big((x*0)\wedge(y*0)\big) = I(0) = \{0\} \subseteq I$. Since $I(x) \nsubseteq I$ therefore (ii) of Theorem 9 yields $y \in I(y) \subseteq I$. Hence $x^\perp \subseteq I$. □

13. THEOREM. (*cf.* [4; Corollary of Theorem 6]) *For $B, C \subseteq A$, the following holds:*

(i) *$B \subseteq C$ implies $C^\perp \subseteq B^\perp$,*
(ii) *$B \subseteq B^{\perp\perp}$,*
(iii) *$B^\perp = B^{\perp\perp\perp}$,*
(iv) *$B^\perp \cap B^{\perp\perp} = \{0\}$,*
(v) *B is a polar if and only if $B = B^{\perp\perp}$,*
(vi) *$B \subseteq C^\perp$ if and only if $C \subseteq B^\perp$.*

Proof.
(i) Clear.
(ii) If $x \in B$, then $x \perp y$ for each $y \in B^\perp$. Hence $x \in B^{\perp\perp}$.
(iii) It follows from (i) and (ii).
(iv) Since $0 \perp x$ for each $x \in A$ we conclude that $0 \in B^\perp \cap B^{\perp\perp}$. If $x \in B^\perp \cap B^{\perp\perp}$, then $x*0 = (x*0)\wedge(x*0) = 0$. Hence $x = 0$.
(v) If B is a polar, then $B = C^\perp$ for some $C \subseteq A$ and (iii) yields $B = C^\perp = C^{\perp\perp\perp} = B^{\perp\perp}$. Conversely, if $B = B^{\perp\perp}$, then $B = C^\perp$, where $C = B^\perp$.
(vi) It follows from (i) and (ii). □

14. THEOREM. (*cf.* [4; Theorem 5]) *$B \subseteq A$ implies $B^\perp = \cap\{I \in \mathbf{I}_P(A) \mid B \nsubseteq I\}$.*

Proof. Denote $C = \{I \in \mathbf{I}_P(A) \mid\ B \nsubseteq I\}$. Assume that $y \in B^\perp$, $I \in C$ and $x \in B \setminus I$. Then $(x*0)\wedge(y*0) = 0$ and in view of (iii) of Theorem 9, we obtain $y \in I$. Hence $B^\perp \subseteq \cap C$.

Conversely, assume that $y \notin B^\perp$, i.e. there exists $x \in B$ such that $(x*0) \wedge (y*0) > 0$. In view of [4; Theorem 4], there exists $I \in \mathbf{I}_P(A)$ such that

$((x*0)\wedge(y*0)) \notin I$ and (i) and (ii) of Theorem 6 yield $x \notin I$ and $y \notin I$. Since $x \in B \setminus I$ therefore $I \in C$ and $y \notin I$ implies $y \notin \cap C$. Hence $\bigcap C \subseteq B^{\perp}$. □

15. COROLLARY. (*cf.* [4; Corollary of Theorem 5] *and* [7; Lemma 7]) *Each polar in A is an ideal of A.*

16. THEOREM. (*cf.* [2; Lemma 5] *and* [4; Theorem 2]) *$B \subseteq A$ implies $B^{\perp} = \{x \in A \mid\ I(x) \cap I(B) = \{0\}\}$.*

P r o o f. It is well known that the identity $x \wedge \Big(\bigvee\limits_{\lambda\in\Lambda} y_\lambda\Big) = \bigvee\limits_{\lambda\in\Lambda}(x\wedge y_\lambda)$ holds in any Brouwerian lattice. Assume that $x \in A$. In view of (i) of Lemma 7 and Theorem 8, we obtain $I(x)\cap I(B) = I(x)\cap\Big(\bigvee\limits_{y\in B} I(y)\Big) = \bigvee\limits_{y\in B}\big(I(x)\cap I(y)\big) = \bigvee\limits_{y\in B} I\big((x*0)\wedge(y*0)\big)$. From this we observe that $I(x)\cap I(B) = \{0\}$ if and only if $(x*0)\wedge(y*0) = 0$ for all $y \in B$. Hence $I(x)\cap I(B) = \{0\}$ if and only if $x \in B^{\perp}$. □

17. COROLLARY. (*cf.* [2; Lemma 4] *and* [4; Corollary of Theorem 2]) *$B \subseteq A$ implies $B^{\perp} = I(B)^{\perp}$. Hence any polar in A is the polar of an ideal.*

18. THEOREM. (*cf.* [7; Lemma 7]) *For each $I \in \mathbf{I}(A)$, $I^{\perp}$ is the pseudocomplement of I in $\mathbf{I}(A)$.*

P r o o f. In view of (ii) and (iv) of Theorem 13, we obtain $I \cap I^{\perp} = \{0\}$. Assume that $J \in \mathbf{I}(A)$, $I\cap J = \{0\}$ and $x \in J$. Then $I \cap I(x) \subseteq I \cap J = \{0\}$ and Theorem 16 yields $x \in I^{\perp}$. Hence $J \subseteq I^{\perp}$. □

19. THEOREM. (*cf.* [4; Theorem 8] *and* [7; Theorem 7]) *$\mathbf{P}(A)$ is a complete Boolean algebra when equipped with the meet $\cap$, a new join operation $\vee'$ defined by $I \vee' J = (I\vee J)^{\perp\perp}$ and complementation ${}^{\perp}$. The mapping $\Phi\colon \mathbf{I}(A) \to \mathbf{P}(A)$ defined by $\Phi(I) = I^{\perp\perp}$ is a lattice epimorphism.*

P r o o f. It is well known to be valid in any Brouwerian lattice. □

20. THEOREM. (*cf.* [4, Theorems 6, 7]) *The following holds:*

(i) $\bigcap\limits_{\lambda\in\Lambda} B_\lambda^{\perp} = \Big(\bigcup\limits_{\lambda\in\Lambda} B_\lambda\Big)^{\perp}$ *for all $B_\lambda \subseteq A$, where $\lambda \in \Lambda$,*

(ii) $\bigcap\limits_{\lambda\in\Lambda} B_\lambda^{\perp} = \Big(\bigvee\limits_{\lambda\in\Lambda} B_\lambda\Big)^{\perp}$ *for all $B_\lambda \in \mathbf{I}(A)$, where $\lambda \in \Lambda$,*

(iii) $\bigcap\limits_{\lambda\in\Lambda} B_\lambda = \Big(\bigvee\limits_{\lambda\in\Lambda} B_\lambda^{\perp}\Big)^{\perp}$ *for all $B_\lambda \in \mathbf{P}(A)$, where $\lambda \in \Lambda$,*

(iv) $\Big(\bigcap\limits_{\lambda\in\Lambda} B_\lambda^{\perp}\Big)^{\perp} = \bigvee\limits_{\lambda\in\Lambda}{}' B_\lambda$ *for all $B_\lambda \in \mathbf{P}(A)$, where $\lambda \in \Lambda$.*

Proof. It is omitted since it is basically a theorem about Brouwerian lattices. □

REFERENCES

[1] HANSEN, M.: *Minimal prime ideals in autometrized algebras*, Czechoslovak Math. J. **44** (1994), 81–90.

[2] HANSEN, M.: *Filets and z-ideals in autometrized algebras*, Czechoslovak Math. J. (To appear).

[3] RACHŮNEK, J.: *Prime ideals in autometrized algebras*, Czechoslovak Math. J. **37** (1987), 65–69.

[4] RACHŮNEK, J.: *Polars in autometrized algebras*, Czechoslovak Math. J. **39** (1989), 681–685.

[5] RACHŮNEK, J.: *Regular ideals in autometrized algebras*, Math. Slovaca **40** (1990), 117–122.

[6] SWAMY, K. L. N.: *A general theory of autometrized algebras*, Math. Ann. **157** (1964), 65–74.

[7] SWAMY, K. L. N.—RAO, N. P.: *Ideals in autometrized algebras*, J. Austral. Math. Soc. **24** (1977), 362–374.

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