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OSCILLATION CRITERIA FOR SECOND ORDER NONLINEAR RETARDED DIFFERENTIAL EQUATIONS

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ABSTRACT. In this paper the oscillatory behaviour of nonlinear delay differential equation of the form

$$\left(|u'(t)|^{\alpha-1}u'(t)\right)' + p(t)f\left[u\left(\tau(t)\right)\right] = 0$$

is investigated. Some new oscillatory criteria are given.

1. Introduction

In the recent papers [1]–[6], [8]–[10], [12]–[14], the oscillatory and asymptotic properties of various types of differential equations

$$\left(|u'(t)|^{\alpha-1}u'(t)\right)' + p(t)f\left[u\left(\tau(t)\right)\right] = 0 \tag{1}$$

have been considered. In this paper we shall study those properties under the following hypotheses (H1)–(H4):

(H1) $\alpha > 0$ is a real constant;
(H2) $p \in C[t_0, \infty)$, $p(t) > 0$;
(H3) $\tau \in C^1[t_0, \infty)$, $\tau'(t) > 0$, $\tau(t) \le t$, $\lim_{t\to\infty} \tau(t) = \infty$;
(H4) $f \in C(-\infty, \infty)$, f is nondecreasing on $(-\infty, \infty)$, $f \in C^1(M)$, $M = (-\infty, 0) \cup (0, \infty)$, $uf(u) > 0$ for $u \neq 0$.

By a solution of (1) we mean a function $u \in C^1[T_u, \infty)$, $T_u \ge t_0$, which has the property $|u'(t)|^{\alpha-1}u'(t) \in C^1[T_u, \infty)$ and satisfies (1) on $[T_u, \infty)$. We consider only those solutions of (1) that satisfy $\sup\{|u(t)| : t \ge T\} > 0$ for

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all $T \geq T_u$. We assume that (1) possesses such a solution. A nontrivial solution of (1) is said to be *oscillatory* if it has arbitrarily large zeros: otherwise it is said to be *nonoscillatory*. Equation (1) is called oscillatory if all its solutions are oscillatory. It is known that the condition $\int^{\infty} p(s)\,\mathrm{d}s = \infty$ is enough for oscillation of (1). In this paper, we are concerned with the case when $\int^{\infty} p(s)\,\mathrm{d}s < \infty$. The aim of this paper is to present some new oscillatory criteria, which are new also for $\alpha = 1$, namely, for the second order nonlinear differential equation

$$u''(t) + p(t)f\big[u(\tau(t))\big] = 0 .$$

Some comparison with existing results is also included. As is customary, all functional inequalities are assumed to hold eventually, that is, they are satisfied for all sufficiently large t.

2. Main results

THEOREM 2.1. *Let* $\alpha \geq 1$. *Let* $f'(u)$ *be nondecreasing on* $(-\infty, 0)$ *and nonincreasing on* $(0, \infty)$. *Further assume that*

$$\int^{\infty} p(s)\big|f\big[c\tau(s)\big]\big|\,\mathrm{d}s = \infty \qquad \text{for all} \quad c \neq 0 \tag{2}$$

and moreover,

$$\int^{\infty} \left(\tau^{\alpha}(s)p(s) - \frac{\alpha^2 \tau^{\alpha-2}(s)\tau'(s)}{4f'\big[\pm\lambda\tau(s)\big]} \right) \mathrm{d}s = \infty \qquad \text{for some} \quad \lambda > 0 . \tag{3}$$

Then equation (1) *is oscillatory.*

Proof. Assume the converse and suppose that equation (1) possesses an eventually positive solution $u(t)$. The case $u(t) < 0$ can be treated similarly. Then

$$\big(|u'(t)|^{\alpha-1}u'(t)\big)' = -p(t)f\big[u(\tau(t))\big] < 0 .$$

Hence, the function $|u'(t)|^{\alpha-1}u'(t)$ is decreasing. Therefore, either

(i) $u'(t) > 0$, eventually

or

(ii) $u'(t) < 0$, eventually.

Since

$$0 > \big(|u'(t)|^{\alpha-1}u'(t)\big)' = \alpha|u'(t)|^{\alpha-1}u''(t) ,$$

we see that $u''(t) < 0$. Then the case $u'(t) < 0$ yields $u(t) \to -\infty$ as $t \to \infty$. This is a contradiction. Therefore, we conclude that $u(t) > 0$, $u'(t) > 0$, $u''(t) < 0$, eventually and

$$\left[\left(u'(t)\right)^{\alpha}\right]' = -p(t) f\left[u\left(\tau(t)\right)\right]. \tag{4}$$

We claim that $u'(t) \to 0$ as $t \to \infty$. To prove it, assume the converse. Then $u'(t) \to 2c$ as $t \to \infty$, $c > 0$. The monotonicity of $u'(t)$ implies $u'(t) \geq 2c$. Integrating the last inequality from t_1 to t, we have

$$u(t) \geq u(t_1) + 2c(t - t_1) \geq ct, \tag{5}$$

eventually. Integrating (4) from t_1 to t and using (5), one gets

$$-\left[u'(t)\right]^{\alpha} + \left[u'(t_1)\right]^{\alpha} = \int_{t_1}^{t} p(s) f\left[u\left(\tau(s)\right)\right] \, \mathrm{d}s > \int_{t_1}^{t} p(s) f\left[c\tau(s)\right] \, \mathrm{d}s.$$

Letting $t \to \infty$, we have

$$\int_{t_1}^{\infty} p(s) f\left[c\tau(s)\right] \, \mathrm{d}s < \infty.$$

This contradiction shows that $u'(t) \to 0$ as $t \to \infty$. Therefore, for any $\lambda > 0$ there exists a t_1 such that $\lambda/2 > u'(t)$, $t \geq t_1$. Integrating the last functional inequality from t_1 to t, we get

$$u(t) \leq u(t_1) + \frac{\lambda}{2}(t - t_1) \leq \lambda t, \qquad t \geq t_2 \geq t_1.$$

Hence for any $\lambda > 0$ and t large enough

$$f'\left[u\left(\tau(t)\right)\right] \geq f'\left[\lambda\tau(t)\right]. \tag{6}$$

On the other hand, since $u'(t)$ is decreasing and $u'(t) \to 0$ as $t \to \infty$, it follows that

$$u'\left(\tau(t)\right) \geq u'(t) \geq \left(u'(t)\right)^{\alpha}, \tag{7}$$

eventually. Define

$$w(t) = \tau^{\alpha}(t) \frac{\left[u'(t)\right]^{\alpha}}{f\left[u\left(\tau(t)\right)\right]}. \tag{8}$$

It is easy to see that $w(t) > 0$ and

$$\begin{aligned} w'(t) &= \alpha\tau^{\alpha-1}(t)\tau'(t) \frac{\left[u'(t)\right]^{\alpha}}{f\left[u\left(\tau(t)\right)\right]} + \tau^{\alpha}(t) \frac{\left[(u'(t))^{\alpha}\right]'}{f\left[u\left(\tau(t)\right)\right]} \\ &\quad - \tau^{\alpha}(t) \frac{\left[u'(t)\right]^{\alpha} f'\left[u\left(\tau(t)\right)\right] u'\left(\tau(t)\right)\tau'(t)}{f^2\left[u\left(\tau(t)\right)\right]} \\ &= \alpha\frac{\tau'(t)}{\tau(t)} w(t) - \tau^{\alpha}(t)p(t) - w(t) \frac{f'\left[u\left(\tau(t)\right)\right] u'\left(\tau(t)\right)\tau'(t)}{f\left[u\left(\tau(t)\right)\right]}. \end{aligned} \tag{9}$$

Combining (6) and (7) together with (9), we see that

$$\begin{aligned} w'(t) &\le -\tau^\alpha(t)p(t) + \alpha\frac{\tau'(t)}{\tau(t)}w(t) - \frac{\tau'(t)f'[\lambda\tau(t)]}{\tau^\alpha(t)}w^2(t) \\ &= -\tau^\alpha(t)p(t) - \frac{\tau'(t)f'[\lambda\tau(t)]}{\tau^\alpha(t)}\left[\left(w(t) - \frac{\alpha\tau^{\alpha-1}(t)}{2f'[\lambda\tau(t)]}\right)^2 - \frac{\alpha^2\tau^{2\alpha-2}(t)}{4\big(f'[\lambda\tau(t)]\big)^2}\right] \\ &\le -\tau^\alpha(t)p(t) + \frac{\alpha^2\tau^{\alpha-2}(t)\tau'(t)}{4f'[\lambda\tau(t)]}\,. \end{aligned} \tag{10}$$

Integrating the above inequality from t_2 to t, we conclude in view of (3) that $w(t) \to -\infty$ as $t \to \infty$. This contradicts positivity of $w(t)$ and the proof is complete. □

Remark 1. There have been usually some conditions of the form (see [1], [2] and [8])

$$f(u)\operatorname{sgn} u \ge |u|^\beta \operatorname{sgn} u \tag{11}$$

imposed on the function f. Since we have relaxed this condition, Theorem 2.1 can be applied also to the equations, where [2; Theorem 1], [1; Theorem 2.3] and [8; Theorem 2.4] fail. We illustrate this fact in the example stated below.

For $\alpha = 1$ Theorem 2.1 gives:

COROLLARY 2.1. *Let $f'(u)$ be nondecreasing on $(-\infty, 0)$ and nonincreasing on $(0,\infty)$. Further assume that (2) holds for any $c \ne 0$ and*

$$\int^\infty \left(\tau(s)p(s) - \frac{\tau'(s)}{4\tau(s)f'[\pm\lambda\tau(s)]}\right) \mathrm{d}s = \infty \qquad \text{for some} \quad \lambda > 0\,. \tag{12}$$

Then equation

$$u''(t) + p(t)f\big[u(\tau(t))\big] = 0 \tag{13}$$

is oscillatory.

Remark 2. In [7] and [3], Chanturia and Kiguradze have shown that if

$$\int^\infty \left(sp(s) - \frac{1}{4s}\right) \mathrm{d}s = \infty\,, \tag{14}$$

then the equation

$$y''(t) + p(t)y(t) = 0 \tag{15}$$

is oscillatory. Corollary 2.1 extends this result to the more general equations.

In the following theorem we weaken conditions imposed on the derivative of the function $f(u)$.

THEOREM 2.2. *Let $\alpha \ge 1$. Let $f'(u)$ be nondecreasing on $(-\infty, -t^*)$ and nonincreasing on (t^*, ∞), where $t^* \ge 0$. Further assume that* (2) *and* (3) *hold. If*

$$\int_{t_1}^{\infty} \left(\int_{x}^{\infty} p(s)\, \mathrm{d}s \right)^{1/\alpha} \mathrm{d}x = \infty\,, \tag{16}$$

then equation (1) *is oscillatory.*

P r o o f . We can proceed exactly as in the proof of Theorem 2.1 to see that equation (1) reduces to (4). Integrating (4) from t ($\ge t_1$) to ∞ and taking into account the monotonicity of $f(u)$, we get

$$u'(t) \ge \left(\int_{t}^{\infty} p(s) f\big(u(\tau(s))\big)\, \mathrm{d}s \right)^{1/\alpha} \ge k \left(\int_{t}^{\infty} p(s)\, \mathrm{d}s \right)^{1/\alpha}, \tag{17}$$

$k = f^{1/\alpha}\big(u(\tau(t_1))\big)$. Integrating the last inequality from t_1 to t, we have

$$u(t) \ge k \int_{t_1}^{t} \left(\int_{x}^{\infty} p(s)\, \mathrm{d}s \right)^{1/\alpha} \mathrm{d}x\,. \tag{18}$$

Condition (16) yields $u(t) \to \infty$ as $t \to \infty$ and then $u(t) \ge t^*$, eventually. Hence (6) is satisfied. Next, we follow all steps of the proof of Theorem 2.1 to finish the proof. □

COROLLARY 2.2. *Let $f'(u)$ be nondecreasing on $(-\infty, -t^*)$ and nonincreasing on (t^*, ∞) for some $t^* \ge 0$. Further assume that* (2), (12) *and* (16) *are satisfied. Then equation* (13) *is oscillatory.*

EXAMPLE 1. Consider the second order nonlinear differential equation

$$u''(t) + p(t) \ln^3\Big(1 + \big|u[\tau(t)]\big|\Big) \operatorname{sgn} u[\tau(t)] = 0\,. \tag{19}$$

By Corollary 2.2, equation (19) is oscillatory, provided that (16) holds and

$$\int^{\infty} p(s) \ln^3\big(1 + c\tau(s)\big)\, \mathrm{d}s = \infty \qquad \text{for any} \quad c > 0\,,$$

$$\int^{\infty} \left(\tau(s)p(s) - \frac{\tau'(s)\big(1 + \lambda\tau(s)\big)}{12\tau(s) \ln^2\big(1 + \lambda\tau(s)\big)} \right) \mathrm{d}s = \infty \qquad \text{for some} \quad \lambda > 0\,.$$

Note that [2; Theorem 1], [1; Theorem 2.3] and [8; Theorem 2.4] cannot be applied to (19) as (11) is not satisfied. On the other hand, Corollary 2.1 also fails for (19) since $f'(u)$ is nonincreasing on (t^*, ∞), where $t^* > 0$.

Now we turn our attention to equations with "opposite" behavior of the function $f'(u)$.

THEOREM 2.3. *Let $f'(u)$ be nonincreasing on $(-\infty, 0)$ and nondecreasing on $(0, \infty)$. Further assume that (2) holds for any $c \neq 0$. Let one of the following conditions be satisfied*

(i) $\alpha > 1$ *and for some* $M > 0$

$$\int^{\infty} \left(\tau^{\alpha}(s)p(s) - M\tau^{\alpha-2}(s)\tau'(s)\right) \mathrm{d}s = \infty ; \tag{20}$$

(ii) $\alpha = 1$ *and* (20) *is satisfied with* $M = 1/4$.

Then equation (1) *is oscillatory.*

P r o o f. Let $\alpha > 1$. Assume that $M > 0$ is such that (20) holds. We admit that $u(t)$ is a positive solution of (1). Proceeding exactly as in the proof of Theorem 2.1 we can verify that $u'(t) > 0$, $u''(t) < 0$ and $u'(t) \to 0$ as $t \to \infty$. Then, there exists a $c > 0$ such that $u[\tau(t)] > c$, eventually. It is easy to see that

$$f'\left[u(\tau(t))\right]u'(\tau(t)) \geq f'(c)\, u'(t) = f'(c)\left(u'(t)\right)^{1-\alpha}\left(u'(t)\right)^{\alpha}. \tag{21}$$

Since $u'(t) \to 0$, then for any $\lambda > 0$, we have $u'(t) \leq \lambda$, eventually. It follows from (21) that

$$f'\left[u(\tau(t))\right]u'(\tau(t)) \geq f'(c)\lambda^{1-\alpha}\left(u'(t)\right)^{\alpha} = K\left(u'(t)\right)^{\alpha},$$

where λ is chosen such that $f'(c)\lambda^{1-\alpha} = \alpha^2/(4M)$. Let $w(t)$ be defined by (8), then $w(t) > 0$ and (9) is fulfilled. On the other hand,

$$\begin{aligned} w'(t) &\leq -\tau^{\alpha}(t)p(t) + \alpha\frac{\tau'(t)}{\tau(t)}w(t) - K\frac{\tau'(t)}{\tau^{\alpha}(t)}w^2(t) \\ &= -\tau^{\alpha}(t)p(t) - K\frac{\tau'(t)}{\tau^{\alpha}(t)}\left[\left(w(t) - \frac{\alpha\tau^{\alpha-1}(t)}{2K}\right)^2 - \frac{\alpha^2\tau^{2\alpha-2}(t)}{4K^2}\right] \\ &\leq -\tau^{\alpha}(t)p(t) + \frac{\alpha^2}{4K}\tau^{\alpha-2}(t)\tau'(t). \end{aligned} \tag{22}$$

Integrating the obtained inequality from t_1 to t (t_1 large enough) and then letting $t \to \infty$, we get desirable contradiction. The case $\alpha = 1$ can be treated similarly. The proof is complete now. □

Now we present another easily verifiable oscillation criterion for (1).

COROLLARY 2.3. *Let* $\alpha > 1$ *and* $f'(u)$ *be nonincreasing on* $(-\infty, 0)$ *and nondecreasing on* $(0, \infty)$. *Further assume that* (2) *holds for any* $c \neq 0$. *If*

$$\liminf_{t\to\infty} \frac{p(t)\tau^2(t)}{\tau'(t)} > 0\,, \tag{23}$$

then equation (1) *is oscillatory.*

Proof. It follows from (23) that there exists a constant $k > 0$ such that

$$p(t) \geq k\frac{\tau'(t)}{\tau^2(t)}\,,$$

eventually. Then, it follows from the last inequality that (20) is satisfied with $M = k/2$. Theorem 2.3 implies oscillation of equation (2.1). □

EXAMPLE 2. Consider the equation

$$\big(|u'(t)|u'(t)\big)' + p(t)\big|u[\tau(t)]\big|u[\tau(t)] = 0\,. \tag{24}$$

It is easy to verify that (23) implies (2) for equation (24). Then, by Corollary 2.3, equation (24) is oscillatory, provided that (23) holds. Note that [1; Theorem 1] fails for (24).

The following consideration is intended to relax the monotonicity conditions imposed onto $f'(u)$ in Theorems 2.1 and 2.3.

Let us consider the following differential equation

$$\big(|u'(t)|^{\alpha-1}u'(t)\big)' + p(t)h\big[u(\tau(t))\big] = 0\,, \tag{25}$$

subject to conditions (H1)–(H3), and

(H5) $h \in C(-\infty, \infty)$, $uh(u) > 0$ for $u \neq 0$.

THEOREM 2.4. *Assume that*

$$h(u)\operatorname{sgn} u \geq f(u)\operatorname{sgn} u\,, \qquad u \neq 0\,, \tag{26}$$

and (H4) *holds. If all assumptions of Theorem* 2.1 *are satisfied, then equation* (25) *is oscillatory.*

Proof. Assume that $u(t)$ is a positive solution of (25). Then $u'(t) > 0$, $u''(t) < 0$ and

$$\big([u'(t)]^\alpha\big)' = -p(t)h\big[u(\tau(t))\big] \leq -p(t)f\big[u(\tau(t))\big]\,.$$

Let $w(t)$ be defined by (8). Then $w(t) > 0$ and

$$w'(t) \leq \alpha\frac{\tau'(t)}{\tau(t)}w(t) - \tau^\alpha(t)p(t) - w(t)\frac{f'\big[u(\tau(t))\big]u'\big(\tau(t)\big)\tau'(t)}{f\big[u(\tau(t))\big]}\,.$$

The rest of the proof is similar to the proof of Theorem 2.1 and is omitted. □

THEOREM 2.5. *Let* (H4) *and* (26) *hold. Assume that all assumptions of Theorem* 2.3 *are satisfied. Then equation* (25) *is oscillatory.*

It remains an open problem how to obtain oscillatory criteria similar to Theorem 2.1 and 2.3 for (1) with $0 < \alpha < 1$. The following theorem provides a partial answer.

THEOREM 2.6. *Assume that*

$$\int_{t_0}^{\infty} \frac{\mathrm{d}u}{|f(\pm u)|^{1/\alpha}} < \infty$$

and

$$\int_{t_0}^{\infty} \tau'(s)\left(\int_{s}^{\infty} p(x)\,\mathrm{d}x\right)^{1/\alpha} \mathrm{d}s = \infty .$$

Then equation (1) *is oscillatory.*

P r o o f . Assume that $u(t)$ is a positive solution of (1). Proceeding similarly as in the proof of Theorem 2.1, it can be shown that $u'(t) > 0$ and $u''(t) < 0$. Integrating (1) from t to s $(\geq t)$, we obtain

$$-\left[u'(s)\right]^{\alpha} + \left[u'(t)\right]^{\alpha} = \int_{t}^{s} p(x) f\left[u(\tau(x))\right]\,\mathrm{d}x \geq f\left[u(\tau(t))\right] \int_{t}^{s} p(s)\,\mathrm{d}s .$$

Taking into account properties of $u'(t)$ and letting $s \to \infty$, we have

$$\left(u'[\tau(t)]\right)^{\alpha} \geq \left(u'(t)\right)^{\alpha} \geq f\left[u(\tau(t))\right] \int_{t}^{\infty} p(s)\,\mathrm{d}s . \tag{27}$$

It is easy to see that the case $\int^{\infty} p(x)\,\mathrm{d}x = \infty$ leads to a contradiction. It follows from (27) that

$$\frac{u'\left[\tau(t)\right]\tau'(t)}{f^{1/\alpha}\left[u(\tau(t))\right]} \geq \tau'(t)\left(\int_{t}^{\infty} p(x)\,\mathrm{d}x\right)^{1/\alpha},$$

which on integration from t_1 to t gives

$$\int_{u[\tau(t_1)]}^{u[\tau(t)]} \frac{\mathrm{d}s}{f^{1/\alpha}(s)} \geq \int_{t_1}^{t} \tau'(s)\left(\int_{s}^{\infty} p(x)\,\mathrm{d}x\right)^{1/\alpha} \mathrm{d}s . \tag{28}$$

The left side of (28) is bounded, however, the right side of (28) tends to ∞ as $t \to \infty$. The proof is complete. □

Remark 3. Theorem 2.6 cannot be applied to equation (1) with $f(u) = u$ and $\alpha \geq 1$. In this case Theorem 2.1 may be successful.

REFERENCES

[1] AGARWAL, R. P.—SHIEN, S. L.—YEH, C.-C.: *Oscillation criteria for second-order retarded differential equations*, Math. Comput. Modelling **26** (1997), 1–11.

[2] CHERN, J. L.—LIAN, W. C.—YEH, C.-C.: *Oscillation criteria for second order half-linear differential equations with functional arguments*, Publ. Math. Debrecen **48** (1996), 209–216.

[3] CHANTURIJA, T. A.—KIGURADZE, I. T.: *Asymptotic Properties of Nonautonomous Ordinary Differential Equations*, Nauka, Moscow, 1990. (Russian)

[4] ELBERT, A.: *A half-linear second order differential equation.* In: Colloq. Math. Soc. János Bolyai 30, North-Holland, Amsterdam, 1979, pp. 153–180.

[5] ELBERT, A.: *Oscillation and nonoscillation theorems for some nonlinear ordinary differential equation.* In: Lecture Notes in Math. 964, Springer-Verlag, New York, 1982, pp. 187–212.

[6] HONG, H.-L.—LIAN, W.-C.—YEH, C.-C.: *Oscillation criteria for half-linear differential equations with functional argument*, Nonlinear World **3** (1996), 849–855.

[7] LADDE, G. S.—LAKSHMIKANTHAM, V.—ZHANG, B. G.: *Oscillation Theory of Differential Equations with Deviating Arguments*, Marcel Dekker Inc., New York-Basel, 1987.

[8] LI, H. A.—YEH, C.-C.: *Oscillation of nonlinear functional-differential equations of second order*, Appl. Math. Lett. **11** (1998), 71–77.

[9] LI, H. J.—YEH, C.-C.: *Sturmian comparison theorem for half-linear second order differential equation*, Proc. Roy. Soc. Edinburgh Sect. A **125** (1995), 1193–1204.

[10] LI, H. J.—YEH, C.-C.: *An integral criterion for oscillation of nonlinear differential equations*, Math. Japon. **41** (1995), 185–188.

[11] MIRZOV, D. D.: *On the oscillation of a system of nonlinear differential equations*, Differ. Uravn. **9** (1973), 581–583.

[12] PENG, M.—GE, W.—HUANG, L.—XU, Q.: *A correction on the oscillatory behavior of solutions of certain second order nonlinear differential equations*, Appl. Math. Comput. **104** (1999), 207–215.

[13] WONG, P. J. Y.—AGARWAL, R. P.: *Oscillation theorems and existence criteria of asymptotically monotone solutions for second order differential equations*, Dynam. Systems Appl. **4** (1995), 477–496.

[14] WONG, P. J. Y.—AGARWAL, R. P.: *Oscillatory behavior of solutions of certain second order nonlinear differential equations*, J. Math. Anal. Appl. **198** (1996), 337–354.

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