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ON 7- AND 8-DECOMPOSABLE FINITE GROUPS

ALI REZA ASHRAFI* — WUJIE SHI**

(Communicated by Pavol Zlatos)

ABSTRACT. Let $G$ be a finite group and $\mathcal{N}_G$ denote the set of all non-trivial proper normal subgroups of $G$. An element $K$ of $\mathcal{N}_G$ is said to be $n$-decomposable if $K$ is a union of $n$ distinct conjugacy classes of $G$. $G$ is called $n$-decomposable, if $\mathcal{N}_G \neq \emptyset$ and every element of $\mathcal{N}_G$ is $n$-decomposable. In this paper, we will completely describe all 7- and 8-decomposable finite groups.

1. Introduction and preliminaries

Let $G$ be a finite group and let $\mathcal{N}_G$ be the set of all non-trivial proper normal subgroups of $G$. An element $K$ of $\mathcal{N}_G$ is said to be $n$-decomposable if $K$ is a union of $n$ distinct conjugacy classes of $G$. If $\mathcal{N}_G \neq \emptyset$ and every element of $\mathcal{N}_G$ is $n$-decomposable, then we say that $G$ is $n$-decomposable.

In [1], [2] and [3], the first author characterize the structure of $n$-decomposable finite groups for $n \leq 6$. In this paper we continue this problem and characterize the non-perfect 7- and 8-decomposable finite groups. To this end some deeper results in the field of the quantitative structure of finite groups are needed. For the motivation of this problem and background material the reader is encouraged to consult [5] [6], [10], [18] [21] and their references.

Let $G$ be a group. Denote by $\pi_e(G)$ the set of all orders of elements in $G$. Following Wujie Shi [21], a finite group $G$ is called EPO-group if every non-identity element of $G$ has prime order. In [20], Wujie Shi and Wenze Yang discussed finite EPO-groups and got an interesting result:

**Theorem 1.1.** (Wujie Shi and Wenze Yang, [20]) The characteristic property of $A_5$ is:

1. the order of the group contains at least three different prime factors,
2. the order of every non-identity element in the group is a prime.
COROLLARY 1.2. If $G$ is a non-abelian finite simple group and the order of every non-identity element of $G$ is prime, then $G$ is isomorphic to $A_5$.

Let $G$ be a finite simple group and set $\pi(G) = \{p : p$ is a prime and $p \mid |G|\}$. Following D. Gorenstein, a finite simple group $G$ is called a $K_3$-group if $|\pi(G)| = 3$. For the sake of completeness we mention below the following theorem of Herzog on the structure of simple $K_3$-groups.

THEOREM 1.3. (Herzog, [13]) If $G$ is a simple $K_3$-group, then $G$ is isomorphic to one of the simple groups $A_5$, $A_6$, $U_3(3)$, $U_4(2)$, $PSL(2,7)$, $PSL(2,8)$, $PSL(2,17)$ and $PSL(3,3)$.

Throughout this paper, as usual, $G'$ denotes the derived subgroup of $G$, $Z(G)$ is the center of $G$, $x^G$, $x \in G$, denotes the conjugacy class of $G$ with the representative $x$, and $G$ is called non-perfect if $G' \neq G$. Also, $\psi(G)$ denotes the number of composite integers of $\pi_e(G)$. All groups considered are assumed to be finite. Our notation is standard and taken mainly from [8] and [14].

2. Main results

Suppose $n$ is a positive integer such that there are non-abelian simple groups $A$ and $B$, not necessarily different, with exactly $n$ conjugacy classes and $G = A \times B$. Then $G$ is a perfect $n$-decomposable finite group. Thus, there are $n$-decomposable perfect finite groups. However, the investigation of such finite groups does not seem to be simple. Hence, in this paper we restrict our attention to the non-perfect finite group.

LEMMA 2.1. Let $G$ be a 7- or 8-decomposable non-solvable non-perfect finite group. Then $G'$ is simple.

Proof. Since $G'$ is a maximal normal subgroup of $G$, $|G : G'| = p$, $p$ is prime, and $G'$ is a minimal normal subgroup of $G$, which is not abelian. So $G'$ is a direct product of $k$ isomorphic non-abelian simple groups, say $H_1, \ldots, H_k$. If $k \geq 2$ and $H_1$ is not a $K_3$-group, then $|\pi_e(H_1 \times H_2)| \geq 11$, which is a contradiction. Thus, $G'$ is simple or $H_1$ is a $K_3$-group. Suppose $G'$ is not simple. Then, by Theorem 1.3, $H_1$ is isomorphic to $A_5$, $A_6$, $U_3(3)$, $U_4(2)$, $PSL(2,7)$, $PSL(2,8)$, $PSL(2,17)$ or $PSL(3,3)$. Using a simple calculation with GAP on element orders of these groups, we can see that $H_1 \cong A_5$. Our main proof will consider a number of cases:

Case 1. $G$ is 7-decomposable.

It is an easy fact that $|\pi_e((A_5)^r)| = 8$ for $r \geq 3$. Thus $G'$ is simple or $G' \cong A_5 \times A_5$. Suppose $G' \cong A_5 \times A_5$. Since $\pi_e(G') = \{1, 2, 3, 5, 6, 10, 15\}$, elements of the same order of $G'$ must be conjugate in $G$. On the other hand,
|Aut(G')| = 28800, which implies that \( p = 2 \) and \( |G| = 7200 \). But \( G' \) has exactly three conjugacy classes of elements of order 2 with lengths 15, 15 and 225, respectively. This shows that 7200 must be divisible by 255, which is a contradiction.

**Case 2.** \( G \) is 8-decomposable.

Choose elements \( x, y \) and \( z \) of \( G' \) such that \( \operatorname{ord}(x) = 2 \), \( \operatorname{ord}(y) = 3 \) and \( \operatorname{ord}(z) = 5 \). We first assume that \( G' \cong A_5 \times A_5 \). Then there is at most one conjugacy class of \( G \) containing all of elements of \( G' \) with a prime order. Now a similar argument as in Case 1 leads to a contradiction. Thus \( G' \cong (A_5)^r \) for \( r \geq 3 \). Since \( |\pi_e((A_5)^r)| = 8 \), the elements of order 2, as well as the elements of order 3, in \( G' \) must be conjugate in \( G \). By a well-known result in character theory, since \( G' = A_5 \times A_5 \times \cdots \times A_5 \), every conjugacy class of \( G' \) is a direct product of the conjugacy classes of the group \( A_5 \). But \( A_5 \) has a unique conjugacy class of elements of order 2 with length 15, so \( G' \) has exactly \( \binom{r}{1} \) conjugacy classes of elements of order 2 with length 15, \( \binom{r}{2} \) conjugacy classes of elements of order 2 with length 15^2, \( \binom{r}{3} \) conjugacy classes of elements of order 2 with length 15^3, \( \ldots \) and, \( \binom{r}{r} \) conjugacy classes of elements of order 2 with length 15^r. Therefore,

\[
|x^G| = 15 \binom{r}{1} + 15^2 \binom{r}{2} + \cdots + 15^r \binom{r}{r} = 16^r - 1 = 2^{4r} - 1.
\]

A similar argument shows that \( |y^G| = 21^r \). This implies that for some integers \( u \) and \( v \), we have:

\[
(2^{4r} - 1)u = 2^{2r} \cdot 3^r \cdot 5^r \cdot p,
\]

\[
(21^r - 1)v = 2^{2r} \cdot 3^r \cdot 5^r \cdot p.
\]

If \( p \notin \{2, 3, 5\} \), then \( G \cong (A_5)^r \times \mathbb{Z}_p \), which is a contradiction. For \( p = 2, 3 \), the first and second equation does not have an integer solution, respectively. Thus \( p = 5 \). Since \( 3^r | v \), \( y = 3^r y_1 \). If \( y_1 \geq 5 \), then \( (21^r - 1)v > 2^{2r} \cdot 3^r \cdot 5^{r+1} \).

Also, if \( y_1 = 2 \), then \( 2 \cdot (21^r - 1) > 5 \cdot 20^r \) for \( r \geq 19 \), and if \( y_1 = 4 \), then \( 4 \cdot (21^r - 1) > 5 \cdot 20^r \) for \( r \geq 5 \). For other values of \( r \), there is no solution for the second equation. This completes the proof. \( \square \)

**Lemma 2.2.** Let \( G \) be a \( n \)-decomposable non-solvable non-perfect finite group and \( |\mathcal{N}_G| \geq 2 \). Then \( |\mathcal{N}_G| = 2 \), \( n \) is a prime number and \( G \cong \mathbb{Z}_n \times B \), where \( B \) is a non-abelian simple group with exactly \( n \) conjugacy classes.

**Proof.** Let \( A \) and \( B \) be elements of \( \mathcal{N}_G \). Then by [1; Theorem 2], \( G \cong A \times B \). It is easy to see that \( A \) and \( B \) are simple groups. By [18; p. 88], \( A \) and \( B \) are the only proper non-trivial normal subgroups of \( G \). So \( |\mathcal{N}_G| = 2 \). If \( A \) and \( B \) are non-abelian simple groups, then \( G' = G \), which is a contradiction.

255
Therefore, one of \( A \) or \( B \), say \( A \), is abelian. Since \( A \) is simple, \( n \) is a prime number and \( A \cong \mathbb{Z}_n \), proving the lemma.

Suppose \( \omega(G') \) denotes the number of orbits of \( G' \) under the action of \( \text{Aut}(G') \). In the following lemma, we show that \( n \) is an upper bound for \( \omega(G) \) in the case that \( G \) is \( n \)-decomposable. In fact, we have:

**Lemma 2.3.** Let \( G \) be a \( n \)-decomposable non-solvable non-perfect finite group with the unique normal subgroup \( G' \). Then \( G \) is isomorphic to a subgroup of \( \text{Aut}(G') \). Moreover, if \( G' \) is simple, then \( n \geq \omega(G') \).

**Proof.** Define \( \alpha : G \to \text{Aut}(G') \) by \( \alpha(g) = T_g : G' \to G' \), where \( T_g(a) = gag^{-1} \) for all \( a \in G' \). It is obvious that \( \alpha \) is well defined. We show that \( \alpha \) is one-to-one. Suppose \( \alpha(g) = I_{G'} \), where \( g \) is a non-identity element of \( G \). Then \( G' \subseteq C_G(g) \) and so \( C_G(g) \subseteq G \). If \( G' = C_G(g) \), then \( g \in Z(C_G(g)) = Z(G') \). But \( G' \) is the unique normal subgroup of \( G \), so \( Z(G') = G' \). Hence \( G' \) is abelian and \( G \) is solvable, a contradiction. Thus \( g \in Z(G) \). Since \( G' \) is unique and \( G \) is non-abelian, \( G' = Z(G) \). This leads to a contradiction. Therefore \( \alpha \) is one-to-one and \( G \) is isomorphic to a subgroup of \( \text{Aut}(G') \). Now it is easy to see that for every elements \( a, b \in G' \), \( a^G = b^G \) if and only if \( a \) and \( b \) lie in the same orbit under the action of \( G \), proving the lemma.

Suppose \( T \) is the set of all groups \( L_2(q) \), where \( q = p^m \), \( p \). \( m \) are primes and \( S \) is the set of all groups \( L_2(p) \), where \( p \) is prime. In the following lemma, we investigate the \( 7 \)- and \( 8 \)-decomposable finite groups with \( G' \in T \cup S \).

**Lemma 2.4.** Suppose \( G \) is a \( 7 \)- or \( 8 \)-decomposable finite group with \( G' \in T \cup S \). Then \( G \cong \text{PSL}(2, 27) : 3, \text{Aut}(\text{PSL}(2, 11)) \) or \( \text{Aut}(\text{PSL}(2, 13)) \).

**Proof.** Let \( G \) be a \( 7 \)- or \( 8 \)-decomposable finite group with \( G' \in T \). If \( 2 \mid q \), then by Lemma 2.3 and a theorem of Kohl, [17; Theorem 2.5], \( n > \omega(G') \geq \omega(G') = 3 + \frac{2m-2}{m} \). This shows that \( m = 2, 3 \) and so \( G' \cong A_5 \) or \( \text{PSL}(2, 8) \), which contradicts Table I. Next we assume that \( q \) is an odd integer. In this case, by the previously mentioned theorem of Kohl

\[
\omega(G') = \begin{cases} 
1 + \frac{(p+1)^2}{4} & \text{if } m - 2, \\
\frac{p^{m+1} - (m+1)p + 12}{2m} & \text{if } m \neq 2,
\end{cases}
\]

and so, by Lemma 2.3 and Table I, \( G' \cong \text{PSL}(2, 27) \). Finally, we assume that \( G' \in S \). Then by the Kohl’s results, \( p \) is odd and \( \omega(G') = \frac{p+3}{2} \). This shows that \( p = 11, 13 \) and \( G \cong \text{Aut}(\text{PSL}(2, 11)) \) or \( \text{Aut}(\text{PSL}(2, 13)) \), which concludes the lemma.  

256
ON 7- AND 8-DECOMPOSABLE FINITE GROUPS

THEOREM 2.5. Let $G$ be a non-perfect 7-decomposable finite group. Then $G$ is isomorphic to an abelian group of order 49, $\text{Aut}(\text{PSL}(2,11)), \mathbb{Z}_7 \times A_6$, $\text{Aut}(\text{Sz}(8))$ or a Frobenius group of order $\frac{1}{6}p^r(p^r - 1)$, $p \geq 5$ is prime, and $r$ is a positive integer, such that the kernel of $G$ is elementary abelian of order $p^r$ and its complement is cyclic.

Proof. We first assume that $G$ is solvable. If $G$ is abelian, then it is clear that $G$ is an abelian group of order 49, as desired. Suppose $G$ is non-abelian. Then $|G : G'| = q$, where $q$ is prime. Since $G'$ is a minimal normal subgroup of $G$, $G'$ is an elementarily abelian subgroup of order $p^r$, $p > 5$ is prime, and $r$ is a positive integer, such that the kernel of $G$ is elementary abelian of order $p^r$ and its complement is cyclic.

Next we assume that $G$ is non-solvable. If $|N_G| = 2$, then by Lemma 2.2, $G \cong \mathbb{Z}_7 \times A_6$. So, we can restrict our investigation to the case that $G'$ is the unique normal subgroup of $G$, which is simple by Lemma 2.1. It is clear that $\pi(G') \leq 6$. If $|\pi(G')| = 6$, then $G'$ is an EPO-group and by Corollary 1.2, $G' \cong A_5$, which is a contradiction. Suppose $|\pi(G')| = 3$. Then by Theorem 1.3, $G'$ is isomorphic to $A_5, A_6, U_3(3), U_4(2), \text{PSL}(2,7), \text{PSL}(2,8), \text{PSL}(2,17)$ or $\text{PSL}(3,3)$ and by Lemma 2.3, $G$ is isomorphic to a subgroup of $\text{Aut}(G')$. But, $G'$ cannot be isomorphic to the groups $A_5$ and $\text{PSL}(2,7)$ since these groups have exactly five and six conjugacy classes, respectively. Suppose $G' \cong A_6$. Since $|\text{Aut}(A_6) : A_6| = 4$ and $G$ is a subgroup of $\text{Aut}(A_6)$ with prime index, $G$ is isomorphic to $S_6 = A_6.2_1, A_6.2_2$ or $A_6.2_3$, in ATLAS notation. [9]. But by Table I, such a group is 5- or 6-decomposable, which is a contradiction. On the other hand, by this table, $L_2(8)$ is a 5-decomposable subgroup of $\text{Aut}(L_2(8)), L_2(17)$ is a 10-decomposable subgroup of $\text{Aut}(L_2(17)), L_3(3)$ is a 9-decomposable subgroup of $\text{Aut}(L_3(3)), U_3(3)$ is a 10-decomposable subgroup of $\text{Aut}(U_3(3))$ and $U_2(2)$ is a 15-decomposable subgroup of $\text{Aut}(U_2(2))$, also $|\text{Aut}(G') : G'| = p, p = 2, 3$, which are impossible. Thus $|\pi(G')| = 4, 5$. In our main proof, we consider two separate cases:

Case 1. $|\pi(G')| = 5$.
In this case $\psi(G') = 1$ and by [22], $G$ is isomorphic to $\text{PSL}(2,q), q = 5, 7, 8, 9, 11, 13, 16, \text{PSL}(3,4), \text{Sz}(8), \text{PSL}(2,3^n)$, where $\frac{3^n - 1}{2}$ and $\frac{3^n + 1}{4}$ are primes, or $\text{PSL}(2,2^n)$, where $2^n - 1$ and $2^n + 1$ are primes. But, by a calculation, the orders of all of these groups have at most four prime divisors, which is a contradiction.

Case 2. $|\pi(G')| = 4$.
In this case $\psi(G') = 1, 2$. We first assume that $\psi(G') = 1$. Apply the previously mentioned result of Shi and Yang. By Lemma 2.4, Table I and [9], $|\text{Aut}(\text{Sz}(8)) : \text{Sz}(8)| = 3$ and $\text{Aut}(\text{Sz}(8))$ is 7-decomposable. Also, by Table I,
Aut(PSL(2,11)) is another 7-decomposable group with $\psi(G') = 1$. Next we suppose that $\psi(G') = 2$. Applying [10; Theorem 2], [24; Theorem 2] and [7; Theorem 2], it is enough to investigate the simple groups PSL(2, q). Suppose $G' \cong PSL(2, q)$, then by Lemma 2.4 and Table I, $G$ is not 7-decomposable. This completes the proof.

**Theorem 2.6.** Let $G$ be a non-perfect 8-decomposable finite group. Then $G$ is isomorphic to Aut(PSL(2, 13)), PSL(2, 27) : 3, PSL(3, 4) : 2 (including PSL(3, 4).21, PSL(3, 4).22 and PSL(3, 4).23), PSL(3, 4) : 3, $S_7$ or a Frobenius group of order $\frac{1}{2}2^r(2^r - 1)$, $r$ is a positive integer, such that the kernel of $G$ is elementary abelian of order $2^r$ and its complement is cyclic.

**Proof.** It is clear that such a group cannot be abelian. If $G$ is a non-abelian solvable group, then using a similar argument as in Theorem 2.5, we can see that $G$ is a Frobenius group of order $\frac{1}{2}p^r(p^r - 1)$, $p$ is odd prime and $r$ is a positive integer. Suppose that $G$ is non-solvable. Then by Lemmas 2.2 and 2.3, $|\mathcal{N}_G| = 1$ and $G'$ is simple. Also, by Corollary 1.2, Theorem 1.3 and Table I, $G'$ cannot be an EPO-group or a $K_3$-group. So, $4 \leq |\pi(G')| \leq 6$. If $|\pi(G')| = 6$, then $\psi(G') = 1$. But in this case, by [22] and [5], such a group has at most four prime divisors, which is a contradiction. In our main proof, we consider two separate cases:

Case 1. $|\pi(G')| = 5$.

Since $G$ is not EPO-group, $\psi(G') = 1, 2$. Also by Lemma 2.4 and [22], there is no group $G$ with $\psi(G') = 1$. Thus $\psi(G') = 2$. By Table I, PSL(3, 4) : 2, PSL(3, 4) : 3 and Aut(PSL(2, 13)) are solutions for our problem. So by [11; Theorem A], it is enough to investigate the cases that $G'$ is isomorphic to the Suzuki group $Sz(q)$ or a projective special linear group PSL(2, q) for some special values of $q$. By Lemma 2.4, if $G' \cong PSL(2, p^m)$, where $p$ and $m$ are primes, then $G' \cong PSL(2, 27)$, which is a contradiction. If $G' \cong PSL(2, p)$, where $p$ is a prime with $p > 13$, then by the previously mentioned theorem of Koh 1, we obtain a contradiction. Finally, assume that $G' \cong Sz(q)$, where $q = 2^{2m+1}$ is such that each of $q - 1, q - (2q)^{\frac{1}{2}} + 1$ and $q + (2q)^{\frac{1}{2}} + 1$ is either a prime or a product of two distinct primes. By [17; Theorem 3.4], $\omega(Sz(q)) = \omega(PSL(2, q)) + 2$ and by Lemma 2.3 and [17; Theorem 2.5], $8 \geq \omega(Sz(q)) = 2 + \omega(PSL(2, q)) = 5 + \frac{2^{2m+1}-2}{2m+1}$. This shows that $G' \cong Sz(8)$ and by Table I, we get our final contradiction.

Case 2. $|\pi(G')| = 4$.

Using a tedious calculation for applying the [24; Theorem 2], [7; Theorem 2], [17; Theorem 2.5], Lemma 2.4 and Table I, we can see that $G \cong Aut(PSL(2, 13))$, PSL(2, 27) : 3, PSL(3, 4) : 2 or PSL(3, 4) : 3, which completes the proof.
Table I: The fusion maps of some simple groups into their automorphism groups.

<table>
<thead>
<tr>
<th>Class</th>
<th>Fusion</th>
<th>Fusion into</th>
<th>Fusion into</th>
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<tr>
<td>$A_5$-Classes</td>
<td>$S_5$</td>
<td>1a 2a 3a 5a 5b</td>
<td>1A 2A 3A 5A 5A</td>
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<tr>
<td>$A_6$-Classes</td>
<td>$S_6$</td>
<td>1a 2a 3a 3b 4a 5a 5b</td>
<td>1A 2A 3A 3B 4A 5A 5A</td>
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<tr>
<td>$A_6$-Classes</td>
<td>$A_6.2_2$</td>
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<td>1A 2A 3A 3A 4A 5A 5B</td>
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<tr>
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<td>$A_6.2_3$</td>
<td>1a 2a 3a 3b 4a 5a 5b</td>
<td>1A 2A 3A 3A 4A 5A 5A</td>
</tr>
<tr>
<td>$A_7$-Classes</td>
<td>$S_7$</td>
<td>1a 2a 3a 3b 4a 5a 6a 7a 7b</td>
<td>1A 2A 3A 3B 4A 5A 6A 7A 7B</td>
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<tr>
<td>$PSL(2,7)$-Classes</td>
<td>$Aut(PSL(2,7))$</td>
<td>1a 2a 3a 4a 7a 7b</td>
<td>1A 2A 3A 4A 7A 7A</td>
</tr>
<tr>
<td>$PSL(2,8)$-Classes</td>
<td>$Aut(PSL(2,8))$</td>
<td>1a 2a 3a 7a 7b 8c 9a 9b 9c</td>
<td>1A 2A 3A 7A 7A 9A 9A 9A</td>
</tr>
<tr>
<td>$PSL(2,11)$-Classes</td>
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<td>1a 2a 3a 5a 5b 6a 11a 11b</td>
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<tr>
<td>$PSL(2,13)$-Classes</td>
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<td>1a 2a 3a 6a 7a 7b 7c 13a 13b</td>
<td>1A 2A 3A 6A 7A 7B 7C 13A 13A</td>
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<tr>
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<td>$PSL(2,16).2$</td>
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<td>$PSL(2,16)$-Classes</td>
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<td>1A 2A 3A 5A 5B 15A 15B 15A 15B</td>
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<td>$Aut(PSL(2,19))$</td>
<td>1a 2a 3a 5a 5b 9a 9b 9c 10a</td>
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<td>1A 2A 3A 8A 8B 9A 9B 9C</td>
</tr>
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<td>$PSL(2,27) : 2$</td>
<td>1a 2a 3a 3b 7a 7b 7c 13a 13b</td>
<td>1A 2A 3A 3A 7A 7B 7C 13A 13B</td>
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<tr>
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<td>$PSL(2,27) : 2$</td>
<td>1a 2a 3a 3b 7a 7b 7c 13a 13b</td>
<td>1A 2A 3A 3A 7A 7B 7C 13A 13B</td>
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ON 7- AND 8-DECOMPOSABLE FINITE GROUPS

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261


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