Anatolij Dvurečenskij

Book Reviews

Mathematica Slovaca, Vol. 55 (2005), No. 3, 381--382

Persistent URL: http://dml.cz/dmlcz/136920

Math. Slovaca, **55** (2005), No. 3, 381–382

Mathematica Slovaca

BOOK REVIEWS

Dalla Chiara, M. Giuntini, R.—Greechie, R.:
REASONING IN QUANTUM THEORY. SHARP AND UNSHARP QUANTUM LOGIC.
Trends in Logic-Studia Logica Library Vol. 22.
Kluwer Academic Publishers, Dordrecht-Boston-London 2004, xxi + 295 pp.
ISBN 1-4020-1978-5 (HB), ISBN 1-4020-1979-3 (e-book)

Twenties and thirties of the last century were very exciting in physics and mathematics as well as in mathematical logic. There appeared a new physics, which we call today quantum physics, where the traditional Newton laws fail. This was confirmed by measurements, and measurements are intimately connected with mathematical statistics and statistics with probability theory. Axiomatic of probability theory was done by Kolmogorov in 1933, but it was soon observed that quantum mechanical events do not satisfy the Kolmogorov axioms as it follows e.g. from the famous Heisenberg uncertainty principle. In 1920, Łukasiewicz published a two-page article on three-valued logic. Today this many-valued logic was developed into nowadays fuzzy logic.

All these three inspirations, quantum physics, probability theory and many-valued logics are the background of the present monograph under review written by prominent experts in quantum theories, quantum logics and mathematical logic. The aim of the book is to present various logical investigations of quantum phenomena, including the last hit, quantum computation.

The book consists of two parts, first one from 7 chapters and the second one from 10 chapters.

They present some abstract notions needed for the reading and by sketching the historical underpinnings of the subject. Some basic notions like posets, Hilbert space, axioms of quantum theory, Birkhoff and von Neumann's inseminal ideas that propositions connected with quantum systems can be viewed as forming a kind of logic more appropriately modeled by projections on a Hilbert space than by a Boolean algebra are presented.

The books gives the abstract axiomatization of sharp quantum theory, as well as the notion of event. The events form together a σ-complete orthomodular poset. The abstract notion of state and observable are given.

Book calls the readers often back to a Hilbert space. The event structure of $\mathcal{C}(H)$ is studied in more details as a principal case of orthomodular lattices where states are described by Gleason's theorem. It is shown when a generalized Hilbert space is a Hilbert space which was done by Solèr.

The authors show how fuzzy ideas entered Hilbert space quantum theory (QT). This is given by effects of a Hilbert space H which are Hermitian operators on H laying among zero and unity operators. The structure of such operators, $\mathcal{E}(H)$, is a so-called Brower-Zadeh poset. In this context, sharp and unsharp events are understood as projection operators and effect operators.

Effect algebras play an important role. On the example of a truncated sum of fuzzy sets, the authors present MV-algebras and quantum MV-algebras.

An important place is devoted to abstract axiomatic foundation of unsharp quantum theory. Here is presented a very important methods of construction of OMP's and OMP's based on pasting of finite Boolean algebras, Greechie's diagrams.

In the second part, the main accent is put to the explanation of quantum logics as logic. It starts with an orthomodular quantum logic and with a weaker orthologic. An algebraic and Kripkean realization of Hilbert event-state systems is given.

The book speaks about metalogical properties and anomalies of quantum logic. Quantum logic can be axiomatized in many ways: the Hilbert-Bernas style and the Gentzen style. The authors present a quantum logic-calculus in the Goldblatt style, and they explain about the metalogical intractability of orthomodularity. It is explained that the orthomodularity is not an elementary first-order property. It is still unknown whether every orthomodular lattice is embeddable into a complete orthomodular lattice.

We can read on the first-order quantum logic and quantum set theories, and partial logics. The authors start with partial a Boolean algebra. It is shown that partial classical logics are weaker than classical logic. Here a very interesting application of the Gleason theorem is given.

At the end, unsharp quantum logic is studied. It is shown that they can be regarded as natural logical abstractions from the effect-state systems. Such a logic is a paraconsistent quantum logic. More stronger examples of unsharp quantum logic called Brower-Zadeh logics (also fuzzy intuitionistic logics). There are two different forms weak Brower-Zadeh logic and strong Brower-Zadeh logic. In addition, the intuitive meaning of the corresponding quantum logic connectives in the sense of Łukasiewicz are explained.

The last chapter is dedicated to a new form of unsharp quantum logic that has been naturally suggested by the theory of unsharp computation. An analogue of the classical bit is here a qubit represented as any unit vector in a Hilbert space. Quantum logical gates, quantum computational semantics are presented in details.

In Conclusions, the authors summarize answers to the following questions:

(a) Why quantum logics?
(b) Are quantum logics helpful to solve the difficulties of QT?
(c) Are quantum logics "real logics"?
(d) Does quantum logic confirm the thesis that "logic is empirical"?

The authors argues positively to these deep questions.

The book is accomplished with the bibliography with 180 items. The monograph is written in a very nice and fresh style trying also to show that quantum like logic can be found in other disciplines and also in the past. The authors are keeping the regard of the reader permanently on his trip trough the book. The audience is supposed from mathematicians, logicians, quantum theorists, and specialists from quantum computing, both experts and students. The monograph is welcome in the quantum logic realm, and it surely enriches the quantum structure literature from this important logical point of view.

Anatolij Dvurečenskij, Bratislava