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ON BF-ALGEBRAS

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ABSTRACT. In this paper we introduce the notion of BF-algebras, which is a generalization of B-algebras. We also introduce the notions of an ideal and a normal ideal in BF-algebras. We investigate the properties and characterizations of them.

1. Introduction

The concept of B-algebras was introduced by J. Neggers and H. S. Kim [6]. They defined a *B-algebra* as an algebra $(A;*,0)$ of type $(2,0)$ (i.e., a nonempty set A with a binary operation $*$ and a constant 0) satisfying the following axioms:

(B1) $x * x = 0$,
(B2) $x * 0 = x$,
(B) $(x * y) * z = x * [z * (0 * y)]$.

In [4], Y. B. Jun, E. H. Roh, and H. S. Kim introduced BH-algebras, which are a generalization of $BCK/BCI/B$-algebras. An algebra $(A;*,0)$ of type $(2,0)$ is a *BH-algebra* if it obeys (B1), (B2), and

(BH) $x * y = 0$ and $y * x = 0$ imply $x = y$.

Recently, Ch. B. Kim and H. S. Kim [5] defined a *BG-algebra* as an algebra $(A;*,0)$ of type $(2,0)$ satisfying (B1), (B2), and

(BG) $x = (x * y) * (0 * y)$.

For other generalizations of B-algebras see [11] (BZ-algebras) and [8] (β-algebras). Here we define $BF/BF_1/BF_2$-algebras. We introduce the notions of an

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ideal and a normal ideal in BF-algebras. We then consider the properties and characterizations of them.

2. BF-algebras

Definition 2.1. A *BF-algebra* is an algebra $(A;*,0)$ of type $(2,0)$ satisfying (B1), (B2), and the following axiom:
(BF) $0*(x*y)=y*x$.

Remark 2.2. If $(A;*,0)$ is a B-algebra, then it satisfies (BF), (BG), and (BH . For a proof see [9, Proposition 1.5(b)] and [1, Proposition 2.2(ii), Lemma 3.5(i)].

Example 2.3. Let $\mathbb{R}$ be the set of real numbers and let $\mathbf{A}$ $(\mathbb{R};*,0)$ be the algebra with the operation $*$ defined by

$$x*y=\begin{cases} x & \text{if } y=0,\\ y & \text{if } x=0,\\ 0 & \text{otherwise.}\end{cases}$$

Then $\mathbf{A}$ is a BF-algebra.

Example 2.4. Let $A=[0;+\infty)$ $(=\{x\in\mathbb{R}: x\geq 0\})$. Define the binary operation $*$ on A as follows:

$$x*y=|x-y| \qquad \text{for all} \quad x,y\in A.$$

Then $(A;*,0)$ is a BF-algebra.

Proposition 2.5. *If $\mathbf{A}=(A;*,0)$ is a BF-algebra, then*
(a) $0*(0*x)=x$ *for all* $x\in A$;
(b) *if* $0*x=0*y$, *then* $x=y$ *for any* $x,y\in A$;
(c) *if* $x*y=0$, *then* $y*x=0$ *for any* $x,y\in A$.

Proof. Let $\mathbf{A}$ be a BF-algebra and $x\in A$. By (BF) and (B2) we obtain $0*(0*x)=x*0=x$, that is, (a) holds. Also (b) follows from (a). Let now $x,y\in A$ and $x*y=0$. Then $0=0*0=0*(x*y)=y*x$. This gives (c). □

Proposition 2.6. *Any BF-algebra $(A;*,0)$ that satisfies the identity $(x*z)*(y*z)=x*y$ is a B-algebra.*

Proof. This follows immediately from Proposition 2.5(a) and [10, Theorem 2.2].

Definition 2.7. A BF-algebra is called a *BF_1-algebra* (resp. a *BF_2-algebra*) if it obeys (BG) (resp. (BH)).

Every B-algebra is a BF_1/BF_2-algebra (see Remark 2.2). The BF-algebra $(\mathbb{R}; *, 0)$ given in Example 2.3 is not a BF_1-algebra, since $(1*2)*(0*2) = 2 \neq 1$. Example 2.4 is a BF_2-algebra which is not a BF_1-algebra.

Proposition 2.8. *An algebra* $\mathbf{A} = (A; *, 0)$ *of type* $(2,0)$ *is a* BF_1*-algebra if and only if it obeys the laws* (B1), (BF), *and* (BG).

Proof. Suppose that (B1), (BF), and (BG) are valid in $\mathbf{A}$. Let $x \in A$. Substituting $y = x$, (BG) becomes $x = (x*x)*(0*x)$. Hence applying (B1) and (BF) we conclude that $x = 0*(0*x) = x*0$. Consequently, (B2) holds. Therefore $\mathbf{A}$ is a BF_1-algebra. The converse is obvious. □

Proposition 2.9. *Let* $\mathbf{A} = (A; *, 0)$ *be an algebra of type* $(2,0)$. *Then* $\mathbf{A}$ *is a* BF_2*-algebra if and only if* $\mathbf{A}$ *satisfies* (B2), (BF), *and the following axiom:*
(BH') $x * y = 0 \iff x = y$.

Proof. Let $\mathbf{A}$ be a BF_2-algebra. By definition, (B2) and (BF) are valid in $\mathbf{A}$. Suppose that $x*y = 0$ for $x, y \in A$. Proposition 2.5(c) yields $y*x = 0$. From (BH) we see that $x = y$. If $x = y$, then $x*y = 0$ by (B1). Thus (BH') holds in $\mathbf{A}$.

Let now $\mathbf{A}$ satisfies (B2), (BF), and (BH'). (BH') implies (B1) and (BH). Therefore $\mathbf{A}$ is a BF_2-algebra. □

Theorem 2.10. *In a* BF*-algebra* $\mathbf{A}$ *the following statements are equivalent:*
(a) $\mathbf{A}$ *is a* BF_1*-algebra;*
(b) $x = [x*(0*y)]*y$ *for all* $x, y \in A$*;*
(c) $x = y*[(0*x)*(0*y)]$ *for all* $x, y \in A$.

Proof.
(a) $\Longrightarrow$ (b): Let $\mathbf{A}$ be a BF_1-algebra and $x, y \in A$. To obtain (b), substitute $0*y$ for y in (BG) and then use Proposition 2.5(a).

(b) $\Longrightarrow$ (c): We conclude from (b) that $0*x = [(0*x)*(0*y)]*y$. Hence $0*(0*x) = y*[(0*x)*(0*y)]$ by (BF). But $0*(0*x) = x$, and we have (c).

(c) $\Longrightarrow$ (a): Let (c) hold. (BF) clearly forces

$$0*x = [(0*x)*(0*y)]*y. \tag{1}$$

Using (1) with $x = 0*a$ and $y = 0*b$ we have

$$0*(0*a) = [(0*(0*a))*(0*(0*b))]*(0*b).$$

Hence applying Proposition 2.5(a) we deduce that $a = (a*b)*(0*b)$. Consequently, $\mathbf{A}$ is a BF_1-algebra. □

Theorem 2.11. *Let* $\mathbf{A} = (A; *. 0)$ *be a* BF *-algebra. Then:*

(a) $\mathbf{A}$ *is a* BG*-algebra;*
(b) *For* $x, y \in A$, $x * y = 0$ *implies* $x = y$*;*
(c) *The right cancellation law holds in* $\mathbf{A}$*, i.e., if* $x * y = z * y$*, then* $x = z$ *for any* $x, y, z \in A$*;*
(d) *The left cancellation law holds in* $\mathbf{A}$*, i.e., if* $y * x = y * z$*, then* $x = z$ *for any* $x, y, z \in A$.

Proof.

(a) is a direct consequence of the definitions.

(b): Let $x, y \in A$ and $x * y = 0$. By (BG), $x = (x * y) * (0 * y) = 0 * (0 * y)$. From Proposition 2.5(a) we conclude that $x = y$.

(c) is obvious, since the right cancellation law holds in every BG-algebra (see [5, Lemma 2.4]).

(d) follows from (c) and (BF). □

Proposition 2.12. *Every* BF_1*-algebra is a* BF_2*-algebra. Every* BF_2*-algebra satisfying the axiom* (BG) *is a* BF_1*-algebra.*

Proof. The first statement is a consequence of Theorem 2.11(b). The second part of Proposition 2.12 follows from the definitions.

Theorem 2.13. *Let* $\mathbf{A} = (A; *, 0)$ *be a* BF_1*-algebra. Then* $(A; *)$ *is a quasi group.*

Proof. Let $\mathbf{A} = (A; *, 0)$ be a BF_1-algebra and $x, y \in A$. We take $z_1 = x*(0*y$ and $z_2 = (0 * x) * (0 * y)$. By Theorem 2.10, we have $x = z_1 * y$ and $x - y * z_2$. Now, Theorem 2.11 implies that $(A; *)$ is a quasigroup.

The interrelationships between some classes of algebras mentioned before are visualized in Figure 1. (An arrow indicates proper inclusion, that is, if $\mathcal{X}$ and $\mathcal{Y}$ are classes of algebras, then $\mathcal{X} \to \mathcal{Y}$ means $\mathcal{X} \subset \mathcal{Y}$.) The implications (a and (d) follow easily from the definitions. By [5, Proposition 2.8], we get (e) The implications (b) and (c) follow from Theorem 2.11 and Proposition 2.12, respectively.

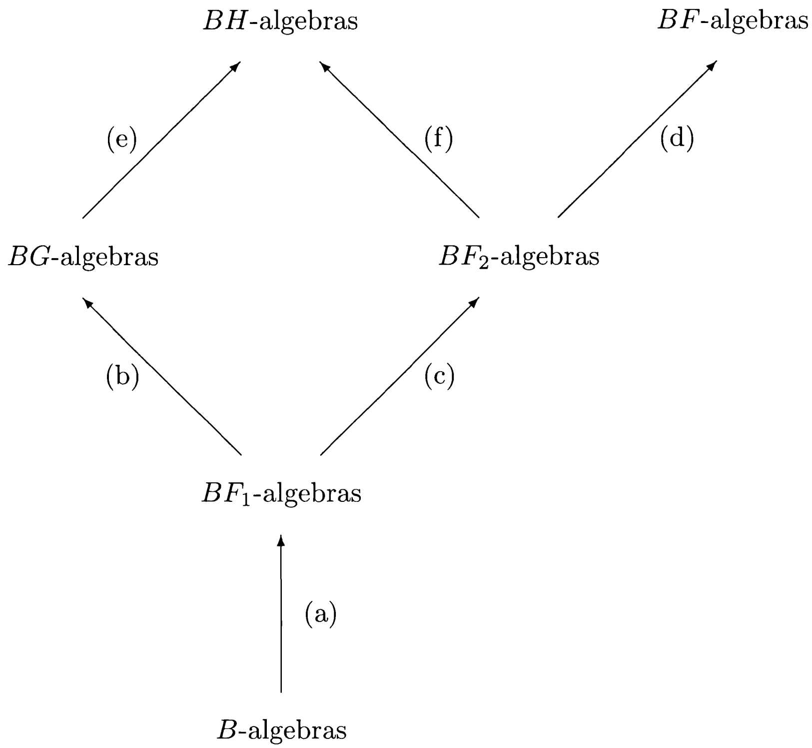

FIGURE 1

3. Ideals in BF-algebras

In BF-algebras (similarly as in $BCK/BCI/BH$-algebras; see [3], [2], and [4]), we define the notion of an ideal.

From now on, $\mathbf{A}$ always denotes a BF-algebra $(A; *, 0)$.

DEFINITION 3.1. A subset I of A is called an *ideal* of $\mathbf{A}$ if it satisfies:

(I_1) $0 \in I$,

(I_2) $x * y \in I$ and $y \in I$ imply $x \in I$ for any $x, y \in A$.

We say that an ideal I of $\mathbf{A}$ is *normal* if for any $x, y, z \in A$, $x * y \in I$ implies $(z * x) * (z * y) \in I$.

An ideal I of $\mathbf{A}$ is said to be *proper* if $I \neq A$.

Obviously, $\{0\}$ and A are ideals of $\mathbf{A}$. A is normal, but $\{0\}$ is not normal in general. (See the example below.)

Example 3.2. Let $A = \{0, 1, 2, 3\}$ and $*$ be defined by the following table:

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	0
2	2	3	0	2
3	3	0	2	0

Then $I = \{0\}$ is not a normal ideal in the BF-algebra $(A; *, 0)$. Indeed, $1 * 3 = 0 \in I$, but $(2 * 1) * (2 * 3) = 3 * 2 = 2 \notin I$.

Lemma 3.3. *Let I be a normal ideal of a BF-algebra $\mathbf{A}$ and $x, y \in A$. Then:*
(a) $x \in I \implies 0 * x \in I$,
(b) $x * y \in I \implies y * x \in I$.

Proof.
(a): Let $x \in I$. Then $x = x * 0 \in I$. Since I is normal, $(0 * x) * (0 * 0) \in I$. Hence $0 * x \in I$.
(b): Let $x * y \in I$. By (a), $0 * (x * y) \in I$. Applying (BF) we have $y * x \in I$. □

Definition 3.4. A nonempty subset N of A is called a *subalgebra* of $\mathbf{A}$ if $x * y \in N$ for any $x, y \in N$.

It is easy to see that if N is a subalgebra of $\mathbf{A}$, then $0 \in N$.

Lemma 3.5. *Let N be a subalgebra of $\mathbf{A}$ and let $x, y \in A$. If $x * y \in N$, then $y * x \in N$.*

Proof. Let $x * y \in N$. By (BF), $y * x = 0 * (x * y)$. Since $0 \in N$ and $x * y \in N$, we see that $0 * (x * y) \in N$. Consequently, $y * x \in N$. □

Example 3.6. Let $A = \{0, 1, 2, 3\}$. We define the binary operation $*$ on A as follows:

$*$	0	1	2	3
0	0	1	2	3
1	1	0	1	1
2	2	1	0	1
3	3	1	1	0

Then $\mathbf{A} = (A; *, 0)$ is a BF-algebra. The set $N = \{0, 1\}$ is a subalgebra of $\mathbf{A}$. N is not an ideal, since $2 * 1 = 1 \in N$, but $2 \notin N$. It is easy to see that the set $I = \{0, 2, 3\}$ is an ideal of $\mathbf{A}$, but it is not a subalgebra.

Proposition 3.7. *If I is a normal ideal of $\mathbf{A}$, then I is a subalgebra of $\mathbf{A}$ satisfying the following condition:*
(NI) *if $x \in A$ and $y \in I$, then $x * (x * y) \in I$.*

P r o o f. Let $x \in A$ and $y \in I$. Lemma 3.3(a) shows that $0 * y \in I$. Since I is normal, we conclude that $(x*0)*(x*y) \in I$, i.e., $x*(x*y) \in I$. Thus (NI) holds. Let now $x, y \in I$. Therefore $x * (x * y) \in I$. By Lemma 3.3(b), $(x * y) * x \in I$. From the definition of ideal we have $x * y \in I$. Thus I is a subalgebra satisfying (NI). □

Remark 3.8. The converse of Proposition 3.7 does not hold. Indeed, the subalgebra $\{0, 1\}$ of the BF-algebra $\mathbf{A}$ (see Example 3.6) satisfies (NI), but it is not an ideal.

In [7], J. N e g g e r s and H. S. K i m introduced the notion of a normal subalgebra of a B-algebra. Let $\mathbf{A} = (A; *, 0)$ be a B-algebra and N be a subalgebra of $\mathbf{A}$. N is said to be a *normal subalgebra* if
(NS) $(x * a) * (y * b) \in N$ for any $x * y$, $a * b \in N$.

Remark 3.9. In [9], it is proved that if N is a subalgebra of $\mathbf{A}$, then N is normal if and only if N satisfies (NI).

In B-algebras the following result holds:

Proposition 3.10. *Let $\mathbf{A}$ be a B-algebra and let $N \subseteq A$. Then N is a normal subalgebra of $\mathbf{A}$ if and only if N is a normal ideal.*

P r o o f. Let N be a normal subalgebra of $\mathbf{A}$. Clearly, $0 \in N$. Suppose that $x * y \in N$ and $y \in N$. Then $0 * y \in N$. Since N is a subalgebra, we have $(x * y) * (0 * y) \in N$. But $(x * y) * (0 * y) = x$, because every B-algebra satisfies (BG) (see Remark 2.2). Therefore $x \in N$, and thus N is an ideal. Let now $x, y, z \in A$ and $x * y \in N$. By (NS), $(z * x) * (z * y) \in N$. Consequently, N is normal. The converse follows from Proposition 3.7 and Remark 3.9. □

Definition 3.11. Let $\mathbf{A} = (A, *, 0_A)$ and $\mathbf{B} = (B, *, 0_B)$ be BF-algebras. A mapping $\varphi\colon A \to B$ is called a *homomorphism* from $\mathbf{A}$ into $\mathbf{B}$ if $\varphi(x * y) = \varphi(x) * \varphi(y)$ for any $x, y \in A$.

Observe that $\varphi(0_A) = 0_B$. Indeed, $\varphi(0_A) = \varphi(0_A * 0_A) = \varphi(0_A) * \varphi(0_A) = 0_B$. We denote by $\ker\varphi$ the subset $\{x \in A : \varphi(x) = 0_B\}$ of A (it is the kernel of the homomorphism φ).

Lemma 3.12. *Let $\varphi\colon A \to B$ be a homomorphism from $\mathbf{A}$ into $\mathbf{B}$. Then $\ker\varphi$ is an ideal of $\mathbf{A}$.*

P r o o f. Obviously, $0_A \in \ker\varphi$, that is, (I_1) holds. Let $x * y \in \ker\varphi$ and $y \in \ker\varphi$. Then $0_B = \varphi(x * y) = \varphi(x) * \varphi(y) = \varphi(x) * 0_B = \varphi(x)$. Consequently, $x \in \ker\varphi$. Therefore, (I_2) is satisfied. Thus I is an ideal of $\mathbf{A}$. □

The next example shows that the kernel of a homomorphism is not always a normal ideal. Let $\mathbf{A}$ be the algebra given in Example 3.2. Clearly, $\mathrm{id}_A\colon A \to A$ is a homomorphism and the ideal $\ker(\mathrm{id}_A) = \{0\}$ is not normal.

The example below will demonstrate that there is a homomorphism φ of BF-algebras with $\ker\varphi = \{0\}$ which it is not one-to-one.

Example 3.13. Let $\mathbf{A} = (A; *, 0)$ be the BF-algebra, where $A = \{0, 1, 2\}$ and $*$ is given by the table

$*$	0	1	2
0	0	1	2
1	1	0	0
2	2	0	0

Let $\varphi\colon A \to A$ be defined by $\varphi(0) = 0$ and $\varphi(1) = \varphi(2) = 1$. It is obvious that φ is not one-to-one, but $\ker\varphi = \{0\}$.

Proposition 3.14. *Let* $\mathbf{A}$ *and* $\mathbf{B}$ *be* BF_2*-algebras and let* $\varphi\colon A \to B$ *be a homomorphism from* $\mathbf{A}$ *into* $\mathbf{B}$*. Then:*
(a) $\ker\varphi$ *is a normal ideal;*
(b) φ *is one-to-one if and only if* $\ker\varphi = \{0_A\}$.

P r o o f.
(a): By Lemma 3.12, $\ker\varphi$ is an ideal of $\mathbf{A}$. Let $x, y, z \in A$ and $x * y \in \ker\varphi$. Then $0_B = \varphi(x * y) = \varphi(x) * \varphi(y)$. From (BH') it follows that $\varphi(x) = \varphi(y)$. Consequently, $\varphi((z * x) * (z * y)) = (\varphi(z) * \varphi(x)) * (\varphi(z) * \varphi(x)) = 0_B$, and hence $(z * x) * (z * y) \in \ker\varphi$.
(b): Obviously, if φ is one-to-one, then $\ker\varphi = \{0_A\}$. On the other hand, suppose that $x, y \in A$ and $\varphi(x) = \varphi(y)$. Then $\varphi(x * y) = \varphi(x) * \varphi(y) = \varphi(x) * \varphi(x) = 0_B$. Hence $x * y \in \ker\varphi = \{0_A\}$, and so $x * y = 0_A$. By (BH'), $x = y$. Therefore, φ is one-to-one. □

Next we construct quotient BF-algebras via normal ideals. Let $\mathbf{A} = (A; *, 0)$ be a BF-algebra and I be a normal ideal of $\mathbf{A}$. For any $x, y \in A$, we define

$$x \sim_I y \iff x * y \in I.$$

By (I_1), $x * x = 0 \in I$, that is, $x \sim_I x$ for any $x \in A$. This means that $\sim_I$ is reflexive. From Lemma 3.3(b) we deduce that $\sim_I$ is symmetric. To prove that

$\sim_I$ is transitive, let $x \sim_I y$ and $y \sim_I z$. Then $x * y \in I$ and $y * z \in I$. Since I is normal,

$$(z * x) * (z * y) \in I. \tag{2}$$

We have

$$z * y \in I, \tag{3}$$

because $y * z \in I$. Hence, we conclude from (2) and (3) that $z * x \in I$, and thus that $x * z \in I$, so that finally $x \sim_I z$ as well. Consequently, $\sim_I$ is an equivalence relation on A.

Theorem 3.15. *Let I be a normal ideal of a BF-algebra $\mathbf{A}$. Then $\sim_I$ is a congruence relation of $\mathbf{A}$.*

Proof. Let $x, y, z, t \in A$. Suppose that $x \sim_I y$ and $z \sim_I t$. Then $x * y \in I$ and $z * t \in I$. Since I is normal, (2) holds, and hence $[0 * (z * x)] * [0 * (z * y)] \in I$. From (BF) we deduce that $(x * z) * (y * z) \in I$. Thus

$$x * z \sim_I y * z. \tag{4}$$

As $z * t \in I$ we have $(y * z) * (y * t) \in I$. Therefore

$$y * z \sim_I y * t. \tag{5}$$

From (4) and (5) we conclude that $x*z \sim_I y*t$. Consequently, $\sim_I$ is a congruence relation of $\mathbf{A}$. □

Let I be a normal ideal of $\mathbf{A}$. For $x \in A$, we write x/I for the congruence class containing x, that is, $x/I = \{y \in A : x \sim_I y\}$. We note that

$$x \sim_I y \qquad \text{if and only if} \qquad x/I = y/I.$$

Denote $A/I = \{x/I : x \in A\}$ and set $x/I *' y/I = x * y/I$. The operation $*'$ is well-defined, since $\sim_I$ is a congruence relation of $\mathbf{A}$. It is easy to see that $\mathbf{A}/I = (A/I, *', 0/I)$ is a BF-algebra. The algebra $\mathbf{A}/I$ is called the *quotient BF-algebra of $\mathbf{A}$ modulo I*. There is a natural map φ_I, called the *quotient map*, from $\mathbf{A}$ onto $\mathbf{A}/I$ defined by

$$\varphi_I(x) = x/I \qquad \text{for all} \quad x \in A.$$

Clearly, φ_I is a homomorphism of $\mathbf{A}$ onto $\mathbf{A}/I$. Observe that $\ker(\varphi_I) = I$. Indeed,

$$x/I = 0/I \iff x \sim_I 0 \iff x * 0 \in I \iff x \in I.$$

Theorem 3.16. *Let $\mathbf{A}$ and $\mathbf{B}$ be BF_2-algebras and let $\varphi\colon A \to B$ be a homomorphism from $\mathbf{A}$ onto $\mathbf{B}$. Then $\mathbf{A}/\ker\varphi$ is isomorphic to $\mathbf{B}$.*

Proof. By Proposition 3.14(a), $I = \ker\varphi$ is a normal ideal of $\mathbf{A}$. Define a mapping $\psi\colon A/I \to B$ by $\psi(x/I) = \varphi(x)$ for all $x \in I$. Let $x/I = y/I$. Then $x \sim_I y$, that is, $x*y \in I$. Hence $\varphi(x)*\varphi(y) = 0_B$. By (BH') we have $\varphi(x) = \varphi(y)$. Consequently, $\psi(x/I) = \psi(y/I)$. This means that ψ is well defined. It is easy to see that ψ is a homomorphism from $\mathbf{A}/I$ onto $\mathbf{B}$. Observe that $\ker\psi = \{0_A/I\}$. Indeed, $x/I \in \ker\psi \iff \psi(x/I) = 0_B \iff \varphi(x) = 0_B \iff x \in I \iff x/I = 0_A/I$. From Proposition 3.14(b) it follows that ψ is one-to-one. Thus ψ is an isomorphism from $\mathbf{A}/I$ onto $\mathbf{B}$. □

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REFERENCES

[1] CHO, J. R.—KIM, H. S.: *On B-algebras and quasigroups*, Quasigroups Related Systems **7** (2001), 1–6.
[2] HUANG, Y.: *Irreducible ideals in BCI-algebras*, Demonstratio Math. **37** (2004), 1–8.
[3] ISEKI, K.—TANAKA, S.: *Ideal theory of BCK-algebras*, Math. Japon. **21** (1976), 351–366.
[4] JUN, Y. B.—ROH, E. H.—KIM, H. S.: *On BH-algebras*, Sci. Math. Jpn. **1** (1998), 347–354.
[5] KIM, Ch. B.—KIM, H. S.: *On BG-algebras*, Mat. Vesnik (To appear)
[6] NEGGERS, J.—KIM, H. S.: *On B-algebras*, Mat. Vesnik **54** (2002), 21–29.
[7] NEGGERS, J.—KIM, H. S.: *A fundamental theorem of B-homomorphism for B-algebras*, Int. Math. J. **2** (2002), 207–214.
[8] NEGGERS, J.—KIM, H. S.: *On β-algebras*, Math. Slovaca **52** (2002), 517–530.
[9] WALENDZIAK, A.: *A note on normal subalgebras in B-algebras*, Sci. Math. Jpn. **62** (2005), 49–53.
[10] WALENDZIAK, A.: *Some axiomatizations of B-algebras*, Math. Slovaca **56** (2006), 301–306.
[11] ZHANG, X.—YE, R.: *BZ-algebra and group*, J. Math. Phys. Sci. **29** (1995), 223–233.

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