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Fuzzy transforms in image compression and fusion

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Abstract. An overview of direct and inverse fuzzy transforms of three types is given and applications to data processing are considered. The construction and some important properties of fuzzy transforms are presented on the theoretical level. Three applications of F-transform to data processing have been chosen: compression and reconstruction of data, removing noise and data fusion. All of them successively exploit the filtering property of the inverse fuzzy transform.

1. Introduction

Nowadays, the boom caused by successful applications of fuzzy sets is over and we can make a backward analysis of what has been contributed to general mathematics. With this purpose, we introduced in [4] the notion of a fuzzy transform (F-transform, for short) which explains modeling with fuzzy IF-THEN rules as a specific transformation. According to that, a dependence hidden in original data is transformed into a look-up-table of a fuzzy function. Therefore, the success of fuzzy set theory has been explained by an analogy with the success of classical transforms (Fourier, Laplace, integral, wavelet etc.).

The aim of this paper is to overview the main properties of F-transform and its applications published in different papers. We give a brief introduction to the theory and demonstrate applications to image processing, namely to image compression and reconstruction, and to image fusion. We will describe three different types of F-transform: ordinary (based on arithmetic operations and described in [4]) and two other ones (in some sense dual) based on lattice operations and described in [7].

Moreover, we will explain a construction of approximation models on the basis of all these F-transforms (Sections 2, 3 and Subsection 3.2) and show, how they can be applied to data compression and decompression (Section 4). We also present two new applications, namely removing noise (Section 5) and data fusion (Section 6).

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2. Fuzzy Partition and the Direct F-Transform

Approximate representation of a continuous function by a system of fuzzy IF-THEN rules proved that a required accuracy can be achieved even if we have a rough information about a behavior of a function. By this we mean that, given average values of a function within intervals which cover the universe, we can reconstruct the function with the respective level of accuracy.

We begin our analysis by postulating requirements on the domain of functions. This domain consists of elements of a fuzzy partition of some interval $[a, b]$ of real numbers (universe of discourse). Having this in mind, we propose to construct a function by (fuzzy set)-to-point characterization instead of point-to-point characterization (more details can be found in [7]).

We will consider functions with one variable because of space limitation. However, F-transform of a function of two or more variables can be easily obtained as a straightforward extension of definitions given below (see [9, 7] for more details).

Definition 1 ([7]). *Let $[a, b]$ be a real interval and $x_1 < \cdots < x_n$ be fixed nodes within $[a, b]$ such that $x_1 = a$, $x_n = b$ and $n \geq 2$. We say that fuzzy sets $A_1, \ldots, A_n$ identified with their membership functions $A_1(x), \ldots, A_n(x)$ and defined on $[a, b]$*[1] *constitute a* fuzzy partition *of $[a, b]$ if they fulfil the following conditions for $k = 1, \ldots, n$:*

(1) $A_k : [a, b] \longrightarrow [0, 1]$, $A_k(x_k) = 1$;
(2) $A_k(x) = 0$ if $x \notin (x_{k-1}, x_{k+1})$;
(3) $A_k(x)$ is continuous;
(4) $A_k(x)$, $k = 2, \ldots, n$, monotonically increases on $[x_{k-1}, x_k]$ and $A_k(x)$, $k = 1, \ldots, n-1$, monotonically decreases on $[x_k, x_{k+1}]$;
(5) for all $x \in [a, b]$

$$\sum_{k=1}^{n} A_k(x) = 1. \tag{1}$$

The membership functions $A_1(x), \ldots, A_n(x)$ are called basic functions.

We say that a fuzzy partition $A_1(x), \ldots, A_n(x)$, $n > 2$, is *uniform* if the nodes $x_1, \ldots, x_n$ are equidistant, i.e. $x_k = a + h(k-1)$, $k = 1, \ldots, n$, where $h = (b-a)/(n-1)$, and two more properties are fulfilled for $k = 2, \ldots, n-1$:

(6) $A_k(x_k - x) = A_k(x_k + x)$, for all $x \in [0, h]$,
(7) $A_k(x) = A_{k-1}(x - h)$, for all $x \in [x_k, x_{k+1}]$ and $A_{k+1}(x) = A_k(x - h)$, for all $x \in [x_k, x_{k+1}]$.

Let $C[a, b]$ be the set of continuous functions on interval $[a, b]$. The following definition (see also [5, 7]) introduces the fuzzy transform of a function $f \in C[a, b]$.

Definition 2. *Let $A_1, \ldots, A_n$ be basic functions which constitute a fuzzy partition of $[a, b]$ and f be any function from $C[a, b]$. We say that the n-tuple of real numbers $[F_1, \ldots, F_n]$ given by*

$$F_k = \frac{\int_a^b f(x) A_k(x) dx}{\int_a^b A_k(x) dx}, \qquad k = 1, \ldots, n, \tag{2}$$

[1]Membership function of a fuzzy set on $[a, b]$ is a mapping from $[a, b]$ to $[0, 1]$.

is the (integral) F-transform of f with respect to $A_1, \ldots, A_n$.

We will denote the F-transform of a function $f \in C[a,b]$ with respect to $A_1, \ldots, A_n$ by $\mathbf{F}_n[f]$. Then, according to Definition 2, we can write $\mathbf{F}_n[f] = [F_1, \ldots, F_n]$. The elements $F_1, \ldots, F_n$ are called *components of the F-transform.*

The F-transform with respect to $A_1, \ldots, A_n$ establishes a linear mapping from $C[a,b]$ to $\mathbb{R}^n$ so that

$$\mathbf{F}_n[\alpha f + \beta g] = \alpha \mathbf{F}_n[f] + \beta \mathbf{F}_n[g]$$

for $\alpha, \beta \in \mathbb{R}$ and functions $f, g \in C[a,b]$. This linear mapping is denoted by $\mathbf{F}_n$ where n is dimension of the image space.

At this point we will refer to [7] for some useful properties of the F-transform components. The most important property concerns the following problem: how accurately is the original function f represented by its F-transform? We will show in this contribution that under certain assumptions on the original function, the components of its F-transform are *weighted mean values* of the given function where the weights are given by the basic functions.

Lemma 1. *Let f be a continuous function on $[a,b]$ and $A_1, \ldots, A_n$, $n \geq 3$, be basic functions which constitute a uniform fuzzy partition of $[a,b]$. Let $F_1, \ldots, Fn$, be the F-transform components of f with respect to $A_1, \ldots, A_n$. Then for each $k = 1, \ldots, n-1$, and for each $t \in [x_k, x_{k+1}]$ the following estimations hold:*

$$|f(t) - F_k| \leq 2\omega(h, f), \qquad |f(t) - F_{k+1}| \leq 2\omega(h, f)$$

where $h = \frac{b-a}{n-1}$ and

$$\omega(h, f) = \max_{|\delta| \leq h} \max_{x \in [a, b-\delta]} |f(x+\delta) - f(x)|$$

is the modulus of continuity of f on $[a,b]$.

Theorem 1. *Let f be a continuous function on $[a,b]$ and $A_1, \ldots, A_n$ be basic functions which constitute a fuzzy partition of $[a,b]$. Then the k-th component F_k ($k = 1, \ldots, n$) minimizes the function*

$$\Phi(y) = \int_a^b (f(x) - y)^2 A_k(x) dx.$$

Lemma 2. *Let function f be a continuous on $[a,b]$ and basic functions $A_1, \ldots, A_n$, $n \geq 3$, constitute a uniform fuzzy partition of $[a,b]$. Let $F_1, \ldots, F_n$, be the F-transform components of f with respect to $A_1, \ldots, A_n$. Then*

$$\int_a^b f(x) dx = h(\frac{1}{2}F_1 + F_2 + \cdots + F_{n-1} + \frac{1}{2}F_n).$$

Let us now consider a discrete case, when the original function f is known (may be computed) only at some nodes $p_1, \ldots, p_l \in [a,b]$. We assume that the set $P = \{p_1, \ldots, p_l\}$ of them *sufficiently dense with respect to the fixed partition*, i.e.

$$(3) \qquad (\forall k)(\exists j) \quad A_k(p_j) > 0.$$

Then the (discrete) F-transform of f is introduced as follows.

Definition 3 ([7]). *Let a function f be given at nodes $p_1, \ldots, p_l \in [a, b]$ and $A_1, \ldots, A_n$, $n < l$, be basic functions which constitute a fuzzy partition of $[a, b]$. We say that the n-tuple of real numbers $[F_1, \ldots, F_n]$ is the discrete F-transform of f with respect to $A_1, \ldots, A_n$ if*

$$F_k = \frac{\sum_{j=1}^{l} f(p_j) A_k(p_j)}{\sum_{j=1}^{l} A_k(p_j)}. \tag{4}$$

Similarly to the integral F-transform, we may show that the components of the discrete F-transform are the *weighted mean values* of the given function where the weights are given by the basic functions.

2.1. Inverse F-transform

The inverse F-transform (with respect to $A_1, \ldots, A_n$) takes an n-dimensional vector of reals $\mathbf{r} = (r_1, \ldots, r_n)$ and produces a linear combination of basic functions with coefficients given by $\mathbf{r}$. In the case when $\mathbf{r}$ is the F-transform with respect to $A_1, \ldots, A_n$ of some function $f \in C[a, b]$, i.e. $\mathbf{r} = \mathbf{F}_n[f]$, the above mentioned linear combination is an inversion formula.

Definition 4 ([7]). *Let $A_1, \ldots, A_n$ be basic functions which constitute a fuzzy partition of $[a, b]$ and f be a function from $C[a, b]$. Let $\mathbf{F}_n[f] = [F_1, \ldots, F_n]$ be the integral F-transform of f with respect to $A_1, \ldots, A_n$. Then the function*

$$f_{F,n}(x) = \sum_{k=1}^{n} F_k A_k(x) \tag{5}$$

is called the inverse F-transform.

The theorem below shows that the inverse F-transform $f_{F,n}$ can approximate the original continuous function f with an arbitrary precision.

Theorem 2 ([7]). *Let f be a continuous function on $[a, b]$. Then for any $\varepsilon > 0$ there exist n_ε and a fuzzy partition $A_1, \ldots, A_{n_\varepsilon}$ of $[a, b]$ such that for all $x \in [a, b]$*

$$|f(x) - f_{F,n_\varepsilon}(x)| \leq \varepsilon \tag{6}$$

where f_{F,n_ε} is the inverse F-transform of f with respect to the fuzzy partition $A_1, \ldots, A_{n_\varepsilon}$.

In the discrete case, we define the inverse F-transform only at nodes where the original function is given:

$$f_{F,n}(p_j) = \sum_{k=1}^{n} F_k A_k(p_j).$$

Analogously to Theorem 2, we may show that the inverse discrete F-transform $f_{F,n}$ can approximate the original function f at common nodes with an arbitrary precision (see [7]).

3. F-Transforms Expressed by Residuated Lattice Operations

Two other fuzzy transforms which are based on operations of a residuated lattice on $[0, 1]$ will be analyzed in this section (see [7]). They are based on the same principle as the ordinary F-transform which has been introduced above and to which we will refer using this adjective. Both new F-transforms extend and generalize the ordinary one.

Let us recall the concept of residuated lattice [1] which will be the basic algebra of operations in the sequel.

Definition 5. *A residuated lattice is an algebra*

$$\mathcal{L} = \langle L, \vee, \wedge, *, \rightarrow, \mathbf{0}, \mathbf{1} \rangle.$$

with four binary operations and two constants such that

(i) $\langle L, \vee, \wedge, \mathbf{0}, \mathbf{1} \rangle$ *is a lattice;*
(ii) $\langle L, *, \mathbf{1} \rangle$ *is a commutative monoid;*
(iii) the operation $\rightarrow$ *is a residuation operation with respect to* $*$, *i.e.*

$$a * b \leq c \quad \textit{iff} \quad a \leq b \rightarrow c.$$

The well known examples of residuated lattices are boolean algebras, Gödel, Łukasiewicz and product algebras. In the particular case $L = [0, 1]$, the multiplication $*$ is called t-norm. In the foregoing text we will deal with some fixed residuated lattice $\mathcal{L}$ on $[0, 1]$.

3.1. Direct F$^\uparrow$ and F$^\downarrow$-transforms

Let the universe be the interval $[0, 1]$. We will weaken the notion of *fuzzy partition* of $[0, 1]$ and assume that fuzzy sets $A_1, \ldots, A_n$, $n \geq 2$ constituting the partition of $[0, 1]$ fulfil the following (only one!) *covering property*:

$$(\forall x)(\exists i) \quad A_i(x) > 0. \tag{7}$$

As above, the membership functions $A_1(x), \ldots, A_n(x)$ are called the *basic functions.* In the sequel, we fix the value of n $(n \geq 2)$ and some fuzzy partition of $[0, 1]$ by basic functions $A_1, \ldots, A_n$.

We assume that a finite subset $P = \{p_1, \ldots, p_l\}$ of $[0, 1]$ is fixed. Moreover, we assume that P is sufficiently dense with respect to the fixed partition, i.e. (3) holds.

Definition 6. *Let a function* f *be defined at nodes* $p_1, \ldots, p_l \in [0, 1]$ *and* $A_1, \ldots, A_n$, $n < l$, *be basic functions which constitute a fuzzy partition of* $[a, b]$. *We say that the* n*-tuple of real numbers* $[F_1^\uparrow, \ldots, F_n^\uparrow]$ *is a (discrete)* $F^\uparrow$*-transform of* f *w.r.t.* $A_1, \ldots, A_n$ *if*

$$F_k^\uparrow = \bigvee_{j=1}^{l} (A_k(p_j) * f(p_j)) \tag{8}$$

and the n*-tuple of real numbers* $[F_1^\downarrow, \ldots, F_n^\downarrow]$ *is the (discrete)* $F^\downarrow$*-transform of* f *w.r.t.* $A_1, \ldots, A_n$ *if*

$$F_k^\downarrow = \bigwedge_{j=1}^{l} (A_k(p_j) \rightarrow f(p_j)). \tag{9}$$

Denote the $F^{\uparrow}$-transform of f w.r.t. $A_1, \ldots, A_n$ by $\mathbf{F}_n^{\uparrow}[f]$ and the $F^{\downarrow}$-transform of f w.r.t. $A_1, \ldots, A_n$ by $\mathbf{F}_n^{\downarrow}[f]$. Then we may write:

$$\mathbf{F}_n^{\uparrow}[f] = [F_1^{\uparrow}, \ldots, F_n^{\uparrow}], \qquad \mathbf{F}_n^{\downarrow}[f] = [F_1^{\downarrow}, \ldots, F_n^{\downarrow}].$$

Analogously to Theorem 1 we may show that components of the lattice based F-transforms are *lower mean values* (respectively, *upper mean values*) of an original function which give the least (greatest) elements to certain sets.

3.2. Inverse $F^{\uparrow}$ ($F^{\downarrow}$)-Transforms

In the construction of inverse $F^{\uparrow}$- and $F^{\downarrow}$-transforms we use the fact that the operations $*$ and $\rightarrow$ are mutually adjoint in a residuated lattice.

Definition 7. *Let a function f be defined at nodes $p_1, \ldots, p_l \in [a, b]$ and let $\mathbf{F}_n^{\uparrow}[f] = [F_1^{\uparrow}, \ldots, F_n^{\uparrow}]$ be the $F^{\uparrow}$-transform of f and $\mathbf{F}_n^{\downarrow}[f] = [F_1^{\downarrow}, \ldots, F_n^{\downarrow}]$ be the $F^{\downarrow}$-transform of f w.r.t. basic functions $A_1, \ldots, A_n$. Then the following functions, defined at the same nodes as f, are called the* inverse $F^{\uparrow}$-transform

$$f_{F,n}^{\uparrow}(p_j) = \bigwedge_{k=1}^{n} (A_k(p_j) \rightarrow F_k^{\uparrow}), \tag{10}$$

and the inverse $F^{\downarrow}$-transform

$$f_{F,n}^{\downarrow}(p_j) = \bigvee_{k=1}^{n} (A_k(p_j) * F_k^{\downarrow}), \tag{11}$$

The following theorem shows that the inverse $F^{\uparrow}$- and $F^{\downarrow}$-transforms approximate the original function from above and from below.

Theorem 3. *Let function f be defined at nodes $p_1, \ldots, p_l \in [0, 1]$. Then for all $j = 1, \ldots, l$*

$$f_{F,n}^{\downarrow}(p_j) \leq f(p_j) \leq f_{F,n}^{\uparrow}(p_j). \tag{12}$$

4. Application of the F-Transform to Image Compression and Reconstruction

A method of lossy image compression and reconstruction on the basis of fuzzy relations has been proposed in a number of papers (see e.g. [2, 3]). When analyzing these methods, we have realized that they can be expressed using F-transforms based on lattice operations. In more details and in a language of F-transforms, a compressed image is obtained as a result of the $F^{\uparrow}$-transform (with a respective choice of the operation $*$) and a reconstructed image is obtained as a result of the inverse $F^{\uparrow}$-transform. It was challenging for us to compare the above cited method (which is widely propagated by the authors of [2]) with the analogous one based on the ordinary F-transform which is propagated in this contribution. Below, we will see that all considered examples witness the advantage of the ordinary F-transform (2) over the lattice based F-transform (cf. (8).

On Figure 1 we illustrate the proposed compression method and reconstruction based on the ordinary F-transform and the $F^{\uparrow}$-transform (with a choice of the Łukasiewicz algebra). The computation is made using a software created by the second author of this contribution. The advantage of the ordinary F-transform over the lattice based F-transform is visible.

a)

b) c)

FIGURE 1. The original image "Woman" a) is compressed and reconstructed by the ordinary F-transform method (picture b)) and by the lattice based $F^{\uparrow}$-transform method (picture c)). The compression ratio $\rho = 0.25$.

5. Application of the F-Transform to Removing of Noise

The problem of removing noise relates to nonlinear signal processing methods. The latter have developed rapidly in recent decades to address various problems characterized, e.g., by non-stationary statistics. In this contribution we characterize an additive noise which can be removed by applying the inverse F-transform to the original function.

We will consider a noise, represented by a function $s(x)$ and such that $f(x) + s(x)$ is the representation of the noised function f. We will refer to this type of a noise as *additive* noise. On the basis of linearity of the direct (ordinary) F-transform, this noise can be removed if its components of the direct F-transform are equal to zero. This simple fact is formulated in the following theorem.

Theorem 4 ([6]). *Let $f(x)$ and $s(x)$ be continuous functions on $[a, b]$ and $A_1(x), \ldots, A_n(x)$, $n > 2$, be basic functions which constitute a fuzzy partition of $[a, b]$. Assume* *that the components of the direct F-transform of s w.r.t.* $A_1, \ldots, A_n$ *are equal to zero. Then*

$$f_{F,n}(x) = (f+s)_{F,n}(x), \quad x \in [x_2, x_{n-1}]$$

where $f_{F,n}$, $(f+s)_{F,n}$ *are the inverse F-transforms of functions* f *and* $(f+s)$ *respectively.*

Below, we describe properties of an additive noise which guarantee that the noise is removable, see [6] for more details.

Theorem 5. *Let* $A_1, \ldots, A_n$ *be a uniform fuzzy partition of interval* $[a, b]$ *such that* $h = (b-a)/(n-1)$ *and* $n > 2$. *Let us consider a function* $s(x)$ *defined on* $[a, b]$ *and such that*

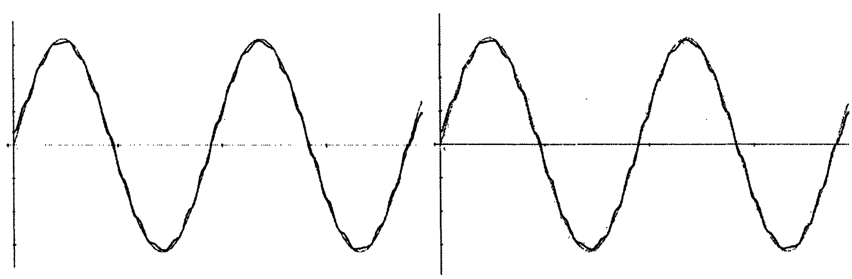
$$s(x+h) = s(x), \qquad x \in [x_1, x_{n-2}]$$

and

$$\int_{x_{k-1}}^{x_k} s(x)\mathrm{dx} = 0, \qquad 2 \le k \le n-1.$$

Then components $S_2, \ldots, S_{n-1}$ *of the direct F-transform of s are equal to zero.*

On Figure 2 we illustrate the effect of removing noise by computing two inverse F-transforms: of an original function and of the same function distorted by a random noise. It is seen that both inverse F-transforms are the same.



a) b)

FIGURE 2. Two inverse F-transforms (thick lines) are the same for an original function (thin line) and for the same function distorted by a random noise (noisy graph on Picture b)).

6. Application of the F-Transform to Image Fusion

Image fusion aims at integration complementary multiview information into one new image with the best possible quality. The term "quality" depends on the demands of specific application.

Mathematically, if u is an ideal image (considered as a function with two variables) and $C_1, \ldots, C_N$ are acquired channels then the relation between each C_i and u is expressed by

$$C_i(x, y) = D_i(u(x, y)) + E_i(x, y)$$

where D_i is an unknown operator describing the image degradation and E_i is an additive random noise. To fuse images from channels means to obtain an image $\hat{u}$ which gives in some sense better representation of u than each individual channel C_i. Different fusion methodologies are influenced by peculiarities of degradation operators D_i. In this contribution, we assume that every point (x, y) of the image can be acquired undistorted in (at least) one channel. Image fusion then consists of comparing the channels in image domain, identifying the channel in which the pixel (or the region) is depicted undistorted and, finally, of combining the undistorted parts.

To find the fused image, we propose the following algorithm (for the simplicity we assume that all images are represented by functions with one variable on the same domain $[a, b]$ and a uniform fuzzy partition of $[a, b]$ is fully determined by the number of basic functions):

Step 1. Choose $\varepsilon > 0$ and let $k = 1$. Denote each channel function C_i by f_i^k, $i = 1, \dots, N$.

Step 2. Let $n = 2^k$. Compute the direct and inverse fuzzy transforms of all N functions f_i^k. Denote the direct fuzzy transforms $\mathbf{F}_n[f_i^k]$ and the inverse fuzzy transforms $f_{F,n,i}^k$, $i = 1, \dots, N$.

Step 3. Compute differences and denote $f_i^{k+1} = f_i^k - f_{F,n,i}^k$, $i = 1, \dots, N$. If $\max_{[a,b]} |f_i^{k+1}| > \varepsilon$, let $k = k + 1$ and go to *Step 2.* Otherwise, denote $k_{max} = k$.

Step 4. Compute the new "sharp" direct fuzzy transform $\mathbf{F}_n[s^k] = [S_1^k, \dots, S_n^k]$ where for each $j = 1, \dots, n$, S_j^k is that value among $\mathbf{F}_n[f_i^k]_j$ whose absolute value is the largest.

Step 5. Compute the new "sharp" inverse fuzzy transform s^k with the components given by $\mathbf{F}_n[s^k]$.

Step 6. Let $k = k - 1$ and until $k \geq 1$ repeat *Step 4.* and *Step 5.*

Step 7. Compute $\sum_{k=1}^{k_{max}} s^k$ and take it as a fused image.

Justification. By Lemma 1, the smaller the modulus of continuity the higher the quality of approximation of an original function by its inverse fuzzy transform. Therefore, for the same partition the accuracy of approximation of each difference function f_i^k by its inverse fuzzy transform is better than the accuracy of approximation of the original function C_i by its inverse fuzzy transform.

If a certain part of a function is affected by degradation, then by Theorem 1, the respective fuzzy transform component is close to zero. Therefore, on *Step 4.* we chose components with maximal absolute values.

On Figure 3 we show two-channel image fusion based on F-transform technique: In one channel, the figure is in focus and the background is out of focus, while in the other channel it is vice versa. Image fusion is performed via combining the channel regions which are in focus.

Finally, the following value is proposed as a measure of degradation of each channel $i = 1, \dots, N$:

$$M_i^{deg} = \sum_{k=2}^{k_{max}} \sum_{j=1}^{2^k} |\mathbf{F}_n[f_i^k]_j|.$$

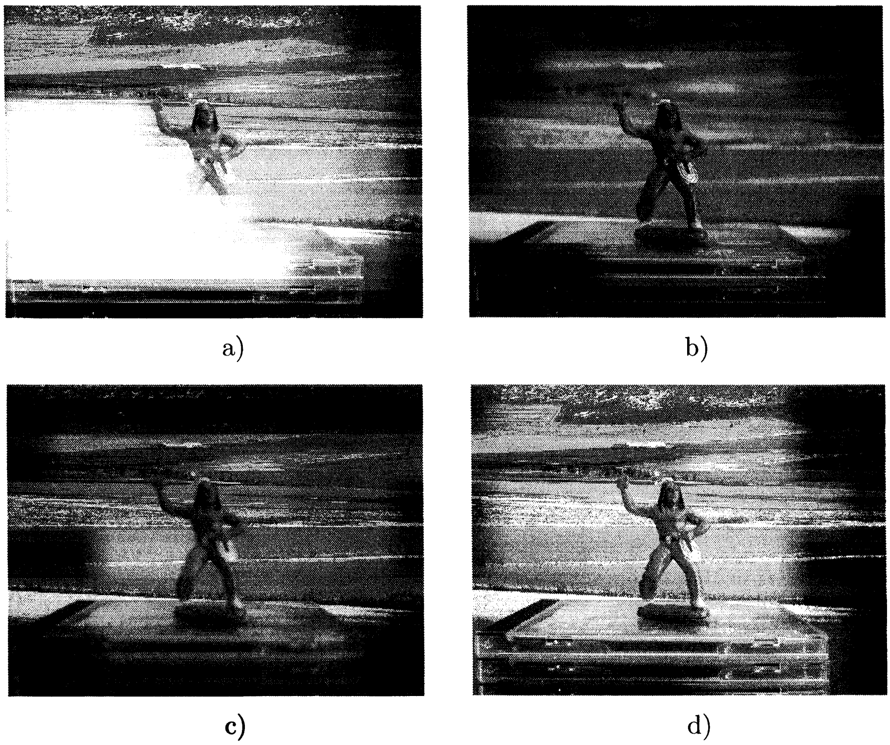


FIGURE 3. The original image is given on Picture a). Two channel images: Picture b) - the figure is in focus) and Picture c)- the background is in focus) are fused with the result on Picture d).

By Lemma 2, we obtain the following estimation

$$M_i^{deg} \leq \sum_{k=3}^{k_{max}} \int_a^b |f_i^{k-1}(x) - f_i^{k-2}(x)| dx.$$

Therefore, the bigger M_i^{deg} the less degradation has the channel C_i.

7. Conclusion

We gave an overview of direct and inverse fuzzy transforms of three types and their applications. We have shown that components of direct F-transforms are weighted mean values which keep aggregated information about an original function in local areas of its domain. On the basis of this, the inverse F-transform produces approximation of the original function and has exceptional filtering properties.

Three applications of F-transform to data processing have been presented: compression and reconstruction of the data, removing noise and newly also data fusion. All of them successfully exploit the mentioned filtering property.

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