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FLOCKING CONTROL OF MULTI–AGENT SYSTEMS WITH APPLICATION TO NONHOLONOMIC MULTI–ROBOTS

Qin Li and Zhong-Ping Jiang

In this paper, we revisit the artificial potential based approach in the flocking control for multi-agent systems, where our main concerns are migration and trajectory tracking problems. The static destination or, more generally, the moving reference point is modeled by a virtual leader, whose information is utilized by some agents, called active agents (AA), for the controller design. We study a decentralized flocking controller for the case where the set of AAs is fixed. Some results on the velocity consensus, collision avoidance, group configuration and robustness are proposed. Further, we apply the proposed controller to the observer based flocking control of a team of nonholonomic mobile robots.

Keywords: multi-agent systems, flocking control, nonholonomic mobile robots, decentralized control

AMS Subject Classification: 93A14, 93C15

1. INTRODUCTION

A flock can be seen as a "loose" but connected formation which does not require the group to be in a unique geometric pattern (see [11]). Many existing results on flocking control of multi-agent systems rely on the concept called (artificial) potential fields or potential functions. The idea based on this concept is to relate the desired geometric patterns (or configurations) to the local or global extremes of an elaborately cooked potential function of the group, and then design the gradient-based control strategy to drive the group to minimize the potential function. The problem of flocking control for particle vehicles with single or double integrator models is worthy of study not only because it can provide high level control strategies for flocking control of multi-vehicle teams with more complex dynamics, but also due to its value in determining the effects of information flow in the distributed control of coupled systems. In the early paper [8], virtual leaders of the group are introduced and pair-wise potential not only exist between real agents in the group but also between a real agent and the virtual leader. The aim of adding a virtual leader is to help shape the potential function for the group so that it can be stabilized at the desired geometric pattern (not only a flock). In [11], the author describes a smooth pair-wise potential function whose gradient specifies a kind of attractive/repulsive force between neighboring agents which is continuous with respect to the relative distance. It is proved in [11] that the control law combining the potential's gradient term with velocity matching term coincides with the Reynolds rules but will generically lead to regular fragmentation of the group. The work [13] relaxes the requirement on the smoothness of the pair-wise potentials but similar controllers as in [11] are adopted. And the system stability is analyzed by the nonsmooth version of LaSalle Invariance Principle.

In this paper, we propose control strategies aimed at migration and trajectory tracking of a group of agents. A virtual leader is used to represent the stationary destination of the migration or a moving reference point on the trajectory being tracked by the group. Along the line of [8, 11] and [13], we revisit the design of gradient-based control laws in the artificial potential framework, which has advantage on the inter-agent collision avoidance issue. It is assumed that some of the agents, called *active agents* (AA), in the group utilize the position and velocity information of the virtual leader as well as their neighboring agents in the controllers, and that the other agents only use that information of their neighbors. The velocity consensus and the configuration convergence of the group by the proposed controllers are analyzed.

The paper is composed of two parts. In the first part, we design a flocking controller for particle agents with double integrator model. At the current stage, we only discuss the case in which the AAs in the group are fixed. We show that, by our controller, the velocities of the group reach consensus; inter-agent collision is avoided; and the configuration of the group almost converges to some local minimum of the collective potentials of the group. As a special case, we give a result on the geometric property of the group with only one AA. Also, we establish the results on velocity consensus, collision avoidance and configuration convergence for the system with some kind of disturbance.

In the second part, the controller designed for the mass point model is applied to the flocking control of a group of unicycles. Specially, we study the case where each unicycle in the group cannot measure its velocity information. The passive observer developed in [1] is used to observe the linear and angular velocities for each agent. And the estimated data are transmitted between each pair of neighboring unicycles for the use of controller design.

The rest of the paper is organized as follows: In Section 2, we introduce some basics of graph theory and the properties of the potential functions used in this work. In Section 3, we present our results on the flocking control of particle model. In Section 4, we describe the flocking control design for multiple unicycles based on the results obtained in Section 3. Simulation results are presented in Section 5, while concluding remarks are made in Section 6.

2. PRELIMINARIES

In this work, we frequently use the map $\|\cdot\|_\sigma : \mathbb{R}^n \to \mathbb{R}^+$,

$$\|x\|_\sigma = \frac{1}{\sigma}(\sqrt{1+\sigma\|x\|^2} - 1) \tag{1}$$

to measure the inter-agent or leader-agent distance, where the parameter $\sigma > 0$, and $\|\cdot\|$ is the Euclidean norm. This map has the following properties: a) $\|\cdot\|_\sigma \in C^2$ on $\mathbb{R}^n$; b) $\|x\|_\sigma = 0 \Leftrightarrow x = \mathbf{0}_n$; c) It is strictly increasing with respect to (for short, w.r.t.) $\|x\|$; d) The gradient $\nabla\|x\|_\sigma = \frac{x}{\sqrt{1+\sigma\|x\|^2}}$, and $\big\|\nabla\|x\|_\sigma\big\| \leq \frac{1}{\sqrt{\sigma}}$; e) $\big|\|x_2\|_\sigma - \|x_1\|_\sigma\big| \leq \frac{1}{\sqrt{\sigma}}\|x_2 - x_1\|$, $\forall\, x_1, x_2 \in \mathbb{R}^n$. The map $\|\cdot\|_\sigma$ was previously used in [11] (called σ-norm therein) to construct smooth potential functions.

2.1. Graph theory

First, we recall some basics of graph theory from the past literature, see, e. g. [2]. An undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ consists of a vertex set $\mathcal{V}$ and an edge set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$. For any $i, j \in \mathcal{V}$, $(i, j) \in \mathcal{E}$ if and only if j is a neighbor of i. A path from vertex i to j is a sequence of edges $(v_1, v_2), (v_2, v_3), \ldots, (v_{n-2}, v_{n-1}), (v_{n-1}, v_n)$, where $n \geq 2, v_1 = i, v_n = j$, and $v_1, \ldots, v_n$ are distinct.

In this work, we use $\mathcal{G}_p(\mathcal{V}, \mathcal{E}(t))$, or simply $\mathcal{G}_p(t)$, to denote the group induced undirected graph for a group of N agents, where the vertex set $\mathcal{V}$ and the edge set $\mathcal{E}(t), t \geq t_0$, are defined as:

$$\mathcal{V} = \{1, 2, \ldots, N\}, \tag{2}$$

$$\mathcal{E}(t) = \{(i, j) : \|x_i(t) - x_j(t)\|_\sigma \leq r_{nb},\ i, j \in \mathcal{V}\}, \quad t \geq t_0 \tag{3}$$

where N is the number of agents in the group, r_{nb} is a positive real number less than r_s, which denotes the physical sensing and communication range of each agent.

The adjacency matrix $A(t) \in \mathbb{R}^{N\times N}$ and the Laplacian $L(t) \in \mathbb{R}^{N\times N}$ of the graph $\mathcal{G}_p(t)$ are defined as:

$$A_p(t) = [a_{ij}(t)],\ \text{with } a_{ij}(t) = \begin{cases} a^*_{ij} > 0, & \text{if } (i, j) \in \mathcal{E}(t) \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

where $a^*_{ij} = a^*_{ji}$, $\forall\, i, j \in \mathcal{V}$; and

$$L_p(t) = [l_{ij}(t)],\ \text{with } l_{ij}(t) = \begin{cases} \sum_{k\neq i} a_{ik}(t), & \text{if } i = j \\ -a_{ij}(t), & \text{otherwise.} \end{cases} \tag{5}$$

Obviously, $A_p(t)$ and $L_p(t)$ are both symmetric, and $L_p(t)$ is positive semi-definite.

Throughout this paper, we call an agent *active agent* (AA) of the group if it utilizes the position and velocity information of the virtual leader in its controller. The set of the AA's at time $t, t \geq t_0$, is denoted by $\mathcal{W}(t)$. In addition, we define matrices

$$B(t) = \text{diad}\{b_1(t), \ldots, b_N(t)\}, \tag{6}$$

$$L_a(t) = L_p(t) + B(t). \tag{7}$$

with

$$b_i(t) = \begin{cases} b^*_i > 0, & \text{if } i \in \mathcal{W}(t) \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

2.2. Potential functions

In this subsection, we introduce potential functions that characterize, respectively, the inter-agent and leader-agent attraction and repulsion.

2.2.1. Inter-agent potential

The inter-agent potential function $\psi_a(\cdot) : (d_{sa}, +\infty) \to [0, +\infty)$, $d_{sa} \geq 0$, is a C^2 function with the following properties: for some positive numbers d_a, r_a satisfying $d_{sa} < d_a < r_a < r_{nb}$,

a) $\frac{\mathrm{d}\psi_a(x)}{\mathrm{d}x} < 0, x \in (d_{sa}, d_a)$; $\frac{\mathrm{d}\psi_a(x)}{\mathrm{d}x} > 0, x \in (d_a, r_a)$; $\frac{\mathrm{d}\psi_a(x)}{\mathrm{d}x} = 0, x \in [r_a, +\infty)$;

b) $\lim_{x \to d_{sa}} \psi_a(x) = +\infty$;

c) $\psi_a(x)$ has a unique minimum at $x = d_a$.

Inspired by the work [11], an example of inter-agent potential can be chosen as:

$$\psi_a(x) = \int_{d_a}^{x} 10 \cdot \left(-\frac{1}{(\xi - d_{sa})^2} + \frac{1}{(d_a - d_{sa})^2} \right) \varrho_h \left(\frac{\xi}{r_a} \right) \mathrm{d}\xi, \quad x \in (d_{sa}, +\infty), \quad (9)$$

where $\varrho_h(z)$ is a bump function defined as:

$$\varrho_h(z) = \begin{cases} 1, & z \in [0, h) \\ \frac{1}{2}\left[1 + \cos\left(\pi \frac{z-h}{1-h}\right)\right], & z \in [h, 1] \\ 0, & z \in (1, +\infty). \end{cases} \quad (10)$$

Here d_{sa} is called safety distance which can be selected to account for inter-agent collision avoidance for the agents with non-point models. In the rest of the paper, inter-agent collision is said to be avoided if and only if the distance, measured in σ-norm, between any pair of agents is greater than d_{sa}. d_a is the critical distance for the repulsive and attractive virtual force between a pair of agents (see the definition after (20) below). r_a is crucial for the choice of dwell time to be introduced in Section 3.

2.2.2. Leader-agent potentials

The leader-agent potential function $\psi_l(\cdot) : [0, +\infty) \to [0, +\infty)$ is a C^2 function with the following properties:

a) $\frac{\mathrm{d}\psi_l(x)}{\mathrm{d}x} = 0$, for $x = 0$; $\frac{\mathrm{d}\psi_l(x)}{\mathrm{d}x} > 0$, for all $x > 0$;

b) $\lim_{x \to +\infty} \psi_l(x) = +\infty$;

c) For any given $x_* > 0$, $\exists\, \varepsilon(x_*) > 0$ such that $\frac{\mathrm{d}\psi_l(x)}{\mathrm{d}x} > \varepsilon$, $\forall\, x \geq x_*$.

It is easy to see that $\psi_l(x)$ has a unique minimum at $x = 0$. An example of function ψ_l is $\frac{x^2}{2} + C$, with $C \in \mathbb{R}^+$.

Throughout this paper, we use $\mathbb{N}, \mathbb{R}^+, \mathbb{Z}^+$ to denote, respectively, the set of natural numbers, nonnegative real numbers and nonnegative integers. $\mathcal{L}_p^m[t_0, +\infty)$ is used to denote the set of all piecewise continuous functions $u : [t_0, +\infty) \to \mathbb{R}^m$ such that $\left(\int_{t_0}^{+\infty} \|u(t)\|^p \, \mathrm{d}t\right)^{1/p} < +\infty$, [6]. In addition, we use $\mathbf{1}_N$ to represent the $N \times 1$ vector with all the elements being 1.

3. FLOCKING CONTROLLER FOR DOUBLE–INTEGRATOR MODEL

In this section, we consider the model of each agent in the group as:

$$\dot{x}_i(t) = v_i(t), \quad \dot{v}_i(t) = u_i(t), \quad i \in \mathcal{V}, \tag{11}$$

where $x_i(t) \in \mathbb{R}^n$ and $v_i(t) \in \mathbb{R}^n$ ($n = 2, 3$) are the position and velocity of the ith robot respectively; and $u_i(t)$ is the control input (acceleration) of the ith robot. The model for the virtual leader is in the same form as that of the agent, i. e.,

$$\dot{x}_l(t) = v_l(t), \quad \dot{v}_l(t) = u_l(t) \tag{12}$$

where "l" stands for the word "leader". Here the virtual leader represents a static destination or a moving reference point for the group.

We emphasize that in this work, for simplicity of derivation, we only discuss the flocking behavior of a group of robots with fixed AAs, i. e. we make the assumption:

Assumption 1. The set of active agents in the group $\mathcal{W}$ is nonempty and fixed.

Remark 1. Updating rules for the set of AAs have been developed to deal with some connectivity guaranteeing issues [9].

Since, under Assumption 1, the set $\mathcal{W}(t)$ and the matrix $B(t)$ in (6) are time-invariant, we drop the argument t in their expressions.

It is known that the mobility and limited sensing range of the agents in the group raises the issue that the neighboring relationship of the group may be time-varying. For this reason, to start with our discussion, we need to define the following time-dependent agent sets:

Definition 1. Agent sets $\mathcal{S}_i(t), \mathcal{N}_i(t), \mathcal{I}_i(t)$, $i \in \mathcal{V}, t \in [t_0, +\infty)$ are defined as

$$\mathcal{S}_i(t) = \{j \in \mathcal{V} : \|x_i(t) - x_j(t)\| < r_s\}, \tag{13}$$

$$\mathcal{N}_i(t) = \{j \in \mathcal{V} : \|x_i(t) - x_j(t)\| < r_{nb}\}, \tag{14}$$

$$\mathcal{I}_i(t) = \{j \in \mathcal{V} : \|x_i(t) - x_j(t)\| < r_a\}, \tag{15}$$

where r_{nb} and r_s are defined in Subsection 2.1; and r_a is as in Subsection 2.2. Obviously, we have the relation: $r_a < r_{nb} < r_s$.

Note that in [13], the solutions of the switching closed-loop system are discussed using the tool of differential inclusion. But in this way, one cannot specify the single rate of change of the state when the system switches since it can only be said to lie in a set. In view of this, in the analysis of the closed-loop system, we introduce dwell time in the system dynamics. Indeed, our control strategy is that each agent determines its neighbor set at every moment in the time sequence

$$\mathcal{T} := \{t_0, t_1, \ldots\} \quad \text{with} \quad t_{k+1} - t_k = \tau_d > 0, \tag{16}$$

and for all $t \in [t_k, t_{k+1}), k \in \mathbb{Z}^+$, agent $i, i \in \mathcal{V}$ implements the decentralized control law

$$\begin{aligned} u_i^{af}(t) = & \sum_{j \in \mathcal{N}_i(t_k)} f_a(d_{ij}) n_{ji} + g(b_i) f_l(d_{il}) n_{li} \\ & - \sum_{j \in \mathcal{N}_i(t_k) \bigcap \mathcal{S}_i(t)} a_{ij}^*(v_i - v_j) - b_i(v_i - v_l) + u_l, \end{aligned} \tag{17}$$

where a_{ij}^* and b_i have been defined in (4) and (8); and

$$f_a(d_{ij}) = \frac{\mathrm{d}\psi_a(d_{ij})}{\mathrm{d}d_{ij}}, \quad f_l(d_{il}) = \frac{\mathrm{d}\psi_l(d_{il})}{\mathrm{d}d_{il}}, \tag{18}$$

$$d_{ij} = \|x_i - x_j\|_\sigma, \;\; d_{il} = \|x_i - x_l\|_\sigma, \;\; n_{ji} = -\nabla_{x_i} d_{ij}, \;\; n_{li} = -\nabla_{x_i} d_{il}, \tag{19}$$

$$g(y) = \begin{cases} 1, & y > 0, \\ 0, & y = 0. \end{cases} \tag{20}$$

Note that $f_a(d_{ij})n_{ji}, f_l(d_{il})n_{li}$ are sometimes called, respectively, the virtual force applied on agent i by agent j and the virtual leader.

In the third term of (17), we use "$j \in \mathcal{N}_i(t_k) \bigcap \mathcal{S}_i(t)$" since, taking the sensing capability of the agents into consideration, it is possible that some agent in the set $\mathcal{N}_i(t_k)$ moves out of the sensing range of agent i at some $t \in [t_k, t_{k+1})$. (For the first term, we can just use "$j \in \mathcal{N}_i(t_k)$" due to the property of the function $f_a(\cdot)$ that $f_a(d_{ij}) = 0$ for $d_{ij} \geq r_{nb}$.) However, in the following Lemma 1, we show that if τ_d is chosen small enough, then for all $t \in [t_k, t_{k+1})$, $j \in \mathcal{S}_i(t)$ for any $j \in \mathcal{N}_i(t_k)$, and $j \notin \mathcal{I}_i(t)$ for any $j \notin \mathcal{N}_i(t_k)$.

First, we define the collective inter-agent potential $V_a(x)$ and leader-agent potential $V_l(x, x_l)$ as follows

$$V_a(x) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j \neq i} \psi_a(d_{ij}), \quad V_l(x, x_l) = \sum_{i \in \mathcal{W}} \psi_l(d_{il}), \tag{21}$$

where $x = [x_1^\top, \ldots, x_N^\top]^\top$. In addition, we define functions $V(x, x_l) : \mathbb{R}^{(N+1)n} \to \mathbb{R}^+$, $H(v, v_l) : \mathbb{R}^{(N+1)n} \to \mathbb{R}^+$, $J(x, x_l, v, v_l) : \mathbb{R}^{2(N+1)n} \to \mathbb{R}^+$ as

$$\begin{aligned} V(x, x_l) &= V_a(x) + V_l(x, x_l), && (22) \\ H(v, v_l) &= \frac{1}{2} \|v - \mathbf{1}_N \otimes v_l\|^2, && (23) \\ J(x, x_l, v, v_l) &= V(x, x_l) + H(v, v_l), && (24) \end{aligned}$$

where $v = [v_1^\top, \ldots, v_N^\top]^\top$. In the following, with a little abuse of notation, we sometimes use $V_a(t)$, $V_l(t)$, $H(t), V(t), J(t)$ to denote the composite functions $V_a(x(t))$, $V_l(x(t), x_l(t))$, $H(v(t), v_l(t)), V(x(t), x_l(t)), J(x(t), x_l(t), v(t), v_l(t))$ respectively.

Lemma 1. Suppose $\|x_i(t_0) - x_j(t_0)\|_\sigma > d_{sa}, \forall\, i, j \in \mathcal{V}$ (i.e., the inter-agent collision does not occur initially). If $\tau_d < \min\{r_s - r_{nb}, r_{nb} - r_a\}\sqrt{\sigma}/(2\sqrt{2J(t_0)})$, then $\forall\, t \in [t_k, t_{k+1})$, $\forall\, k \in \mathbb{Z}^+$,

$$\mathcal{I}_i(t) \subset \mathcal{N}_i(t_k) \subset \mathcal{S}_i(t). \tag{25}$$

And $\forall\, i \in \mathcal{V}$, $\forall\, t \in [t_k, t_{k+1})$, $\forall\, k \in \mathbb{Z}^+$, the control law in (17) can be put into the form:

$$\begin{aligned} u_i^{af}(t) = &- \sum_{j \neq i} \nabla_{x_i} \psi_a(d_{ij}) - g(b_i) \nabla_{x_i} \psi_l(d_{il}) \\ &- \sum_{j \in \mathcal{N}_i(t_k)} a_{ij}^*(v_i - v_j) - b_i(v_i - v_l) + u_l, \end{aligned} \tag{26}$$

or compactly,

$$u^{af} = -\nabla_x V_a - \nabla_x V_l - (L_a(t_k) \otimes I_n)(v - \mathbf{1}_N \otimes v_l) + \mathbf{1}_N \otimes u_l. \tag{27}$$

where $u^{af} = [u_1^{af}, \ldots, u_N^{af}]^\top$.

Proof. Since the velocity v is continuous, there exists $\delta > 0$ such that (25) holds for $t \in [t_0, t_0 + \delta)$. By the fact that $f_a(d_{ij}) = 0$ for $d_{ij} \geq r_a$, we see that the control law (17) can be put into the form (26) during this time period. Now, we show that δ can be extended to t_1. By contradiction, suppose this is not true. Then, there exist some agent $j \in \mathcal{N}_i(t_0)$ and some time instant $t_\star \in [t_0, t_1)$ such that either $d_{ij}(t_\star) = r_s$ or $d_{ij}(t_\star) = r_a$. Without loss of generality, assume $d_{ij}(t_\star) = r_s$. Then, it follows that

$$\begin{aligned} r_s - r_{nb} &< \Big| \|x_j(t_\star) - x_i(t_\star)\|_\sigma - \|x_j(t_0) - x_i(t_0)\|_\sigma \Big| \\ &\leq \frac{1}{\sqrt{\sigma}} \|(x_j(t_\star) - x_i(t_\star)) - (x_j(t_0) - x_i(t_0))\| \\ &\leq \frac{1}{\sqrt{\sigma}} \left(\int_{t_0}^{t_\star} \|v_j(t) - v_l(t)\|\, \mathrm{d}t + \int_{t_0}^{t_\star} \|v_i(t) - v_l(t)\|\, \mathrm{d}t \right) \\ &\leq \int_{t_0}^{t_\star} 2\sqrt{\frac{2H(t)}{\sigma}}\, \mathrm{d}t \leq \int_{t_0}^{t_\star} 2\sqrt{\frac{2J(t)}{\sigma}}\, \mathrm{d}t. \end{aligned} \tag{28}$$

But on the interval $[t_0, t_\star)$, the derivative of the function J w.r.t. t along the solutions of (11), (17) and (12) is

$$\begin{aligned} \dot{J} &= \dot{V}_a + \dot{V}_l + \dot{H} \\ &= (\nabla_x V_a)^\top v + (\nabla_x V_l)^\top v + (\nabla_{x_l} V_l)^\top v_l \\ &\quad + \left[-\nabla_x V_a - \nabla_x V_l - (L_a(t_0) \otimes I_N)\tilde{v}\right]^\top \tilde{v}, \end{aligned} \tag{29}$$

where $\tilde{v} := v - \mathbf{1}_N \otimes v_l$. By noticing the equalities

$$\begin{aligned}(\nabla_x V_a)^\top(\mathbf{1}_N \otimes v_l) &= v_l^\top \sum_{i=1}^{N} \nabla_{x_i} V_a = 0,\\ (\nabla_x V_l)^\top(\mathbf{1}_N \otimes v_l) &= v_l^\top \sum_{i=1}^{N} \nabla_{x_i} V_l = -v_l^\top \nabla_{x_l} V_l,\end{aligned} \tag{30}$$

we arrive at

$$\dot{J} = -\tilde{v}^\top (L_a(t_0) \otimes I_N)\tilde{v} \le 0, \quad \forall\, t \in [t_0, t_\star). \tag{31}$$

This, combining with (28), gives that $\tau_d \ge \min\{r_s - r_{nb}, r_{nb} - r_a\}\sqrt{\sigma}/(2\sqrt{2J(t_0)})$, a contradiction.

By (31) and the continuity of J, we know that $J(t_1) \le J(t_0)$. By induction, suppose (25), (26) hold for the interval $[t_{m-1}, t_m)$, $m \in \mathbb{N}$, and $J(t_m) \le J(t_0)$. Then, following the same reasoning as above, we obtain that (25), (26) are true for the interval $[t_m, t_{m+1})$, and $J(t_{m+1}) \le J(t_0)$. □

Before presenting the main results in this section, we make a connectivity assumption of the group, which says that any non-AA agent has a direct or indirect link with some AA at all times.

Assumption 2. For all $t \ge t_0$, there is a path connecting any agent in $\mathcal{V} \backslash \mathcal{W}$ to some agent in $\mathcal{W}$ in the group induced graph $\mathcal{G}_p(t)$.

By the results in [4], we know that under Assumption 2, the symmetric matrix $L_a(t)$, defined in (7), is positive definite for any $t \ge t_0$. Since the group can only have finite neighboring topologies, we have

$$\lambda_m := \min_{t \ge t_0}\{\lambda_{\min}(L_a(t)) : \text{Assumption 2 holds at } t\} \tag{32}$$

is strictly positive, where $\lambda_{\min}(L_a(t))$ denotes the minimum eigenvalue of the matrix $L_a(t)$.

Next, for the proof of the following Theorem 2, we introduce a generalized Barbalat lemma, which is an extension of the celebrated Barbalat lemma [6] and a result in [10]; also see [5].

Definition 2. The function $f(\cdot) : \mathbb{R} \to \mathbb{R}$ is said to be piecewise uniformly continuous over $[t_0, +\infty)$ w.r.t. an infinite sequence $\{\hat{t}_i\}_{i=0}^{\infty}$, with $\hat{t}_0 = t_0$ and $\inf \hat{t}_i - \hat{t}_{i-1} \ge \hat{\tau} > 0$, if $\forall\, \varepsilon > 0$, $\exists\, \hat{\delta}_\varepsilon > 0$, such that $\forall\, t \in [\hat{t}_{i-1}, \hat{t}_i), i \in \mathbb{N}$ and $\forall\, \tilde{t} \in B_{\hat{\delta}_\varepsilon}(t) \bigcap [\hat{t}_{i-1}, \hat{t}_i)$, $|f(\tilde{t}) - f(t)| < \varepsilon$, where $B_{\hat{\delta}_\varepsilon}(t)$ is the open ball centered at t with the radius $\hat{\delta}_\varepsilon$.

Lemma 2. Let $f(\cdot):\mathbb{R}\to\mathbb{R}$ be piecewise uniformly continuous over $[t_0,+\infty)$ w.r.t. $\{\hat{t}_i\}_{i=0}^{\infty}$, and $h(\cdot):\mathbb{R}\to\mathbb{R}$ satisfy $\lim_{t\to+\infty} h(t)=0$. Suppose that $\lim_{t\to+\infty}\int_{t_0}^{t}(f(s)+h(s))\,\mathrm{d}s$ exists and is finite. Then $\lim_{t\to+\infty} f(t)=0$.

Proof. If it is not true, there exist $y_1>0$ and an infinite sequence $\{\hat{T}_i\}_{i=1}^{\infty}, \hat{T}_i \geq t_0, \hat{T}_i\to+\infty$ such that for any $\hat{T}_i, i\in\mathbb{N}$, $f(\hat{T}_i)>y_1$. From $\lim_{t\to+\infty} h(t)=0$, there exists T_1 such that $|h(t)|<y_1/3$ for all $t\geq T_1$. Let $T_1<\hat{t}_{k_1}<\hat{T}_{k_2}, k_1,k_2\in\mathbb{N}$. Let $y_2=\min\{\frac{\hat{\tau}}{2},\hat{\delta}_{y_1/3}\}$, and without loss of generality, assume that $[\hat{T}_{k_2},\hat{T}_{k_2}+y_2]\subseteq[\hat{t}_{i-1},\hat{t}_i)$ for some $i\in\mathbb{N}$ (otherwise, $[\hat{T}_{k_2}-y_2,\hat{T}_{k_2}]\subseteq[\hat{t}_{i-1},\hat{t}_i)$). Since, $f(t)$ is piecewise uniformly continuous on $[t_0,+\infty)$, we have $|f(\hat{T}_{k_2}+s)-f(\hat{T}_{k_2})|<y_1/3$ for all $0\leq s\leq y_2$. Thus we have for all $t\in[\hat{T}_{k_2},\hat{T}_{k_2}+y_2]$,

$$\begin{aligned}
|f(t)+h(t)| &= |f(\hat{T}_{k_2})+(f(t)-f(\hat{T}_{k_2}))+h(t)| \\
&\geq |f(\hat{T}_{k_2})|-|(f(t)-f(\hat{T}_{k_2}))|-|h(t)| \\
&> y_1-\frac{y_1}{3}-\frac{y_1}{3}=\frac{y_1}{3}
\end{aligned}$$

Hence,

$$\left|\int_{\hat{T}_{k_2}}^{\hat{T}_{k_2}+y_2}(f(t)+h(t))\,\mathrm{d}t\right|=\int_{\hat{T}_{k_2}}^{\hat{T}_{k_2}+y_2}|f(t)+h(t)|\,\mathrm{d}t>\frac{1}{3}y_1y_2$$

Since $\hat{T}_{k_2}$ can be arbitrarily large, $\int_{t_0}^{t}(f(s)+h(s))\,\mathrm{d}s$ cannot converge to a finite limit as $t\to+\infty$, a contradiction. □

Remark 2. If the function f is uniformly continuous over $[t_0,+\infty)$, then the conclusion in Lemma 2 naturally follows.

Theorem 2. Suppose Assumptions 1, 2 hold, and $\|x_i(t_0)-x_j(t_0)\|_\sigma>d_{sa}, \forall\, i,j\in\mathcal{V}$. By the control law (17), $\lim_{t\to+\infty}\|v_i(t)-v_l(t)\|=0,\ \forall\, i\in\mathcal{V}$; the inter-agent collision is avoided; and for all $i\in\mathcal{V}$, $\nabla_{x_i}(V_a+V_l)$, namely the virtual force applied on agent i, converges to zero.

Proof. Consider the energy function J defined in (24). We know from the proof of Lemma 1 that the derivative of J along the solutions of (11), (17) and (12) $\dot{J}=-\tilde{v}^\top(L_a(t)\otimes I_N)\tilde{v}\leq-\lambda_m\|\tilde{v}\|^2$, where λ_m is defined in (32). Combining this with the non-negativeness of $J(t)$, we have $\forall\, t\geq t_0$, $2\lambda_m\int_{t_0}^{t}H(s)\,\mathrm{d}s=\lambda_m\int_{t_0}^{t}\|\tilde{v}(s)\|^2\,\mathrm{d}s\leq J(t_0)$. On the other hand, since $H(t),V_a(t),V_l(t)\leq J(t)\leq J(t_0), \forall\, t\geq t_0$. It follows that there exist positive constants $c_i, i=1,2,3$ such that $\forall\, i,j\in\mathcal{V}$ and $\forall\, t\geq t_0$,

$$d_{ij}(t)\geq c_1>d_{sa},\quad d_{il}(t)\leq c_2,\quad \|\tilde{v}(t)\|\leq c_3. \tag{33}$$

Note that the first inequality of (33) implies that the inter-agent collision can be avoided for all $t\geq t_0$. Also, from (33) and the properties of functions ψ_a and ψ_l, we have that $\mathrm{d}H(t)/\mathrm{d}t=(-\nabla_x V_a-\nabla_x V_l-(L_a(t)\otimes I_N)\tilde{v})^\top\tilde{v}$ is bounded over $[t_0,+\infty)$, which implies that $H(t)$ is uniformly continuous w.r.t. t on $[t_0,+\infty)$. Then, by

Lemma 2, $\lim_{t\to+\infty} H(t) = 0$, which means that $\forall\, i \in \mathcal{V}, \|\tilde{v}_i\| = \|v_i(t) - v_l(t)\| \to 0$ as $t \to +\infty$.

Consider the new variables $\tilde{x}_i = x_i - x_l$, $\tilde{d}_i = \|\tilde{x}_i\|_\sigma$, $\tilde{d}_{ij} = \|\tilde{x}_i - \tilde{x}_j\|_\sigma$. Clearly we have $\tilde{d}_i = d_{il}$, $\tilde{d}_{ij} = d_{ij}$. Define the functions

$$\tilde{V}_a(\tilde{x}) \;=\; \frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i}\psi_a(\tilde{d}_{ij}) = V_a(x), \quad \tilde{V}_l(\tilde{x}) = \sum_{i\in\mathcal{W}}\psi_l(\tilde{d}_i) = V_l(x, x_l), \tag{34}$$

where $\tilde{x} = [\tilde{x}_1^\top, \ldots, \tilde{x}_N^\top]^\top$, and V_a, V_l are defined in (21). Now let $\tilde{u}(t) := u(t) - \mathbf{1}_N \otimes u_l(t) = -\nabla_{\tilde{x}}\tilde{V}_a - \nabla_{\tilde{x}}\tilde{V}_l - (L_a(t)\otimes I_N)\tilde{v}$. We know that $\int_{t_0}^{+\infty}\tilde{u}(t)\,\mathrm{d}t = \lim_{t\to+\infty}\tilde{v}(t) - \tilde{v}(t_0) = -\tilde{v}(t_0)$, and $\lim_{t\to+\infty}(L_a(t)\otimes I_N)\tilde{v} = 0$. Moreover, it is not difficult to see from (33) that $-\nabla_{\tilde{x}}\tilde{V}_a - \nabla_{\tilde{x}}\tilde{V}_l$ is uniformly continuous w.r.t. t for all $t \geq t_0$. Therefore, by Lemma 2,

$$\lim_{t\to+\infty} -\nabla_{\tilde{x}}\tilde{V}_a - \nabla_{\tilde{x}}\tilde{V}_l = \lim_{t\to+\infty} -\nabla_x V_a - \nabla_x V_l = 0. \tag{35}$$

□

Now, we give a result on the configuration of the group achieved by controller (17) when there is only one AA in the group.

Proposition 1. If the assumptions in Theorem 2 hold, and the leader set $\mathcal{W} = \{q\}, q \in \mathcal{V}$ is a singleton, then by the control law (17), $\lim_{t\to+\infty}\|x_q(t) - x_l(t)\| = 0$.

Proof. By contradiction, suppose the $\lim_{t\to+\infty}\|x_q(t) - x_l(t)\| \neq 0$, then there exist $\varepsilon_1 > 0$ and an infinite time sequence $\{\bar{t}_k\}_{k=1}^{\infty}$ such that $\|x_q(\bar{t}_k) - x_l(\bar{t}_k)\| > \varepsilon_1, \forall\, k \in \mathbb{N}$. Hence, by the properties of the function ψ_l and $\|\cdot\|_\sigma$, we know that there exists $\varepsilon_2 > 0$ such that $\frac{\mathrm{d}\psi_l(d_{ql})}{\mathrm{d}d_{ql}}\Big|_{d_{ql}=d_{ql}(\bar{t}_k)} > \varepsilon_2, \forall\, k \in \mathbb{N}$. Also, it is easy to see that there exists $\varepsilon_3 > 0$ such that $\|n_{lq}(\bar{t}_k)\| = \|x_q(\bar{t}_k) - x_l(\bar{t}_k)\|/\sqrt{1 + \sigma\|x_q(\bar{t}_k) - x_l(\bar{t}_k)\|^2} > \varepsilon_3, \forall\, k \in \mathbb{N}$. However, by (35) and the equality $(\mathbf{1}_N^\top \otimes I_n)\nabla_x V_a = 0$, we have

$$\begin{aligned}
&\lim_{t\to+\infty}(\mathbf{1}_N^\top \otimes I_n)\left(-\nabla_x V_a - \nabla_x V_l\right) = \lim_{t\to+\infty}(\mathbf{1}_N^\top \otimes I_n)\left(-\nabla_x V_l\right) \\
=\;& \lim_{t\to+\infty} -\sum_{i\in\mathcal{W}}\nabla_{x_i}\psi_l(d_{il}) = -\lim_{t\to+\infty}\nabla_{x_q}\psi_l(d_{ql}) = \lim_{t\to+\infty}\frac{\mathrm{d}\psi_l(d_{ql})}{\mathrm{d}d_{ql}}n_{lq} = 0.
\end{aligned}$$

□

Note that Proposition 1 tells us that if the group has one fixed AA and is connected at any time, the control law (17) can drive the group to track, or migrate to, the virtual leader in the sense that the AA converges asymptotically to the virtual leader. When $N = 1$, this is exactly the tracking control case as addressed, for example, in [10] and [5].

Now we investigate a robustness property of the proposed control law (17). Consider the control law

$$\tilde{u}^{af} = u^{af} + \delta_u, \tag{36}$$

where u^{af} is as in (17); and $\delta_u(t) : \mathbb{R} \to \mathbb{R}^{Nn}$ denotes the disturbance.

Theorem 2. Suppose Assumptions 1, 2 hold, and $\|x_i(t_0) - x_j(t_0)\|_\sigma > d_{sa}, \forall\, i, j \in \mathcal{V}$. If further $\delta_u(t) \in \mathcal{L}_2^{Nn}[t_0, +\infty) \bigcap \mathcal{L}_\infty^{Nn}[t_0, +\infty)$, then by the control (36), $\lim_{t\to+\infty} \|v_i(t) - v_l(t)\| = 0, \forall\, i \in \mathcal{V}$; and inter-agent collision is avoided. Futhermore, if $\delta_u(t) \to 0$ as $t \to +\infty$, then for any $i \in \mathcal{V}$, $\nabla_{x_i}(V_a + V_l)$, namely the virtual force applied on agent i, converges to zero.

P r o o f. Let us still consider the energy function defined in (24). Taking the derivative of J w.r.t t along the solutions of (11), (36) and (12) yields

$$\dot{J} = -\tilde{v}^\top (L_a(t) \otimes I_N)\tilde{v} + \delta_u^\top \tilde{v} \le -\lambda_m \|\tilde{v}\|^2 + \tilde{\lambda}\|\tilde{v}\|^2 + \frac{1}{4\tilde{\lambda}}\|\delta_u\|^2, \tag{37}$$

where Young's inequality was used [3], and $0 < \tilde{\lambda} < \lambda_m$. Thus for any $t \ge t_0$,

$$\int_{t_0}^{t} (\lambda_m - \tilde{\lambda})\|\tilde{v}\|^2 \,\mathrm{d}\tau \le J(t_0) - J(t) + \frac{1}{4\tilde{\lambda}} \int_{t_0}^{t} \|\delta_u\|^2 \,\mathrm{d}\tau < +\infty. \tag{38}$$

Note from (38) that $\forall\, t \ge t_0$,

$$J(t) \le J(t_0) + \frac{1}{4\tilde{\lambda}} \int_{t_0}^{t} \|\delta_u\|^2 \,\mathrm{d}\tau < +\infty \tag{39}$$

which, by the similar analysis in Theorem 2, shows that the inter-agent collision is avoided and $\nabla_x V_a, \nabla_x V_l, \tilde{v} \in \mathcal{L}_\infty^{Nn}[t_0, +\infty)$. This, together with the assumption $\delta_u \in \mathcal{L}_\infty^{Nn}[t_0, +\infty)$, gives that $\forall\, t \ge t_0$,

$$\frac{1}{2}\frac{\mathrm{d}\left(\|\tilde{\mathrm{v}}(\mathrm{t})\|^2\right)}{\mathrm{dt}} = \left[-\nabla_x V_a - \nabla_x V_l - (L_a(t) \otimes I_N)\tilde{v} + \delta_u\right]^\top \tilde{v} \in \mathcal{L}_\infty^1[t_0, +\infty). \tag{40}$$

By Lemma 2, (38) and (40) imply that $\forall\, i \in \mathcal{V}$, $\lim_{t\to+\infty} \|v_i(t) - v_l(t)\| = 0$.

The second part of the theorem can be obtained via the similar analysis in Theorem 2, with the additional attention to the condition that $\delta_u(t)$ tends to 0 as t goes to $+\infty$. □

4. APPLICATION TO FLOCKING CONTROL OF NONHOLONOMIC ROBOTS

In this section, we apply the control laws discussed above to the flocking control of a group of N unicycles. Here, we study the case where each robot can directly obtain its position and orientation, but cannot measure its velocity information. Instead, an observer is used to give the estimate of the velocity information for each robot, which can be transmitted between neighboring robots. The virtual leader we use is a moving point with the dynamics

$$\dot{q}_l = p_l, \quad \dot{p}_l = u_l, \tag{41}$$

where q_l, p_l and u_l are the position, velocity and acceleration of the virtual leader respectively.

4.1. Dynamic model of the robot

The dynamic model of the unicycle $i, i \in \mathcal{V}$ is given as in [1]:

$$\begin{aligned} \dot{\eta}_i &= J(\eta_i) z_i \\ M\dot{z}_i + C(\dot{\eta}_i) z_i + D z_i &= \tau_i, \quad \forall\, i \in \mathcal{V} \end{aligned} \tag{42}$$

with $\eta_i = [q_i^x, q_i^y, \phi_i]$, $z_i = [z_i^r, z_i^l]$, $\tau_i = [\tau_i^r, \tau_i^l]$,

$$J(\eta_i) = \frac{r}{2}\begin{bmatrix} \cos(\phi_i) & \cos(\phi_i) \\ \sin(\phi_i) & \sin(\phi_i) \\ b^{-1} & b^{-1} \end{bmatrix}, \quad M = \begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{11} \end{bmatrix},$$

$$D = \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix}, \quad C(\dot{\eta}_i) = \begin{bmatrix} 0 & c\dot{\phi}_i \\ -c\dot{\phi}_i & 0 \end{bmatrix} \tag{43}$$

where (q_i^x, q_i^y, ϕ) is the position and orientation of the unicycle; z_i^r and z_i^l are the angular velocities of the right and left wheels respectively; and τ_i^r and τ_i^l are the torques applied to the right and left wheels, respectively. The relation between z_i^r, z_i^l and the linear and angular velocities of the robot i, denoted by v_i, ω_i, is

$$[z_i^r, z_i^l]^\top = B[v_i, \omega_i]^\top, \quad \text{with } B = \frac{1}{r}\begin{bmatrix} 1 & b \\ 1 & -b \end{bmatrix} \tag{44}$$

4.2. Observer

The observer proposed in [1] is used here to estimate the velocity information v_i, ω_i (or z_i^r, z_i^l) of robot i. For each robot in the team, the variables directly estimated by the observer are

$$\begin{aligned} X_i &= Q(\eta_i) z_i, \\ Q(\eta_i) &= \begin{bmatrix} n_{11}\cos(c\Delta\phi_i) & \Delta\sin(c\Delta\phi_i) - n_{12}\cos(c\Delta\phi_i) \\ n_{11}\sin(c\Delta\phi_i) & -n_{12}\sin(c\Delta\phi_i) - \Delta\cos(c\Delta\phi_i) \end{bmatrix}, \end{aligned} \tag{45}$$

where

$$n_{11} = m_{11}(m_{11}^2 - m_{12}^2)^{-1}, \quad n_{12} = -m_{12}(m_{11}^2 - m_{12}^2)^{-1}, \quad \Delta = \sqrt{n_{11}^2 - n_{12}^2}.$$

It is straightforward to check that $Q(\eta_i)$ is globally invertible and its elements are bounded.

In the rest of this section, we denote the estimated value by adding "$\wedge$" on the corresponding original variables. The observer dynamics is given by [1]

$$\begin{aligned} \dot{\hat{\eta}}_i &= J(\eta_i)Q^{-1}(\eta_i)\hat{X}_i + K_{1i}(\eta_i - \hat{\eta}_i) \\ \dot{\hat{X}}_i &= -G(\eta_i)\hat{X}_i + Q(\eta_i)M^{-1}\tau_i + K_{2i}(\eta_i - \hat{\eta}_i) \end{aligned} \tag{46}$$

where $G(\eta_i) = Q(\eta_i)M^{-1}DQ^{-1}(\eta_i)$. The feedback gain matrices K_{1i} and K_{2i} are chosen to satisfy

$$K_{1i}^\top P_1 + P_1 K_{1i} = R_1, \quad G(\eta_i)^\top P_2 + P_2 G(\eta_i) = R_2,$$
$$\left(J(\eta_i)Q^{-1}(\eta_i)\right)^\top P_1 - P_2 K_{2i} = 0,$$

where R_1, R_2, P_1, P_2 are positive definite matrices.

Using the observer (46), the estimation errors $\tilde{\eta}_i = \eta_i - \hat{\eta}_i, \tilde{X}_i = X_i - \hat{X}_i$ decay exponentially to zero, i. e., there exist positive constants k_i and γ_i such that

$$\|(\tilde{\eta}_i(t), \tilde{X}_i(t))\| \le k_i \|(\tilde{\eta}_i(t_0), \tilde{X}_i(t_0))\| e^{-\gamma_i(t-t_0)}, \quad \forall t \ge t_0. \tag{47}$$

4.3. Controller

To avoid the non-holonomic constraint in the model (42), for robot $i, i \in \mathcal{V}$, consider a control reference point CRP_i for vehicle $i, i \in \mathcal{V}$, whose position is given by

$$q_i^h = \begin{bmatrix} q_i^x \\ q_i^y \end{bmatrix} + \mu \begin{bmatrix} \cos(\phi_i) \\ \sin(\phi_i) \end{bmatrix}, \quad \mu > 0, \tag{48}$$

i. e. the “hand position” in [7].

Inspired by [12], we study the flocking control of CRP_is based on the results obtained for double integrator agents. In the rest of this section, by “agent” i, $i \in \mathcal{V}$ we mean the control reference point CRP_i; and by “group” we mean the set composed of all CRP_is. Accordingly, the sets in Definition 1 should be redefined by substituting x_i with q_i^h for all $i \in \mathcal{V}$. And the inter-agent collision is said to be avoided if $\|q_i^h(t) - q_j^h(t)\| > d_{sa}$ for all $i, j \in \mathcal{V}$ and all $t \in [t_0, +\infty)$.

By (42) and (44), the velocity and acceleration of CRP_i are

$$\begin{aligned} p_i^h &:= \dot{q}_i^h = \begin{bmatrix} v_i \cos(\phi_i) - \mu\omega_i \sin(\phi_i) \\ v_i \sin(\phi_i) + \mu\omega_i \cos(\phi_i) \end{bmatrix}, \\ u_i^h &:= \dot{p}_i^h = S(\phi_i)[\tau_i - DB\zeta_i - C(\dot{\eta}_i)B\zeta_i] - \xi(v_i, \omega_i, \phi_i), \end{aligned} \tag{49}$$

where $\zeta_i = [v_i, \omega_i]^\top$ and

$$\begin{aligned} S(\phi_i) &= \begin{bmatrix} \cos(\phi_i) & -\mu \sin(\phi_i) \\ \sin(\phi_i) & \mu \cos(\phi_i) \end{bmatrix} B^{-1} M^{-1}, \\ \xi(v_i, \omega_i, \phi_i) &= \begin{bmatrix} v_i\omega_i \sin(\phi_i) + \mu\omega_i^2 \cos(\phi_i) \\ -v_i\omega_i \cos(\phi_i) + \mu\omega_i^2 \sin(\phi_i) \end{bmatrix} \end{aligned}$$

Firstly, following the idea in Section 3, we propose the decentralized control law for the group with a fixed AA set: $\forall\, i \in \mathcal{V}, \forall\, t \in [t_0, +\infty)$:

$$\tau_i^{af}(t) = S^{-1}(\phi_i)\left(\chi_i^{af} + \xi(\hat{v}_i, \hat{\omega}_i, \phi_i)\right) + (D + C(\hat{\omega}_i))B\hat{\zeta}_i, \tag{50}$$

with

$$\begin{aligned} \hat{\zeta}_i &= \begin{bmatrix} \hat{v}_i \\ \hat{\omega}_i \end{bmatrix} = B^{-1} Q^{-1}(\eta_i) \hat{X}_i, \\ \chi_i^{af}(t) &= \sum_{j \in \mathcal{N}_i(t_k)} f_a(d_{ij}^h) n_{ji}^h + g(b_i) f_l(d_{il}^h) n_{li}^h - \sum_{j \in \mathcal{N}_i(t_k) \bigcap \mathcal{S}_i(t)} a_{ij}^* (\hat{p}_i^h - \hat{p}_j^h) \\ &\qquad - b_i(\hat{p}_i^h - p_l) + u_l, \qquad \forall t \in [t_k, t_{k+1}), k \in \mathbb{Z}^+, \end{aligned} \tag{51}$$

where $\hat{p}_i^h = [\hat{v}_i \cos(\phi_i) - \mu\hat{\omega}_i \sin(\phi_i), \hat{v}_i \sin(\phi_i) + \mu\hat{\omega}_i \cos(\phi_i)]^\top$; and the definitions of $d_{ij}^h, d_{il}^h, n_{ji}^h$ and n_{li}^h mimic those of d_{ij}, d_{il}, n_{ji} and n_{li} in Section 3 by substituting $x_i, i \in \mathcal{V}$ with q_i^h.

Define inter-agent and leader-agent potentials V_a^h and V_a^l as

$$V_a^h = \frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i}\psi_a(d_{ij}^h), \quad V_l^h = \sum_{i\in\mathcal{W}}\psi_l(d_{il}^h).$$

By virtue of Theorem 2, we have the following result on flocking behavior of the unicycle team.

Theorem 3. Suppose Assumptions 1, 2 hold, and $\|q_i^h(t_0) - q_j^h(t_0)\|_\sigma > d_{sa}, \forall\, i, j \in \mathcal{V}$. Then, by the observer based control law (50) and (46), $\lim_{t\to+\infty}\|p_i^h(t) - p_l(t)\| = 0, \forall\, i \in \mathcal{V}$; the inter-agent collision is avoided; and for any $i \in \mathcal{V}$, $\nabla_{q_i^h}(V_a^h + V_l^h)$, namely the virtual force applied on the CRP_i, converges to zero.

P r o o f. From (49) and (50), the dynamics of CRP_i can be written as

$$\begin{aligned}\dot{q}_i^h &= p_i^h,\\ \dot{p}_i^h &= \chi_i^{df} + \delta_{ui}^h\end{aligned}$$

where

$$\chi_i^{df}(t) = \sum_{j\in\mathcal{N}_i(t_k)} f_a(d_{ij}^h)n_{ji}^h + g(b_i)f_l(d_{il}^h)n_{li}^h - \sum_{j\in\mathcal{N}_i(t_k)\bigcap\mathcal{S}_i(t)} a_{ij}^*(p_i^h - p_j^h)$$
$$- b_i(p_i^h - p_l) + u_l, \qquad \forall\, t \in [t_k, t_{k+1}), k \in \mathbb{Z}^+, \quad (52)$$

$$\delta_{ui}^h(t) = [\xi(\hat{v}_i, \hat{\omega}_i, \phi_i) - \xi(v_i, \omega_i, \phi_i)] + S(\phi_i)\left(DB(\hat{\zeta}_i - \zeta_i) + C(\hat{\omega}_i)B\hat{\zeta}_i - C(\omega_i)B\zeta_i\right)$$
$$- \sum_{j\in\mathcal{N}_i(t_k)\bigcap\mathcal{S}_i(t)} a_{ij}^*[\hat{p}_i^h - p_i^h) - (\hat{p}_j^h - p_j^h)] - b_{li}(\hat{p}_i^h - p_i^h),$$
$$\forall\, t \in [t_k, t_{k+1}), k \in \mathbb{Z}^+. \quad (53)$$

Denote $\hat{z}_i = Q^{-1}(\eta_i)\hat{X}_i$ and $\tilde{z}_i = [\tilde{z}_i^r, \tilde{z}_i^l]^\top = z_i - \hat{z}_i$. Then, from (45) and (47), we have for all $t \geq t_0$,

$$(n_{11}\tilde{z}_i^r(t) - n_{12}\tilde{z}_i^l(t))^2 + \Delta^2(\tilde{z}_i^l(t))^2 = \|\tilde{X}_i(t)\|^2 \leq k_i^2\|(\tilde{\eta}_i(t_0), \tilde{X}_i(t_0))\|e^{-2\gamma_i(t-t_0)}.$$

Thus, there exist positive constants α_i, β_i such that

$$\left\|[\tilde{z}_i^r(t), \tilde{z}_i^l(t)]\right\| \leq \alpha_i e^{-\beta_i(t-t_0)}, \quad \forall\, t \geq t_0.$$

By similar reasoning, from (44), it is easy to show that there exist positive constants ρ_i, σ_i such that

$$\|[\tilde{v}_i(t), \tilde{\omega}_i(t)]\| \leq \rho_i e^{-\sigma_i(t-t_0)}, \quad \forall\, t \geq t_0,$$

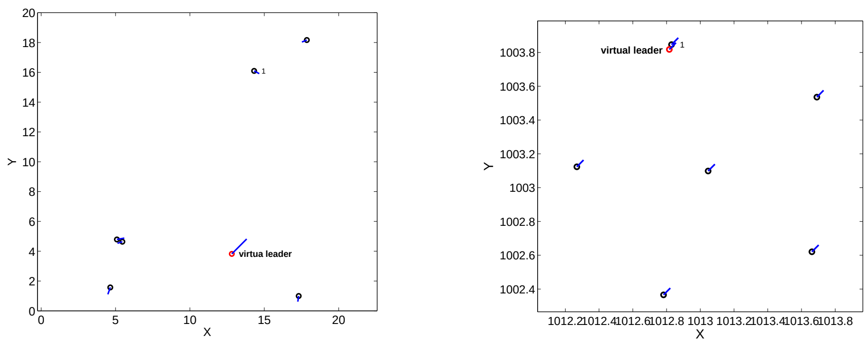
where $[\tilde{v}_i, \tilde{\omega}_i]^\top = [v_i - \hat{v}_i, \omega_i - \hat{\omega}_i]^\top$. Therefore, after some simple manipulations, it follows that $\delta_{ui}^h(t) \in \mathcal{L}_2^2[t_0, +\infty)\bigcap\mathcal{L}_\infty^2[t_0, +\infty)$ and $\lim_{t\to+\infty}\delta_{ui}^h(t) = 0$. By Theorem 2, the results hold. □

5. SIMULATIONS

In this section, we present some simulations to verify our proposed flocking controllers. The parameter σ in $\|\cdot\|_\sigma$ is set to be 1. And we use x_σ to denote the value $(\sqrt{1+\sigma x^2}-1)/\sigma$ for any $x \in \mathbb{R}^+$.

First, the flocking of 6 mass point agents by the controller (17) is shown in Figure 1, where agent 1 (labeled with 1 in the figure) is the only AA of the group. The inter-agent potential is the one defined in (9) with $d_{sa} = 0_\sigma$, $d_a = 1_\sigma$ and, to ensure the Assumption 2 can hold, $r_a = 30_\sigma$. The leader-agent potential is chosen as $\psi_l(x) = 10\left(\frac{1}{2}x^2+1\right)$. In addition, we let $a_{ij}^* = b_i^* = 10$, $\forall\, i,j \in \mathcal{V}$. The initial positions of the group are randomly chosen in the square $[0,20]\times[0,20]$; and the velocities are randomly chosen in $[-0.5,0.5]\times[-0.5,0.5]$. The position for the virtual leader is also randomly chosen in $[0,20]\times[0,20]$, but its velocity is fixed to $[1,1]^\top$.

The line attached to each agent (resp. the virtual leader) indicates the velocity (direction) of that agent (resp. the virtual leader). Note that since agent 1 is the only AA in the group, according to Proposition 1, its position converges to that of the virtual leader.

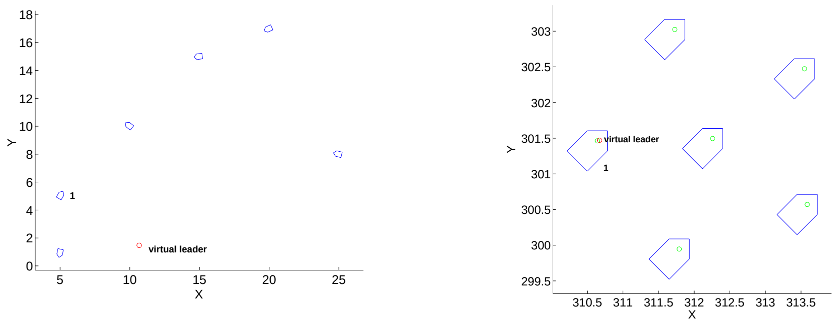


(a) $t = t_0$ (b) $t = t_0 + 1000s$

Fig. 1. Flocking of mass point group, $N = 6$.

Next, we simulate the flocking control for a group of unicycles. The model parameters of the robots are: $m_{11} = m_{22} = 1.2356$, $c = 0.2250$, $b = 0.2$ and $d_{11} = d_{22} = 10$. And the offset of the control reference point $\mu = 0.2$.

The observer-based flocking controller (46)−(50) is applied to a group of 6 unicycles. Also, unicycle 1 is the only AA in the group. The inter-agent potential is also as in the form of (9) but with $d_{sa} = 0.8_\sigma, d_a = 2_\sigma, r_a = 100_\sigma$. The leader-agent potential is chosen the same as for the mass point case. The initial positions are chosen in the square $[0,30]\times[0,30]$ such that the distance between any pair of CRP_is, measured in σ-norm, is greater than d_{sa}. The headings of the group are chosen randomly in $[0,2\pi]$. In addition, the linear and angular velocity are randomly chosen, respectively, in the intervals $[-1,1]$ and $[-0.5,0.5]$. The position of the virtual leader is selected randomly in $[0,30]\times[0,30]$, while its velocity is fixed to $[0.3,0.3]$. The results are shown in the following Figure 2.



(a) $t = t_0$ (b) $t = t_0 + 1000s$

Fig. 2. Flocking of unicycle group, $N = 6$.

6. CONCLUSIONS AND FUTURE WORK

In this paper, we have discussed the migration and trajectory tracking of a group of agents by means of the artificial potential method. The leader-agent potential is responsible for attracting the active agents to the virtual leader, while the inter-agent potential takes effect to generate the attraction and repulsion between neighboring agents. The velocity consensus of the group is due to the involvement of the linear velocity feedback term in the controller. A novel observer-based controller design is proposed for the flocking control of unicycle groups. Future work will be done on how to satisfy Assumption 2 while the group is migrating or tracking, and on extending our control laws to account for the group which has directed sensing or communication topology.

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REFERENCES

[1] K. D. Do, Z. P. Jiang, and J. Pan: A global output-feedback controller for simultaneous tracking and stabilization of unicycle-type mobile robots. IEEE Trans. Robotics and Automation *20* (2004), 3, 589–594.

[2] C. Godsil and G. Royle: Algebraic Graph Theory. Springer-Verlag, New York 2001.

[3] G. H. Hardy, J. E. Littlewood, and G. Pólya: Inequalities. Second edition. Cambridge University Press, Cambridge 1952.

[4] Y. Hong, J. Hu, and L. Gao: Tracking control for multi-agent consensus with an active leader and variable topology. Automatica *42* (2006), 7, 1177–1182.

[5] Z. P. Jiang and H. Nijmeijer: Tracking control of mobile robots: a case study in backstepping. Automatica *33* (1997), 7, 1393–1399.

[6] H. Khalil: Nonlinear Systems. Third edition. Prentice Hall, Englewood Cliffs, NJ 2002.

[7] J. R. T. Lawton, R. W. Beard, and B. J. Young: A decentralized approach to formation maneuvers. IEEE Trans. Robotics and Automation *19* (2003), 6, 933–941.

[8] N. E. Leonard and E. Fiorelli: Virtual leaders, artificial potentials and coordinated control of groups. In: Proc. IEEE Conference on Decision and Control, 2001, pp. 2968–2973.

[9] Q. Li and Z. P. Jiang: Decentralized Control Strategies for Connectivity Guaranteed Tracking of Multi-Agent Systems. In: 7th World Congress. Intelligent Control and Automation, 2008, pp. 323–328.

[10] A. Micaelli and C. Samson: Trajectory Tracking for Unicycle-type and Wwo-steering-wheels Mobile Robots. Technical Report 2097, INRIA, 1993.

[11] R. Olfati-Saber: Flocking for multi-agent dynamic systems: algorithms and theory. IEEE Trans. Automat. Control *51* (2006), 3, 401–420.

[12] W. Ren and E. Atkins: Distributed multi-vehicle coordinated control via local information exchange. Internat. J. Robust and Nonlinear Control *17* (2007), 10–11, 1002–1033.

[13] H. G. Tanner, A. Jadbabaie, and G. J. Pappas: Flocking in fixed and switching networks. IEEE Trans. Automat. Control *52* (2007), 5, 863–868.

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