BOOK REVIEWS

P. Pucci, J. Serrin: THE MAXIMUM PRINCIPLE. Birkhäuser-Verlag, Basel, 2007. ISBN 978-3-7643-8144-8, 237 pages, price EUR 64.95.

The book presents a quite complete survey of maximum principle results for second-order elliptic partial differential equations. The book summarizes the results for linear equations and concentrates on the nonlinear equations and inequalities.

Besides the maximum principles the authors analyze also the related qualitative properties like the comparison, tangency, sweeping, and compact support principles, as well as the boundary point lemma, the Harnack inequality, the phenomenon of dead cores, etc. The theory applies both to the classical solutions of class $C^1(\Omega)$ and to the weak solutions from appropriate Sobolev spaces.

The results are presented in a clear and mathematically precise way in the form of theorems and proofs. The authors treat carefully the bibliographic references. Most of the chapters are concluded by "Notes", where the original sources are cited, and interesting historical facts about the particular results are often mentioned as well as short surveys of the available literature for the particular topic. In the last section of every chapter, the authors list illustrative exercises together with several problems to prove.

The presentation is chronological starting with the work of Eberhard Hopf from 1927 concerning linear elliptic equations and finishing with recent results on nonlinear equations, operators, and inequalities.

The first chapter is introductory. It gives an outline of the book, explains the basic terms and the notation. The second chapter overviews the classical results of Eberhard Hopf both for linear and nonlinear second-order operators. A special attention is paid to Hopf's tangency and comparison principles for fully nonlinear equations of second order and to the uniqueness result for the Dirichlet problem. Further sections of Chapter 2 present specialized results for general quasilinear differential inequalities and for the divergence structure inequalities. The last section of Chapter 2 presents the proofs of Hopf's original statements.

Chapter 3 is devoted to maximum principles for nonlinear divergence structure elliptic partial differential inequalities, where the solution is understood in the sense of distributions. The important p-Laplacian operator serves as a model problem here. This chapter studies in detail the homogeneous inequalities, maximum principles for thin sets, comparison principles in $W^{1,p}(\Omega)$, singular elliptic inequalities, strongly degenerate operators, maximum principles for non-homogeneous elliptic inequalities, the uniqueness of the singular Dirichlet problem, and Sobolev's inequality.

Chapter 4 concerns the boundary value problems for nonlinear ordinary differential equations. General existence theorems, existence on a half-line, and the end point lemma are treated here.

Chapter 5 analyzes the strong maximum principle and the compact support principle together with the corresponding boundary point lemma. A special cases as well as generalized versions are presented.

Chapter 6 concentrates on the maximum principle for non-homogeneous divergence structure partial differential inequalities. Detailed proofs of general theorems are given. The chapter is concluded by an application to the mean curvature equation.

Chapter 7 is devoted to the Harnack inequality. Besides the Harnack inequality itself, the authors explain the relationship of the local boundedness and the weak Harnack inequality, the role of the Hölder continuity, the role of the spatial dimension, and they show the John-Nirenberg theorem.

Finally, Chapter 8 provides various applications and uses of the maximum principles and related results proved before. The applications include the Cauchy-Liouville theorems, the radial symmetry of the solutions, the symmetry requirements for the domain if both the Dirichlet and Neuman data are prescribed, the phenomenon of dead cores, and the strong maximum principle for Riemannian manifolds.

The book provides a nice overview of the classical results, presents up-to-date generalizations of the known results as well as the new results of the authors. Almost all statements are proved which makes the book excellently selfcontained. The proofs are brief, clear, and understandable for a general reader. The book can be recommended both for researchers in the field and for interested students. In addition, the book may serve to wider audience for brief orientation in the topic, as a quick reference, and as a valuable source of related bibliography.

Tomáš Vejchodský

T. Damour, O. Darrigol, B. Duplantier, V. Rivasseau, eds.: EINSTEIN 1905–2005. Poincaré seminar 2005. Birkhäuser-Verlag, Basel, 2006. ISBN 978-3-7643-7435-8, 293 pages, price EUR 119.

This volume contains contributions presented in April 2005 at the Poincaré Seminar, which is held twice a year at the Institute Henri Poincaré in Paris. This seventh session of the seminar was dedicated to the A. Einstein 1905's papers and their legacy. The book is directed towards large audience of physicists and mathematicians. Some contributions could be also appreciated by historians of science.

After the first contribution by O. Darrigol describing the long process of genesis of the (special) theory of relativity, C. Will describes special relativity from a centenary perspective. The geometry of relativistic spacetime is explained in detail by J. Bros and U. Moschella. Next, P. Grangier describes single photon experiments, which present a spectacular realization of A. Einstein's light quanta hypothesis. Einstein's epistemological conceptions and their philosophical roots connected to thinkers like Hume, Kant, Mach, and Pioncaré are discussed by T. Damour. In the last section B. Duplantier contributes an essay on historical, mathematical and physical aspects of the Brownian motion. The volume also contains a previously unpublished lecture on statistical physics by A. Einstein from 1910 translated from German by B. Duplantier and E. Parks.

Vojtěch Pravda