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REMARKS ON DISCRETELY ABSOLUTELY STAR-LINDELÖF SPACES

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Abstract. In this paper, we prove the following statements:
(1) There exists a Hausdorff Lindelöf space X such that the Alexandroff duplicate $A(X)$ of X is not discretely absolutely star-Lindelöf.
(2) If X is a regular Lindelöf space, then $A(X)$ is discretely absolutely star-Lindelöf.
(3) If X is a normal discretely star-Lindelöf space with $e(X) < \omega_1$, then $A(X)$ is discretely absolutely star-Lindelöf.

Keywords: countably compact space, star-Lindelöf space, absolutely star-Lindelöf space, discretely absolutely star-Lindelöf

MSC 2010: 54D20, 54B10, 54D55

1. Introduction

By a space, we mean a T_1 topological space. Recall that a space X is *countably compact* if every countable open cover of X has a finite subcover. Matveev defined in [5] a space X to be *absolutely countably compact* (= acc) if for every open cover $\mathscr{U}$ of X and every dense subspace D of X, there exists a finite subset F of D such that $\mathrm{St}(F,\mathscr{U}) = X$, where $\mathrm{St}(F,\mathscr{U}) = \bigcup\{U \in \mathscr{U} \colon U \cap F \neq \emptyset\}$. He also proved that every Hausdorff acc space is countably compact (see [5]).

A space X is *star-Lindelöf* (see [3], [6] under different names) (*discretely star-Lindelöf*) (see [9], [15]) if for every open cover $\mathscr{U}$ of X, there exists a countable subset (a countable discrete closed subset, respectively) $F \subseteq X$ such that $\mathrm{St}(F,\mathscr{U}) = X$. It is clear that every separable space and every discretely star-Lindelöf space are star-

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Lindelöf as well as every space of countable extent (in particular, every countably compact space or every Lindelöf space).

In [2], a star-Lindelöf space is called * Lindelöf; in [3], a star-Lindelöf space is called strongly star-Lindelöf, and in [15], a discretely star-Lindelöf space is called space in countable web.

A space X is *absolutely star-Lindelöf* (see [1], [6]) (*discretely absolutely star-Lindelöf*) (see [10], [11]) if for every open cover $\mathscr{U}$ of X and every dense subset D of X, there exists a countable subset F of D such that $\mathrm{St}(F, \mathscr{U}) = X$ (F is discrete and closed in X and $\mathrm{St}(F, \mathscr{U}) = X$, respectively).

From the above definitions, it is not difficult to see that every acc space is absolutely star-Lindelöf, every absolutely star-Lindelöf space is star-Lindelöf, every discretely absolutely star-Lindelöf space is absolutely star-Lindelöf and every discretely absolutely star-Lindelöf space is discretely star-Lindelöf.

Throughout the paper, the cardinality of a set A is denoted by $|A|$. The extent $e(X)$ of a space X is the smallest infinite cardinal κ such that every discrete closed subset of X has cardinality at most κ. For a cardinal κ, let κ^+ denote the smallest cardinal greater than κ. Let $\mathfrak{c}$ denote the cardinality of the continuum, ω_1 the first uncountable cardinal and ω the first infinite cardinal. For a pair of ordinals α, β with $\alpha < \beta$, we write $(\alpha, \beta) = \{\gamma \colon \alpha < \gamma < \beta\}$. Other terms and symbols that we do not define will be used as in [4].

2. Some results on discretely absolutely star-Lindelöf spaces

For a space X, recall that the Alexandroff duplicate $A(X)$ of X is constructed in the following way: The underlying set of $A(X)$ is $X \times \{0,1\}$ and each point of $X \times \{1\}$ is isolated; a basic neighbourhood of a point $\langle x, 0\rangle \in X \times \{0\}$ is a set of the form $(U \times \{0\}) \cup ((U \times \{1\}) \setminus \{\langle x, 1\rangle\})$, where U is a neighborhood of x of X. It is well-known that $A(X)$ is compact (countably compact, Lindelöf) iff so is X and $A(X)$ is Hausdorff (regular, Tychonoff, normal) iff so is X. Moreover, Vaughan [12] proved that if X is countably compact, then $A(X)$ is acc. In this section, we show the statements stated in abstract.

Example 2.1. There exists a Hausdorff Lindelöf space X such that $A(X)$ is not discretely absolutely star-Lindelöf.

Proof. Let

$$A = \{a_n \colon n \in \omega\} \quad \text{and} \quad B = \{b_m \colon m \in \omega\},$$
$$A_n = \{\langle a_n, b_m\rangle \colon m \in \omega\} \quad \text{and} \quad Y = \bigcup_{n\in\omega} A_n$$

and let

$$X = Y \cup A \cup \{a\} \quad \text{where } a \notin Y \cup A.$$

We topologize X as follows: every point of Y is isolated; a basic neighborhood of a point $a_n \in A$ for each $n \in \omega$ takes the from

$$U_{a_n}(m) = \{a_n\} \cup \{\langle a_n, b_i \rangle \colon\ i > m\} \quad \text{for } m \in \omega$$

and a basic neighborhood of a takes the from

$$U_a(F) = \{a\} \cup \bigcup \{\langle a_n, b_m \rangle \colon\ a_n \in A \setminus F,\ m \in \omega\} \text{ for a finite subset } F \text{ of } A.$$

Clearly, X is a Hausdorff space by the construction of the topology on X, X is not regular, since the point a can not be separated from the closed subset A by disjoint open subsets of X. Moreover, X is Lindelöf, since $|X| = \omega$.

We show that $A(X)$ is not discretely absolutely star-Lindelöf.

Let us consider the open cover

$$\mathscr{U} = \{\langle a_n, 1 \rangle \colon\ n \in \omega\} \cup \{\langle a, 1 \rangle\} \cup \{\langle a, 0 \rangle \cup A(Y)\} \cup \{\langle a_n, 0 \rangle \cup A(A_n) \colon\ n \in \omega\}$$

and the dense subset

$$D = \bigcup \{A(A_n) \colon\ n \in \omega\} \cup (A \times \{1\}) \cup \{\langle a, 1 \rangle\}$$

of $A(X)$. Let F be any countable subset of D and F discrete closed in X. Then, $\{n\colon\ F \cap A(A_n) \neq \emptyset\}$ is finite. In fact, if $\{n\colon\ F \cap A(A_n) \neq \emptyset\}$ is not finite, then $\langle a, 0 \rangle$ is a limit point of $F \cap A(Y)$ by the definition of the topology of X. This is a contradiction, since F is discrete closed in $A(X)$. Hence, there exists a $n_0 \in \omega$ such that $F \cap A(A_{n_0}) = \emptyset$. Thus,

$$\langle a_{n_0}, 0 \rangle \notin \mathrm{St}(F, \mathscr{U}),$$

since $\langle a_{n_0}, 0 \rangle \cup A(A_{n_0})$ is the only element of $\mathscr{U}$ containing $\langle a_{n_0}, 0 \rangle$, which completes the proof. □

Recall from [8] that a space X has *property* (a) if for every open cover $\mathscr{U}$ of X and every dense subset D of X, there exists a subset F of D such that F is discrete and closed in X and $\mathrm{St}(F, \mathscr{U}) = X$. For a regular Lindelöf space X, we show that $A(X)$ is discretely absolutely star-Lindelöf. For the proof of the statement, we need two lemmas:

Lemma 2.2. *Every T_1 paracompact space has property* (a).

P r o o f. Let X be T_1 paracompact, and let $\mathscr{U}$ be an open cover of X and D a dense subset of X. Then, there exists a locally finite open refinement $\mathscr{V}$ of $\mathscr{U}$. Thus, it is sufficient to show that there exists a subset F of D such that F is discrete closed in X and $X = \mathrm{St}(F, \mathscr{V})$. By transfinite induction, we define a sequence x_α of X and a sequence d_α of D satisfying the following conditions (1) and (2) for each α.

(1) $x_\alpha \notin \mathrm{St}(\{d_\beta \colon \beta < \alpha\}, \mathscr{V})$, and

(2) $d_\alpha \in D \cap \bigcap\{V \in \mathscr{V} \colon x_\alpha \in V\}$.

Pick $d_0 \in D$, if $\mathrm{St}(d_0, \mathscr{V}) = X$, then we finish the transfinite induction. If not, we pick $x_0 \in X \setminus \mathrm{St}(d_0, \mathscr{V})$. Assume that we have defined x_γ and d_γ for $\gamma < \alpha$. If $\mathrm{St}(\{d_\gamma \colon \gamma < \alpha\}, \mathscr{V}) = X$, then we finish the induction. If not, we pick $x_\alpha \in X \setminus \mathrm{St}(\{d_\gamma \colon \gamma < \alpha\}, \mathscr{V})$ and $d_\alpha \in D \cap \bigcap\{V \in \mathscr{V} : x_\alpha \in V\}$. We finish the induction at some α such that

$$\mathrm{St}(\{d_\beta \colon \beta < \alpha\}, \mathscr{V}) = X.$$

Put $F = \{d_\beta \colon \beta < \alpha\}$. Since X is T_1, then $\{d_\beta\}$ is closed for each $\beta < \alpha$. By the choice of the sequences x_α and d_α, clearly,

$$\mathrm{St}(\{x_\gamma \colon \gamma < \beta\}, \mathscr{V}) \subseteq \mathrm{St}(\{d_\gamma \colon \gamma < \beta\}, \mathscr{V}) \quad \text{for each } \gamma < \alpha.$$

Thus,

$$x_\beta \notin \mathrm{St}(\{x_\gamma \colon \gamma < \beta\}, \mathscr{V}) \quad \text{for each } \beta < \alpha.$$

Hence, there is no element of $\mathscr{V}$ containing two distinct elements of $\{x_\beta \colon \beta < \alpha\}$. By our construction, for $V \in \mathscr{V}$, if there is some $\beta < \alpha$ such that $x_\beta \in V$, then V contains the only element d_β of $\{d_\gamma \colon \gamma < \alpha\}$. Thus $F = \{d_\beta \colon \beta < \alpha\}$ is discrete in X. Since all one point subsets of F are closed and $\mathscr{V}$ is locally finite, F is closed in X, which completes the proof. □

Since every regular Lindelöf space is paracompact, then we have the following lemma by Lemma 2.2:

Lemma 2.3. *Every regular Lindelöf space is discretely absolutely star-Lindelöf.*

Since $A(X)$ is regular Lindelöf iff X regular Lindelöf, we have the following theorem by Lemma 2.3.

Theorem 2.4. *If X is a regular Lindelöf space X, then $A(X)$ is discretely absolutely star-Lindelöf.*

In the following, we give an example showing that $A(X)$ need not be discretely absolutely star-Lindelöf for a Tychonoff discretely absolutely star-Lindelöf space X.

Example 2.5. There exists a Tychonoff discretely absolutely star-Lindelöf space X such that $A(X)$ is not star-Lindelöf (hence, is not discretely absolutely star-Lindelöf).

P r o o f. Let $\mathscr{R}$ be a maximal almost disjoint family of infinite subsets of ω with $|\mathscr{R}| = \mathfrak{c}$. Let

$$X = (\mathfrak{c}^+ \times \omega) \cup \mathscr{R}.$$

We topologize X as follows: $\mathfrak{c}^+ \times \omega$ has the usual product topology and is an open subspace of X, and a basic neighborhood of $r \in \mathscr{R}$ takes the from

$$G_{\beta,K}(r) = ((\beta, \mathfrak{c}^+) \times (r \setminus K)) \cup \{r\} \quad \text{for } \beta < \mathfrak{c}^+ \text{ and a finite subset } K \text{ of } \omega.$$

Then, X is discretely absolutely star-Lindelöf and $e(X) = \mathfrak{c}$ (see [8, Example 3.1]). We show that $A(X)$ is not star-Lindelöf. Let us consider the open cover

$$\mathscr{U} = \{\langle r, 0\rangle \cup A(\mathfrak{c}^+ \times r))\colon\ r \in \mathscr{R}\} \cup \{\langle r, 1\rangle\colon\ r \in \mathscr{R}\} \cup \{A(\mathfrak{c}^+ \times \{n\})\colon\ n \in \omega\}.$$

Let F be a countable subset of $A(X)$. Then there exists an $r \in \mathscr{R}$ such that $\langle r, 1\rangle \notin F$, since $|\mathscr{R}| = \mathfrak{c}$. Hence, $\langle r, 1\rangle \notin \mathrm{St}(F, \mathscr{U})$, since $\{\langle r, 1\rangle\colon\ r \in \mathscr{R}\}$ is open and closed in $A(X)$ and $\langle r, 1\rangle$ is isolated for every $r \in \mathscr{R}$, which completes the proof. □

Remark 1. Since every discretely absolutely star-Lindelöf space is absolutely star-Lindelöf and discretely star-Lindelöf, Example 2.5 shows that $A(X)$ of an absolutely star-Lindelöf (discretely star-Lindelöf) space X need not be absolutely star-Lindelöf (discretely star-Lindelöf, respectively).

For a normal space, we have the following consistent example.

Example 2.6. Assuming $2^{\aleph_0} = 2^{\aleph_1}$, there exists a normal discretely absolutely star-Lindelöf space S_1 with $e(X) > \omega$ such that $A(X)$ is not star-Lindelöf (hence, is not discretely absolutely star-Lindelöf).

P r o o f. Let $S = L \cup \omega$ be the same space X (see [13, Example E]). Let κ be regular and $cf(\kappa) \geqslant |S|$. We define

$$X = L \cup (\kappa^+ \times \omega)$$

and topologize X as follows: a basic neighborhood of $l \in L$ in X is a set of the from

$$G_{U,\alpha}(l) = (U \cap L) \cup ((\alpha, \kappa^+) \times (U \cap \omega))$$

for a neighborhood U of l in X and for $\alpha < \omega_1$, and a basic neighborhood of $\langle \alpha, x \rangle \in \kappa^+ \times \omega$ in S_1 is a set of the form

$$G_V(\langle \alpha, x \rangle) = V \times \{x\},$$

where V is a neighborhood of α in κ^+. Then, S_1 is normal and discretely absolutely star-Lindelöf (see [12, Example 2.2]). Similarly to the proof of Example 2.5, it is not difficult to prove that X is not star-Lindelöf. $\square$

Remark 2. The author does not know if there exists a normal discretely absolutely star-Lindelöf space X such that $A(X)$ is not discretely absolutely star-Lindelöf in ZFC.

In Example 2.5, we note that the $e(X) = \mathfrak{c}$. In the following, we give an example showing that $A(X)$ of a discretely star-Lindelöf (or absolutely star-Lindelöf) space X with $e(X) = \omega$ need not be discretely absolutely star-Lindelöf.

Example 2.7. There exist both a Tychonoff absolutely star-Lindelöf space and a Tychonoff discretely star-Lindelöf space X with $e(X) = \omega$ such that $A(X)$ is not discretely absolutely star-Lindelöf.

Proof. Let $X = ((\omega_1 + 1) \times (\omega + 1)) \setminus \{\langle \omega_1, \omega \rangle\}$ be the Tychonoff plank.

First, we show that X is absolutely star-Lindelöf. To this end, let $\mathscr{U}$ be an open cover of X. Let S be the set of all isolated points of ω_1 and let $D = S \times \omega$. Then, D is dense in X and every dense subspace of X includes D. Thus, it is sufficient to show that there exists a countable subset F of D such that $\mathrm{St}(F, \mathscr{U}) = X$. Since ω_1 is countably compact, it follows from [5, Theorem 1.8] that ω_1 is acc. By [5, Theorem 2.3], we see that $\omega_1 \times (\omega + 1)$ is acc. Hence, there exists a finite subset F_1 of D such that

$$\omega_1 \times (\omega + 1) \subseteq \mathrm{St}(F_1, \mathscr{U}).$$

On the other hand, for each $n \in \omega$, there exists a $U_n \in \mathscr{U}$ such that $\langle \omega_1, n \rangle \in U_n$. Pick $d_n \in U_n \cap D$ for each $n \in \omega$, Then,

$$\{\omega_1\} \times \omega \subseteq \mathrm{St}(\{d_n \colon n \in \omega\}, \mathscr{U}).$$

If we put $F = F_1 \cup \{d_n \colon n \in \omega\}$, then F is a countable subset of D such that $X = \mathrm{St}(F, \mathscr{U})$, which shows that X is absolutely star-Lindelöf.

Next, we show that X is discretely star-Lindelöf. To this end, let $\mathscr{U}$ be an open cover of X. Since $\omega_1 \times (\omega + 1)$ is countably compact, then there exists a finite subset F_1 of X such that

$$\omega_1 \times (\omega + 1) \subseteq \mathrm{St}(F_1, \mathscr{U}).$$

We put $F = F_1 \cup \{\langle \omega_1, n\rangle \colon n \in \omega\}$. Then, F is a countable discrete closed subset of X such that $\mathrm{St}(F, \mathscr{U}) = X$, which completes the proof.

Finally, we show that $A(X)$ is not discretely absolutely star-Lindelöf, let us consider the open cover

$$\mathscr{U} = \{A(\omega_1 \times (\omega + 1))\} \cup \{\langle\langle \omega_1, n\rangle, 0\rangle \cup A(\omega_1 \times \{n\}) \colon n \in \omega\} \cup \{\langle\langle \omega_1, n\rangle, 1\rangle \colon n \in \omega\}$$

and the dense subset

$$D = A(\omega_1 \times (\omega + 1)) \cup \{\langle\langle \omega_1, n\rangle, 1\rangle \colon n \in \omega\}$$

of $A(X)$. Let F be a countable subset of D which is discrete closed in $A(X)$. Since $A(\omega_1 \times (\omega + 1))$ is countably compact, then $F \cap A(\omega_1 \times (\omega + 1))$ is finite, hence there exists an $n_0 \in \omega$ such that $F \cap A(\omega_1 \times \{n_0\}) = \emptyset$, therefore

$$\langle\langle \omega_1, n_0\rangle, 0\rangle \notin \mathrm{St}(F, \mathscr{U}),$$

since $\langle\langle \omega_1, n_0\rangle, 0\rangle \cup A(\omega_1 \times \{n_0\})$ is the only element of $\mathscr{U}$ containing $\langle\langle \omega_1, n_0\rangle, 0\rangle$, which completes the proof. □

Remark 3. In Example 2.7, it is not difficult to show that X is not discretely absolutely star-Lindelöf. Thus, the author does not know if there exists a Tychonoff discretely absolutely star-Lindelöf space X with $e(X) = \omega$ such that $A(X)$ is not discretely absolutely star-Lindelöf.

In the following, we give a positive result.

Theorem 2.8. *If X is a normal discretely star-Lindelöf space X with $e(X) < \omega_1$, then $A(X)$ is discretely absolutely star-Lindelöf.*

Proof. We prove that $A(X)$ is discretely absolutely star-Lindelöf. To this end, let $\mathscr{U}$ be an open cover of $A(X)$. Obviously every point of $X \times \{1\}$ is isolated. Let B be the set of all isolated points of X, and let

$$D = (X \times \{1\}) \cup (B \times \{0\}).$$

Then, D is a dense subspace of $A(X)$ and every dense subset of $A(X)$ includes D. Thus, it is sufficient to show that there exists a countable subset $F \subseteq D$ such that F is discrete closed in X and $\mathrm{St}(F, \mathscr{U}) = A(X)$. For each $x \in X$, choose an open neighborhood $W_x = (V_x \times \{0,1\}) \setminus \{\langle x, 1\rangle\}$ of $\langle x, 0\rangle$ satisfying that there exists a $U \in \mathscr{U}$ such that $W_x \subseteq U$, where V_x is an open subset of X containing x. Put $\mathscr{V} = \{V_x \colon x \in X\}$. Then, $\mathscr{V}$ is an open cover of X. Hence, there exists a countable subset $E_0 \subseteq X$ such that E_0 is discrete closed in X and $X = \mathrm{St}(E_0, \mathscr{V})$, since X is discretely star-Lindelöf. For the collection $= \{V_x \colon x \in E_0\}$ of X, since E_0 is discrete closed, there exists a pairwise disjoint open family $\{U_x \colon x \in E_0\}$ in X such that $x \in U_x \subseteq V_x$ for each $x \in E_0$, since E_0 is a discrete closed subset of a normal space X. By normality, there is an open subset U in X such that

$$E_0 \subseteq U \subseteq \overline{U} \subseteq \bigcup_{x \in F_0} U_x.$$

Clearly, $\{U \cap U_x \colon x \in F_0\}$ is a discrete family of nonempty open subsets of X. Let

$$E_1' = \{x \in E_0 \colon x \text{ is not isolated in } X\}.$$

For every $x \in E_1'$, pick $y_x \in U \cap U_x$ such that $x \neq y_x$. Then,

$$\{\{x\} \colon x \in E\} \cup \{\{y_x\} \colon x \in E_1'\}$$

is discrete closed in X and $\langle y_x, 1\rangle \in W_x$ and $\langle x, 0\rangle \in W_x$.

Put $E_1 = E_0 \times \{1\}$. For every $x \in X \setminus (E_0 \cup \{V_x \colon x \in E_1'\})$, there exists $x' \in X$ such that $x \in V_{x'}$ and $V_{x'} \cap E_0 \neq \emptyset$, hence $W_{x'} \cap E_1 \neq \emptyset$. Let

$$E_2 = E_1 \cup \{\langle y_x, 1\rangle \colon x \in E_1'\} \cup ((E_0 \setminus E_1') \times \{0\}).$$

Then, E_2 is a countable discrete closed (in X) subset of D and $X \times \{0\} \subseteq \mathrm{St}(E_2, \mathscr{U})$. Let $E_3 = A(X) \setminus \mathrm{St}(E_2, \mathscr{U})$. Then, E_3 is a discrete and closed subset of $A(X)$. Since $e(X) < \omega_1$, then $e(A(X)) < \omega_1$. Thus we see that E_3 is countable. If we put $F = E_2 \cup E_3$, then F is a countable discrete closed (in X) subset of D and $A(X) = \mathrm{St}(F, \mathscr{U})$, which completes the proof. □

We have the following corollary of Theorem 2.8.

Corollary 2.9. *Every normal discretely star-Lindelöf space X with $e(X) < \omega_1$ can be embedded in a normal discretely absolutely star-Lindelöf space as a closed subspace.*

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