Mathematica Bohemica

Jindřich Bečvář; Zuzana Došlá; Jaroslav Kurzweil; Jean Mawhin; Dagmar Medková; Milan Tvrdý

Remembering Štefan Schwabik (1941–2009)

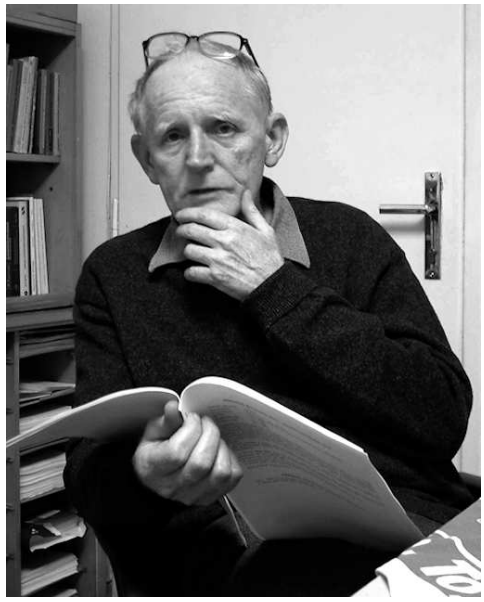
Mathematica Bohemica, Vol. 136 (2011), No. 2, 113–134

Persistent URL: http://dml.cz/dmlcz/141575

136 (2011) MATHEMATICA BOHEMICA No. 2, 113–134

REMEMBERING ŠTEFAN SCHWABIK (1941–2009)

J. Bečvář, Z. Došlá, J. Kurzweil, J. Mawhin, D. Medková, M. Tvrdý



Dagmar Medková: A word from the chief editor

Our colleague, Professor Štefan Schwabik, died suddenly in November 2009. He was not only a prominent mathematician and outstanding pedagogue but also a wonderful personality, always ready to help selflessly anybody who got in trouble. For thirty years he had been closely connected with the journal Mathematica Bohemica (formerly Časopis pro pěstování matematiky). In 1981–1989 he held the post of Deputy Chief Editor, in 1990–2007 the post of Chief Editor. He collaborated with the Editorial Board till the day of his death.

To honor the colleague of ours, to recall him to his friends, and to offer a possibility to get acquainted with his character, life and work to those who had not had the

opportunity of meeting him, we decided to publish, with kind permission of the authors who were his close collaborators and friends, their reminiscences that had been published in Czech or delivered at the Commemorative Conference in 2010.

Jaroslav Kurzweil: Štefan Schwabik—an outstanding mathematician and colleague

Štefan Schwabik became member of the Mathematical Institute of the Czechoslovak Academy of Sciences (now Academy of Sciences of the Czech Republic) in 1964 immediately after graduating in Mathematics from the Faculty of Mathematics and Physics of Charles University in Prague. From a beginner devoted to Mathematics he soon grew to an acknowledged expert in mathematical analysis, particularly in the theory of integral, integral equations, and generalized differential equations. I am, for example, deeply convinced that the monograph "Topics in Banach space integration" (World Scientific, 2005) written together with the Chinese mathematician Ye GuoJu will, after decades, still be a frequently used source of information.

He has been well known in the mathematical community throughout the world: he spent several long stays abroad and became an editor of the journal Real Analysis Exchange. His admiration for Bernard Bolzano projected into his life posture: the necessity of fighting idiotism and dishonesty irritated him deeply. He consistently studied the development of mathematical thinking. He found time to give lectures on history of mathematics at summer schools for high school teachers and to engage himself in the problems of mathematical education at elementary schools.

He met with enough obstacles. He defended his dissertation for the CSc scientific degree in 1972 but had to wait till 1977 for actually being awarded the degree. He defended his dissertation for the DrSc scientific degree in 1991, became Associated Professor after his habilitation in 1993 and was appointed Professor in 2000.

Štefan Schwabik did a lot of work that is never ending and without which the scientific research could not be functioning: writing reviews, reports, and participating in various boards and committees. In 1981 he was appointed the Deputy Editor in Chief of the *Časopis pro pěstování matematiky* (since 1991 Mathematica Bohemica) but he had been actively working for the Journal many years before. When the Chief Editor František Zítek passed away in 1988, Štefan Schwabik took his position and held it till 2007. For many years, he was regularly going to Brno and Opava (he was Professor of the Silesian University) to teach and act as a member of scientific boards and other bodies; his efficiency in managing all these tasks was impressive.

Štefan was very frank and sincere. He loved to socialize and have fun, he had numerous acquaintances with whom he discussed many things. Nevertheless, he also knew when to be silent, and, above all, was dependable and faithful to his friends.

Štefan died suddenly and prematurely at the age of sixty eight years. Let us keep him in grateful memory and remember that we know neither the day nor the hour of our departure.

[Translated from *Pokroky matematiky, fyziky a astronomie* 55 (2010), 69–70, by Jiří Jarník.]

Štefan Schwabik had very close connections with the mathematical community in Brno. He participated in all Czechoslovak conferences EQUADIFF organized in Brno in the years 1972, 1985, 1997, 2009 and substantially contributed to their success, as well as to the success of the Colloquia on Differential and Difference Equations (CDDE) organized in Brno in the years 2000, 2002, and 2006. For many years, Štefan Schwabik was member of the Editorial Board of the journal Archivum Mathematicum *published by the Faculty of Science of the Masaryk University. His name is also connected with the foundation and development of the PhD study programs of the Faculty of Science of the Masaryk University in the fields of Mathematical Analysis, History of Mathematics, and Mathematical Education.*

Therefore, it was natural that the Colloquium on Differential Equations and Integration Theory dedicated to the memory of Štefan Schwabik was organized by his Brno friends and was located in the village Křtiny which lies in a beautiful landscape of Moravian Karst near Brno. In the days of October 14–17, 2010, 75 participants from 11 countries came to Křtiny attracted by Štefan's name and by his memory. During the conference, 15 invited lectures were delivered by J. Kurzweil, J. Jaroš, P. Muldowney, M. Federson, B. Maslowski, J. Malý, R. Hakl, E. Liz, I. Györi, L. Di Piazza, I. Rachůnková, M. Medveď, J. Mawhin, K. Musiał and P. Drábek. In addition, over 20 posters were presented, and enlarged abstracts of all contributions were published in the Abstract Book of CDEIT 2010. In Štefan's honor, a special memorial evening was organized at which the following reminiscences of Jean Mawhin and Milan Tvrdý were presented.

Jean Mawhin: A friend for all seasons

Between my first meeting with Štefan in Brno, at Equadiff 3 of 1972, and the last occasion to enjoy Štefan's wonderful company, in Brno again, at Equadiff 12 of 2009, the world had dramatically changed. But Štefan had definitely remained the same.

He liked mathematical conferences and enjoyed attending them. There are many reasons to attend a conference: to communicate your last results, to show how good a mathematician you are, to learn in priority new exciting results, or, even, to make nice excursions. For Štefan, the main reason was to meet people, to watch people, to

meet old friends and make new ones. For this incredibly lucid observer of mankind in general, and of the mathematical microcosm in particular, a conference was an exceptional laboratory. His passion for photography—the sequence of his cameras is both a political and a technical history of the last forty years—was always, and almost uniquely, a passion for taking snapshots of friends and people.

My first point of contact with Štefan was a common friend, the late Svatopluk Fučík, whose seminal work in nonlinear analysis was familiar to me, before our meeting in Brno. The tragic destiny of Svatopluk deeply affected Štefan and I could soon measure his disponibility and generosity when, after Svatopluk's death, we tried to help his family.

That Equadiff 3 in Brno was the beginning of a long love story with Czechoslovakia and Czech mathematicians. Štefan was most emblematic in this respect. Before the iron curtain fell down, visiting Czechoslovakia was the only way to meet Štefan. It took me some time to learn how difficult and painful the communist period had been for him. I learned it from his friends, not from him. Štefan liked to speak, as you know, but not to tell his life. You could just learn funny anecdotes or events that had touched him, his family, his colleagues, and, of course, his dog. Complaining was orthogonal to his way of thinking and he remained exactly the same despite of the changing environment. Neither his freedom of thinking, nor his lucidity were negotiable.

Another recipe of Štefan was his unique sense of humor, a type of humor that only blossoms in small countries, where the concept of grandeur is replaced by that of derision. I do not know if Štefan, always dressed in an elegant classical way, was aware of his aristocratic look. My wife, like many women, enjoyed very much his company. Margaret told me once that Štefan was born to be a prince, realized that it was impossible, and decided to be a clown. But it took not long for him to become the prince of clowns. He definitely preferred serious things told in a witty way, to empty words told with a solemn voice. He could only be serious in discussing the quality of drinks. He always preferred traditional folk songs to national hymns. He was always ready to initiate or join the funniest jokes. Laughing with him was a fantastic medicine.

Štefan's office in the Mathematical Institute had always been the strong attractor of this venerable institution. If you paid him a visit, it did no take long, through some strange telepathy, to be joined by most of his colleagues. There the prince of clowns became the prince of magicians, changing almost instantaneously mathematical books into excellent drinks. It was also an opportunity to discover Štefan's most recent gadgets.

For several years, Štefan's mathematical interests and mine were not specially close, and our relations were much more friendly than professional. When, in the

second half of the seventies, my interest for integration, and in particular for some integral growing in the meadows of Bohemia, awoke, I realized that Štefan was more than an expert in this area. It did not take long for us to share growing enthusiasm for this integral and a common admiration and affection for its father, Jaroslav Kurzweil. Štefan's feelings for Jaroslav were not restricted to the mathematician. He loved so much that man sharing with him a sense of humor and a philosophy of life expressed differently, but having the same deep human roots. The student had learned from his master more than the generalized differential equations, and their complicity was exceptional. My mathematical conversations with Jaroslav and Štefan were an immense joy and enrichment. Again, the problem was to convince Štefan to speak about his own remarkable achievements. The same was true in history of mathematics, for which Štefan always showed strong interest. It was for him another way of exploring the human side of mathematics, and the same was true for his commitments in various committees and associations.

Each of us keeps and cultivates his own image of Štefan, and each of us agrees that he is unique. He cherished his friends and they cherished him. For me, Štefan remains the brother I never had. He not only was always there for the happiest moments I spent in his country, but he was an essential part of those happiest moments. I now share something more with all my friends in Czech Republic: the unique chance and happiness of having known and enjoyed for many years Štefan's wonderful personality.

Milan Tvrdý:
Scientific contribution of Štefan Schwabik

His scientific interests included real analysis, differential equations, functional analysis and operator theory as well as history of mathematics. In particular, he contributed essentially to the following topics: control theory and integral transforms, differential equations with impulses on surfaces, generalized differential equations, integration theory in $\mathbb{R}^n$, linear Stieltjes integral equations in Banach spaces, integration in Banach space, generalized ODE approach to functional differential equations, variational measures and extensions of integrals.

Control theory (1964–1968). His degree work and the first two papers (cf. [9] and [10]) dealt with classical linear control problems. In particular, he obtained necessary and sufficient conditions ensuring that for a given $T > 0$ and $\widetilde{x} \in \mathbb{R}^n$, the corresponding solution starting at $\widetilde{x}$ can be controlled to the origin at a time $t \leqslant T$.

Differential equations with impulses on surfaces (1968–1971). Next, influenced by Jaroslav Kurzweil, he started a systematic study of generalized ordinary differen-

tial equations, i.e. of the classes of equations that admit not absolutely continuous solutions but preserve substantial properties of ordinary differential equations. In a series of papers [11]–[17] published during the years 1968–1971 (and written in German) he presented results concerning the continuous dependence of solutions of nonlinear generalized differential equations on a parameter and also their stability. A special attention he paid to the case of systems with impulses that turned out to be covered by generalized differential equations. Soon after the papers by Myshkis, Samoilenko, Perestyuk, Halanay and Wexler had appeared he answered many of the basic questions evoked by these pioneering contributions (and about 25 years before the modern boom with the impulse systems started).

Generalized linear differential equations (1971–1979). The results obtained for nonlinear problems covered, of course, the linear problems as well. Nonetheless, during his visit of Prague, Aristide Halanay noted that it would be worthwhile to consider in more detail also special properties of generalized linear differential equations (GLDEs). This was a motive to Štefan Schwabik to turn his attention to these objects. GLDEs are described by the integral equations of the form

$$x(t) = \widetilde{x} + \int_a^t \mathrm{d}[A(s)]\, x(s) + f(t) - f(a), \quad t \in [a, b],$$

where $[a, b]$ is a compact interval in $\mathbb{R}$, A and f are functions of bounded variation and solutions are also functions of bounded variation. At that time we usually wrote these equations in the symbolic form

$$\frac{\mathrm{d}x}{\mathrm{d}\tau} = \mathrm{D}[A(t)\, x + f(t)].$$

The already mentioned paper [16] was the first dealing with GLDEs. In the subsequent series of papers [18]–[30] dealing with GLDEs but also with the operator theory in the space of functions of bounded variation and with Stieltjes integral equations he presented results on the existence and uniqueness of solutions, variation-of-constants formula for GLDEs, boundary value problems and duality theory, Floquet theory, continuous dependence of solutions on a parameter, Perron-Stieltjes (=Kurzweil-Stieltjes) integration of BV functions with respect to BV functions, duality theory in the BV space, extensions to Volterra-Stieltjes or Fredholm-Stieltjes integral equations. By coincidence, in the early seventies, while working on my thesis and inspired by the paper by Halanay and Moro and by the unpublished notes by Kurzweil, I realized that generalized linear differential equations offer a nice opportunity to establish a duality theory for linear nonlocal boundary value problems—in particular, those

with boundary conditions containing the Stieltjes integral. This was a starting impulse for our cooperation—papers [21], [29] and [30] we wrote together. The highlight of this period was our joint monograph [2] (together also with Otto Vejvoda) which summarized the obtained results and presented a rather complete theory of linear differential equations in the space of functions of bounded variation, including the duality theory. The crucial tool was the Perron-Stieltjes (or equivalently Kurzweil-Stieltjes) integral which is a natural generalization of the Lebesgue-Stieltjes integral. (This monograph was preceded by a monograph by Chaim S. Hönig dealing with similar problems, but, in addition, with Banach space valued regulated solutions. On the other hand, the integral used by Hönig was the interior (Dushnik) integral which differs from that of Perron-Stieltjes.)

The papers [31] (1980) and [34] (1982) are echoes of this period.

Generalized differential equations (1980–1992). Papers [39] and [40] from 1987–1988 were devoted to the Sturm-Liouville theory for GLDEs. Coauthor of the latter was Dana Fraňková, a student of Štefan Schwabik. She, inspired by the monograph by Chaim S. Hönig, also turned our attention to possible extensions of the notion of generalized linear differential equations so that they could admit as solutions regulated functions instead of their special cases—functions of bounded variations. The result of this interest were e.g. report [41] written during the stay of Štefan Schwabik in Brazil and paper [45] from 1992.

However, during the eighties Štefan Schwabik started systematic work on the theory of, in general nonlinear, generalized differential equations. His attention was, in particular, attracted by the stability and continuous dependence questions. One of the most important contributions by Štefan Schwabik from this period was the introduction of the notion of the *variational stability* in [36]. Inspired by the notion of the *integral stability* of solutions of ordinary differential equations due to Ivo Vrkoč, for nonlinear generalized differential equations

$$x(t) = \widetilde{x} + \int_a^t \mathrm{D}F(x(\tau), t) \tag{2}$$

Štefan Schwabik defined: the zero solution of (2) is *variationally stable* if for every $\varepsilon > 0$ there is $\delta > 0$ such that

$$\left(\|\widetilde{x}\| < \delta \text{ and } \operatorname{var}_{t_0}^{T} \left(x(s) - \int_{t_0}^{s} \mathrm{D}F(x(\tau), t) \right) < \delta \right) \implies \|x(t)\| < \varepsilon \text{ for } t \in [t_0, T]$$

holds for each solution x of (1) on $[t_0, T]$ which has a bounded variation on $[t_0, T]$. For this type of stability he proved Lyapunov type results (including converse theorems).

Let us note that the integral on the right-hand side of (2) stands for the generalized (nonlinear) Kurzweil integral and the integral equation (2) is usually written in the symbolic form

$$\frac{\mathrm{d}x}{\mathrm{d}\tau} = \mathrm{D}[F(x,t)]. \tag{3}$$

Some of his results have been collected in the two-volume monograph [3], [4] and in the survey [42]. However, the most important and representative was his widely appreciated monograph [5] presenting a very complete theory of generalized differential equations and of their relationship to measure differential equations or differential systems with impulses. With a certain delay, he explained in [60] (2000) also the relationship between generalized differential and difference equations.

Integration theory in $\mathbb{R}^n$ (1990–1996). From the beginning, Štefan Schwabik was deeply interested in the integration theory. His papers and monographs usually contain also particular contributions to generalized integration. Papers [20] and [22] are devoted to Stieltjes type integrals and in 1983 Štefan Schwabik was a coauthor (together with Jiří Jarník and Jaroslav Kurzweil) of the paper [35] on a multiple nonabsolutely convergent integral. In the nineties the integration theory became perhaps the most substantial part of his interests. Papers [44] and [48] (the former jointly with Jaroslav Kurzweil) are devoted to the role of the Perron integral and ACG_*-functions in the theory of differential equations. However, a dominant topics of that period for him were convergence theorems for nonabsolutely convergent integrals, cf. [46]–[48], [51] (jointly with Ivo Vrkoč), [53] and also his joint paper [64] with Jaroslav Kurzweil from 2004. In particular, he introduced the powerful notion of equiintegrability: A system $\mathcal{F}$ of functions $f\colon [a,b] \to \mathbb{R}$ is *equiintegrable* if (i) each function $f \in \mathcal{F}$ is integrable and (ii) for each $\varepsilon > 0$ there is a gauge δ such that

$$\left| \sum_j f(\xi_j)\,(\alpha_j - \alpha_{j-1}) - \int_a^b f \right| < \varepsilon$$

holds for each δ-fine partition (D,ξ) and each $f \in \mathcal{F}$.[1]

The following two results are the most typical:

▷ *If the sequence $\{f_n\}$ is equiintegrable and $f_n \to f$ pointwise, then f is integrable and $\int_a^b f_n \to \int_a^b f$.*

[1] Recall that a couple (D,ξ) is said to be a *partition* of $[a,b]$ if $D = \{\alpha_0, \alpha_1, \ldots, \alpha_m\}$ is a division of $[a,b]$, $\xi = (\xi_1, \ldots, \xi_m) \in [a,b]^m$ and $\alpha_{j-1} \leqslant \xi_j \leqslant \alpha_j$, and functions $\delta\colon [a,b] \to (0,1)$ are called *gauges* and the partition (D,ξ) is *δ-fine* if $[\alpha_{j-1},\alpha_j] \subset (\xi_j - \delta(\xi_j), \xi_j + \delta(\xi_j))$ for all j.

▷ *For the McShane integral the equiintegrability convergence result is equivalent to the Vitali convergence theorem.*

Further highlights of this period are two monographs [6] and [7] written in Czech. The former written together with Petra Šarmanová is a booklet of the popular series on history of mathematics. The latter presents a complete theory of the nonabsolutely convergent integrals in a form accessible to all university students. Its very preliminary version were lecture notes on modern theory of integration that Štefan Schwabik together with David Preiss prepared about 1979 for Czech universities. Unfortunately, their project has not been realized and only few copies of their lecture notes were printed by a primitive technique (ormig) of that time.

Linear Stieltjes integral equations in Banach spaces (1996–2001). In the middle of the nineties Štefan Schwabik started to study abstract Kurzweil-Stieltjes integrals, i.e. integrals of functions with values in a Banach space, see [54], [58]. Basic assumptions followed those of Hönig: the existence and basic properties he obtained for integration of regulated functions with respect to functions of bounded semivariation. On the other hand, the use of the Kurzweil-Stieltjes integral instead of the Dushnik one made the situation more difficult and, of course, it is not covered by Hönig's results. Subsequently, Schwabik was able to get in [55], [56] and [59] basic results for linear Stieltjes integral equations in Banach spaces.

Integration in Banach space (2001–2008). In [52] from 1996 Štefan Schwabik considered spaces $\mathcal{L}$ of Bochner integrable, $\mathcal{S}$ of McShane integrable and $\mathcal{S}^*$ of McShane absolutely integrable functions with values in a Banach space X and proved that the relation $\mathcal{L} = \mathcal{S}^* \subset \mathcal{S}$ is true, the corresponding integrals coincide in $\mathcal{L}$, and $\mathcal{S}^* = \mathcal{S}$ if and only if X is finite-dimensional. In 2001, when a young Chinese mathematician Ye GuoJu (that time from Lanzhou) came to the Institute of Mathematics for one year study stay, they decided to continue in this direction. As a result they published three papers [61]–[63] and started to prepare a monograph on integration in Banach spaces. The monograph [8] entitled "*Topics in Banach space integration*" was published by World Scientific in 2005. In the monograph, the authors presented generalizations of the Kurzweil-Henstock and McShane integrals (both based on Riemann sums) to functions which have values in a Banach space and described their basic properties including several interesting convergence theorems for the McShane integral. In addition, also the Bochner, Dunford and Pettis integrals for Banach space valued functions and the relationships between the various integrals were described. In particular, they proved that a Banach space valued function is McShane integrable if and only if it is both Pettis and Kurzweil-Henstock integrable. Furthermore, strong versions of both the Kurzweil-Henstock and McShane integrals were

treated. For example, the authors proved that the strong McShane integral is equivalent to the Bochner integral and characterized the strong Kurzweil-Henstock and McShane integrals in terms of their primitives.

The McShane integral of Banach space-valued functions defined on an m-dimensional interval I was considered also in a joint paper [65] with Kurzweil. In this paper they proved that a McShane integrable function is integrable over measurable sets contained in I, derived a certain type of absolute continuity of the indefinite McShane integral with respect to the Lebesgue measure and showed that the indefinite McShane integral is countably additive. Using more general partitions with measurable sets instead of intervals they defined a new McShane type integral and proved that it is equivalent to the original McShane integral.

Generalized ODE approach to functional differential equations (2006–2009). In 2006, together with Márcia Federson (Brazil) who came for several months to Prague for a study stay with the Prague school of the integration theory and generalized differential equations, they tried to utilize the ideas of Zdeněk Vorel, Carlos Imaz and Francisco Oliva which allow to transform functional differential equations to differential equations in a Banach space. The scheme of such a transformation is as follows:

Consider an initial value problem for the functional differential equation

$$\dot{y} = f(y_t, t), \quad y_{t_0} = \varphi, \tag{5}$$

where, as usual, $y_t(\vartheta) = y(t+\vartheta)$, $\varphi \in C([-r, 0], \mathbb{R}^n)$ is the initial value function and $f\colon\ C([-r, 0], \mathbb{R}^n) \times [t_0, T] \to \mathbb{R}^n$. Put $X = C([t_0 - r, T], \mathbb{R}^n)$,

$$\widetilde{x}(\vartheta) = \begin{cases} \varphi(\vartheta - t_0) & \text{if } t_0 - r \leqslant \vartheta \leqslant t_0, \\ \varphi(0) & \text{if } t_0 \leqslant \vartheta \leqslant T, \end{cases}$$

and

$$F(y,t)(\vartheta) = \begin{cases} 0 & \text{if } t_0 - r \leqslant \vartheta \leqslant t_0 \quad \text{or} \quad t_0 - r \leqslant t \leqslant t_0, \\ \displaystyle\int_{t_0}^{\vartheta} f(y_s, s)\,\mathrm{d}s & \text{if } t_0 \leqslant \vartheta \leqslant t \leqslant T, \\ \displaystyle\int_{t_0}^{t} f(y_s, s)\,\mathrm{d}s & \text{if } t_0 \leqslant t \leqslant \vartheta \leqslant T. \end{cases}$$

Then, under natural assumptions on f, the given problem (5) is equivalent to

$$x(t) = \widetilde{x} + \int_a^t \mathrm{D}F(x(\tau), t). \tag{6}$$

In general, (6) is an ordinary differential equation in a Banach space X. However, imposing impulses on (5) and changing appropriately the definition of F, the equation (6) becomes a generalized differential equation in a Banach space X. This principle was utilized by Štefan Schwabik, Márcia Federson and their Brazilian students to obtain new interesting results, in particular, on the stability of solutions and the topological dynamics of impulsive problems for functional differential equations (cf. [70], [72], [75]).

In particular, in [75], they proved the existence of a local semidynamical system for an initial value problem for a class of generalized ODEs. Under certain perturbation conditions, they also showed that this class of generalized ODEs admits a discontinuous semiflow called an impulsive semidynamical system. As a consequence, they obtained LaSalle's invariance principle for such a class of generalized ODEs.

Extensions of integrals and variational measures (2008–2009). Quite recently, Štefan Schwabik returned to the study of extensions of integrals and variational measures brought to Prague about 10 years ago by Vladimír Lovicar. As a result he published papers [67], [71], [73], [74]. To explain his contribution, let us recall the notion of the *Saks class* $\mathfrak{S}$ of integrals:

Let $-\infty < a < b < \infty$. Mappings of given sets of real valued functions defined on $[a,b]$ into $\mathbb{R}$ are called *functionals*. If S is an additive functional and $\mathrm{Dom}(S)$ its definition domain, then $F\colon\ [a,b] \to \mathbb{R}$ is said to be the *S-primitive* to $f \in \mathrm{Dom}(S)$ if

$$S(f,I) := S(f \cdot \chi_I) = F[I] = F(d) - F(c)$$

holds for each compact subinterval I of $[a,b]$ with boundary points $c < d$. Furthermore, the additive functional S is said to be an *integral* on $[a,b]$ if, for each $f \in \mathrm{Dom}(S)$, its S-primitive is continuous on $[a,b]$. Let us denote by $\mathfrak{S}$ the set of all integrals in $[a,b]$. Then if T, $S \in \mathfrak{S}$, we say that T contains S ($S \sqsubset T$) if $\mathrm{Dom}(S) \subset \mathrm{Dom}(T)$ and $T(f,I) = S(f,I)$ for all $f \in \mathrm{Dom}(S)$ and all compact subintervals I of $[a,b]$. It is known that the relation $\sqsubset$ is a partial ordering in $\mathfrak{S}$.

Štefan Schwabik presented a general approach to extensions of integrals, like the Cauchy and the Harnack extensions. His results give, as a specimen, the Kurzweil-Henstock integration in the form of the extension of the Lebesgue integral. Furthermore, he introduced and studied two new general extensions in a properly chosen class $\mathfrak{T}$ of integrals containing all the classical integrals like Newton, Riemann, Lebesgue, Perron, Kurzweil-Henstock. Finally, these new extensions lead to approximate Nakanishi-like description of the Kurzweil-Henstock integral based on the Lebesgue integral.

Product integration (1990–2008). Already in 1990 and 1994 Štefan Schwabik was attracted by product integrals (cf. [43] and [49]). About 2002, when he was asked to supervise the doctoral study of A. Slavík, he decided to turn back to this topic and suggested him to choose the history of the product integration as a theme of his thesis. Later, in 2008, they published a joint paper [69] on Kurzweil-Henstock and McShane product integration.

Finally, worth mentioning are also his short, but in any case expert, offsprings to other subjects, like integral transforms (lecture notes [1] he wrote jointly with Jan Kučera in 1969 certainly belong to the best Czech monographs on this nowadays not too fashionable topic), mathematical biology ([32], [57]) or Sobolev inequalities [50].

Jindřich Bečvář: Štefan Schwabik and history of mathematics

Štefan Schwabik was profoundly interested in the history of mathematics and the development of mathematical thinking. He also paid attention to the evolution of teaching mathematics and to the training of prospective students—teachers of mathematics at high schools. During the last twenty years, he regularly attended summer schools, conferences, and seminars at which he gave many knowledgeable lectures about the history of mathematical analysis. Since 1994, he was member of the editorial board of the series *History of Mathematics*, which is the only Czech edition devoted to the publication of research papers and monographs in the fields of history of mathematics and history of teaching mathematics. In the history of mathematics, Štefan published one monograph [6] and thirteen longer papers in various journals and proceedings from conferences and seminars devoted to the history of mathematics (see [76]–[88]).

Štefan Schwabik together with Petra Šarmanová wrote the book *A Short Guide to the History of Integral* which—although not extensive by its size—became a well known reference in the field. The authors explain in attractive style how the surface areas and volumes of bodies were determined from ancient times till the 20th century. In particular, they analyze the long journey leading from the birth of the integral and differential techniques to modern analysis of the present time. Being concise and clear, but mathematically and historically exact, they demonstrate the efforts which must have been invested to arrive at the commonly used integral, including the Lebesgue, Perron, Kurzweil, and Henstock integrals.

Among other works of Štefan Schwabik, we mention a longer paper [79] which he wrote together with I. Netuka. In this work they describe the origin and the evolution of the differential and integral calculi. In the subsequent papers [80], [83], [85],

and [86], Štefan analyzed important milestones in the development of mathematical analysis and its numerous applications, especially the applications coming from the 19th century. In the eighties, he was enchanted by Bernard Bolzano, a Prague mathematician, philosopher, and theologist of Italian-German roots, about whose life and mathematical discoveries he wrote four shorter papers [76]–[78] and [87]. His attention was aimed also at Jan Vilém Pexider and his mathematical results, cf. [84].

Štefan Schwabik was a long term member of the scientific committee of the doctoral study programs in the History of Mathematics and Mathematical Education at the Faculty of Mathematics and Physics of Charles University in Prague, as well as in similar study programs at the Faculty of Science of Masaryk University in Brno. Being an advisor, opponent, or examiner, he was always rigorous and demanding, but at the same time fair-minded. He selflessly helped to those truly interested in the history of mathematics. Under Štefan's supervision, his student Antonín Slavík defended his PhD thesis only after two years of enrollment in the study program. The extended version of his PhD thesis, entitled *Product Integration, Its History and Applications*, was published as the 29th volume of the series History of Mathematics.

Curriculum vitae of Štefan Schwabik

Milestones:

- 1941, March 15, born in Gelnica, Slovakia,
- 1959 finished high school in Košice,
- 1964 finished master study at the Faculty of Mathematics and Physics of the Charles University in Prague by defending his degree work on control theory supervised by Jaroslav Kurzweil,
- 1972 finished doctoral (CSc) study at the Mathematical Institute of the Czechoslovak Academy of Sciences by defending the thesis on generalized differential equations supervised by Jaroslav Kurzweil,
- 1991 Doctor of Sciences (DrSc),
- Since 1964 till the end he was research worker of the Institute of Mathematics of the Czechoslovak Academy of Sciences (from January 1, 1992, Institute of Mathematics of the Academy of Sciences of the Czech Republic), in the period 1996–2001 he was Chairman of the Department of Real and Probabilistic Analysis and of the Seminar on Differential Equations and Integration Theory.

Teaching experience:

- Electrotechnical Faculty of the Czech Technical University, Šafárik University Košice (Slovakia), Faculty of Mathematics and Physics of the Charles University and Universidade de São Paulo (IME),

- ▷ 1993 habilitated at the Faculty of Mathematics and Physics of the Charles University in Prague,
- ▷ 2000 appointed Professor of the Silesian University in Opava,
- ▷ Supervisor of 5 successful PhD (or CSc) thesis.

Moreover, he influenced in a very substantial way his younger collaborators: Petra Šarmanová, coauthor of the booklet on the history of integration and coauthor of the publication devoted to Otakar Borůvka, Ye GuoJu, coauthor of the fundamental monograph on integration in Banach spaces, now Professor of the Hohai University in Nanjing, China, and Marcia Federson, coauthor of several papers on functional differential equations, now Professor of the São Paulo University, campus São Carlos, Brazil.

Further activities:

- ▷ long-term editor in chief of *Mathematica Bohemica* (formerly *Časopis pro pěstování matematiky*, the oldest central European mathematical journal),
- ▷ contributing editor of *Real Analysis Exchange*, member of the editorial boards of *Archivum Mathematicum* and *Mathematica Slovaca*,
- ▷ vice-president of the Grant Agency of the Academy of Science of the Czech Republic,
- ▷ long-term Chairman of the Mathematical Society of the Czechoslovak Union of Mathematicians and Physicists and Vice-Chairman of the Union of Czech Mathematicians and Physicists.

List of publications by Štefan Schwabik

Monographs

[1] Integral transforms (with J. Kučera, in Czech). SPN, Praha, 1969, 120 pp.
[2] Differential and integral equations; Boundary value problems and adjoints (with M. Tvrdý and O. Vejvoda). Academia, Praha, and Reidel, Dordrecht, 1979, pp. 248.
[3] Generalized differential equations. Fundamental results. Rozpravy Československé akademie věd. Matematické a přírodní vědy vol. 95, 6, 1985, pp. 3–103.
[4] Generalized differential equations. Special results. Rozpravy Československé akademie věd. Matematické a přírodní vědy vol. 99, 3, 1989, pp. 3–79.
[5] Generalized ordinary differential equations. World Scientific (Real Analysis Series 5), Singapore, 1992, pp. 382.
[6] A short guide to the history of integral (with P. Šarmanová, in Czech). Prometheus, Praha, 1996, 95 pp.
[7] Integration in $\mathbb{R}$ (Kurzweil's theory) (in Czech). Karolinum, Praha, 1999, 326 pp.
[8] Topics in Banach space integration (with Ye GuoJu). World Scientific (Real Analysis Series 10), Singapore, 2005, 298 pp.

Scientific papers

[9] Extremale Regelungen für die lineare Zeitoptimale Regelungsaufgabe mit einem konvexen Regelungsbereich. Časopis pro pěstování matematiky *91* (1966), 80–88.

[10] On the linear control problem $x' = A\,x + B\,u$. Časopis pro pěstování matematiky *93* (1968), 141–144.
[11] Über ein Differentialgleichungssystem mit unstetigen Lösungen endlicher Variation. Zeitschrift für Angewandte Mathematik und Mechanik, *48* (1968), T31–T32.
[12] Stetige Abhängigkeit von einem Parameter und invariante Mannigfaltigkeiten für verallgemeinerte Differentialgleichungen. Czechoslovak Mathematical Journal *19* (1969), 398–427.
[13] Veralgemeinerte gewöhnliche Differentialgleichungen; Systeme mit Impulsen auf Flächen I. Czechoslovak Mathematical Journal *20* (1970), 468–490.
[14] Veralgemeinerte gewöhnliche Differentialgleichungen; Systeme mit Impulsen auf Flächen II. Czechoslovak Mathematical Journal *21* (1971), 172–197.
[15] Bemerkungen zu Stabilitätsfragen für verallgemeinerte Differentialgleichungen. Časopis pro pěstování matematiky *96* (1971), 57–66.
[16] Verallgemeinerte lineare Differentialgleichungssysteme. Časopis pro pěstování matematiky *96* (1971), 183–211.
[17] Stetige Abhängigkeit von einem Parameter für ein Differentialgleichungssystem mit Impulsen. Czechoslovak Mathematical Journal *21* (1971), 198–212.
[18] On Fredholm-Stieltjes integral equations. Commentationes Mathematicae Universitatis Carolinae *12* (1971), 845–849.
[19] On an integral operator in the space of functions with bounded variation. Časopis pro pěstování matematiky *97* (1972), 297–330.
[20] On the relation between Young's and Kurzweil's concept of Stieltjes integral. Časopis pro pěstování matematiky *98* (1973), 237–251.
[21] On the generalized linear ordinary differential equation (with M. Tvrdý). Časopis pro pěstování matematiky *98* (1973), 206–211.
[22] On a modified sum integral of Stieltjes type. Časopis pro pěstování matematiky *98* (1973), 274–277.
[23] Floquet theory for generalized differential equations (in Czech). Časopis pro pěstování matematiky *98* (1973), 416–418.
[24] Remark on d-characteristic and d_Ξ-characteristic of linear operators in Banach space. Studia Mathematica *48* (1973), 251–255.
[25] Remark on linear equations in Banach space. Časopis pro pěstování matematiky *99* (1974), 115–122.
[26] On Volterra-Stieltjes integral equations. Časopis pro pěstování matematiky *99* (1974), 255–278.
[27] On a integral operator in the space of functions with bounded variation II. Časopis pro pěstování matematiky *102* (1977), 189–202.
[28] Note on Volterra-Stieltjes integral equations. Časopis pro pěstování matematiky *102* (1977), 275–279.
[29] Boundary value problems for systems of generalized linear differential equations (with M. Tvrdý). Czechoslovak Mathematical Journal *29* (1979), 451–477.
[30] On linear problems in the space BV (with M. Tvrdý). In: Equadiff 4, Proceedings (Lecture Notes in Mathematics 703), Springer, Berlin (1979), 366–373.
[31] Differential equations with interface conditions. Časopis pro pěstování matematiky *105* (1980), 391–408.
[32] Periodicity, sensitivity of the periodic solutions and parameter optimization of a theoretical ecosystem model (with M. Dvořáková, N. Kolářová, J. Potůček and M. Straškraba). ISEM Journal *2* (1980), 42–62.
[33] Uniqueness for generalized differential equations. In: Recent trends in mathematics. Teubner, Leipzig, 1983, pp. 263–269.

[34] Generalized Volterra integral equations. Czechoslovak Mathematical Journal *32* (1982), 245–270.
[35] On Mawhin's approach to multiple nonabsolutely convergent integral (with J. Jarník and J. Kurzweil). Časopis pro pěstování matematiky *108* (1983), 356–380.
[36] Variational stability for generalized ordinary differential equations. Časopis pro pěstování matematiky *109* (1984), 389–420.
[37] Generalized differential equations and variational stability. In: Differential equations: qualitative theory (Colloquia Mathematica Societatis János Bolyai 47), eds. B.S. Nagy and L. Hatvani, Amsterdam, 1987, 971–982.
[38] Averaging for generalized differential equations. In: Proceedings of the eleventh international conference on nonlinear oscillations. Budapest, 1987, 318–321.
[39] Generalized Sturm-Liouvile equations. Archivum mathematicum *23* (1987), 95–107.
[40] Generalized Sturm-Liouville equations II (with D. Fraňková). Czechoslovak Mathematical Journal *38* (1988), 531–553.
[41] A survey of some new results for regulated functions. In: 28. seminário brasileiro de análise (Trabajos apresentados). São Paulo, 1988, 201–209.
[42] Generalized differential equations—a survey. Teubner, Leipzig (Teubner Texte zur Mathematik 118), 1990, pp. 59–70.
[43] The Perron product integral and generalized linear differential equations. Časopis pro pěstování matematiky *115* (1990), 368–404.
[44] Ordinary differential equations the solutions of which are ACG_*-functions (with J. Kurzweil). Archivum Mathematicum *26* (1990), 129–136.
[45] Linear operators in the space of regulated functions. Mathematica Bohemica *117* (1992), 79–92.
[46] Convergence theorems for the Perron integral and Sklyarenko's condition. Commentationes Mathematicae Universitatis Carolinae *33* (1992), 237–244.
[47] Henstock's condition for convergence theorems and equiintegrability. Real Analysis Exchange *18* (1992), 190–205.
[48] The Perron integral in ordinary differential equations. Differential and Integral Equations *6* (1993), 836–882.
[49] Bochner product integration. Mathematica Bohemica *119* (1994), 305–335.
[50] Applications of a one-dimensional Sobolev inequality to eigenvalue problems (with R.C. Brown and D.B. Hinton). Differential and Integral Equations *9* (1996), 481–498.
[51] On Kurzweil-Henstock equiintegrable sequences (with I. Vrkoč). Mathematica Bohemica *121* (1996), 189–207.
[52] Abstract Bochner and McShane integrals. Annales Mathematicae Silesianae *10* (1996), 21–56.
[53] On non-absolutely convergent integrals. Mathematica Bohemica *121* (1996), 369–383.
[54] Abstract Perron-Stieltjes integral. Mathematica Bohemica *121* (1996), 425–447.
[55] Linear Stieltjes integral equations in Banach spaces. Mathematica Bohemica *124* (1999), 433–457.
[56] Linear Stieltjes integral equations in Banach spaces II; Operator valued solutions. Mathematica Bohemica *125* (2000), 431–454.
[57] The abstract renewal equation. Electronic Journal of Qualitative Theory of Differential Equations (Proc. 6th Coll. Qualitative Theory of Diff. Equ.), No. 26. (2000, pp. 1–12. http://www.math.u-szeged.hu/ejqtde/6/626.html).
[58] A note on integration by parts for abstract Perron-Stieltjes integrals. Mathematica Bohemica *126* (2001), 613–629.
[59] Operator-valued functions of bounded semivariation and convolutions. Mathematica Bohemica *126* (2001), 745–777.

[60] Generalized ordinary differential equations and discrete systems. Archivum Mathematicum (CDDE 2000 issue) *36* (2000), 383–393.

[61] On the strong McShane integral of functions with values in a Banach space (with Ye GuoJu). Czechoslovak Mathematical Journal *51* (2001), 819–828.

[62] The McShane and the Pettis integral of Banach space-valued functions defined on $\mathbb{R}^m$ (with Ye GuoJu). Illinois Journal of Mathematics *46* (2002), 1125–1144.

[63] The McShane and the weak McShane integral of Banach space-valued functions defined on $\mathbb{R}^m$ (with Ye GuoJu). Mathematical Notes, Miskolc *2* (2001), 127–136.

[64] McShane equi-integrability and Vitali's convergence theorem (with J. Kurzweil). Mathematica Bohemica *129* (2004), 141–157.

[65] On McShane integrability of Banach space-valued functions (with J. Kurzweil). Real Analysis Exchange *29* (2003/2004), 763–780.

[66] Generalized ODE approach to impulsive retarded functional differential equations (with M. Federson). Differential and Integral Equations *19* (2006), 1201–1234.

[67] Variational measures and extensions of the integral. Real Analysis Exchange (Summer Symposium *2007)*, 167–172.

[68] A negative answer to a problem of Fremlin and Mendoza (with Ye GuoJu). Acta Mathematica Scientia, Series B, 27B (2007), 813–820.

[69] Henstock-Kurzweil and McShane product integration; descriptive definitions (with A. Slavík). Czechoslovak Mathematical Journal *58* (2008), 241–269.

[70] Stability for retarded functional differential equations (with M. Federson). Ukrainian Mathematical Journal *60* (2008), 107–126.

[71] Variational measures and the Kurzweil-Henstock integral. Mathematica Slovaca *59* (2009), 1–22.

[72] A new approach to impulsive retarded differential equations: stability results (with M. Federson). Functional Differential Equations *16* (2009), 583–607.

[73] General integration and extensions. Czechoslovak Mathematical Journal *60* (2010), 961–981.

[74] General integration and extensions II. Czechoslovak Mathematical Journal *60* (2010), 983–1005.

[75] Discontinuous local semiflows for Kurzweil equations leading to LaSalle's invariance principle for non-autonomous systems with impulses (with S.M. Afonso, E.M. Bonotto and M. Federson). Journal of Differential Equations *250* (2011), 2969–3001.

History of Mathematics

[76] The mathematician Bernard Bolzano (in Czech). Vesmír *60* (1981), 293–296.

[77] 1981—the bicentenary of Bernard Bolzano (with J. Jarník). Czechoslovak Mathematical Journal *32* (1982), 329–333.

[78] The conclusion of the Bernard Bolzano jubilee year (with J. Jarník, in Czech). Časopis pro pěstování matematiky *107* (1982), 196–202.

[79] The rise and development of calculus (17.–19. century) (with I. Netuka, in Czech). In: Světonázorové problémy matematiky II. SPN, Praha, 1984, 160–190.

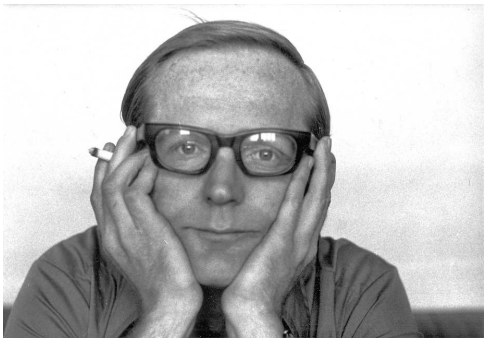
[80] The infinitesimal calculus—basis for further development of mathematics and mechanics (in Czech). In: Vědecká revoluce 17. století. Dějiny přírodních věd a techniky ve školské výuce (Práce z dějin přírodních věd 20), ed. J. Janko, ÚČSSD ČSAV, Praha, 1986, 97–107.

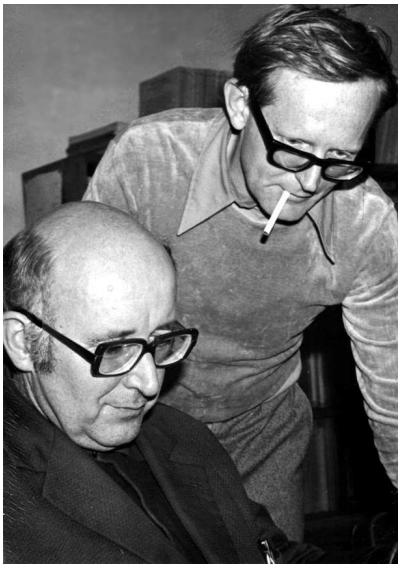
[81] Časopis pro pěstování matematiky and its successors (with M. Brdička, in Czech). In: Jubilejní almanach Jednoty československých matematiků a fyziků, ed. L. Pátý, Jednota československých matematiků a fyziků, Praha, 1987, 30–83.

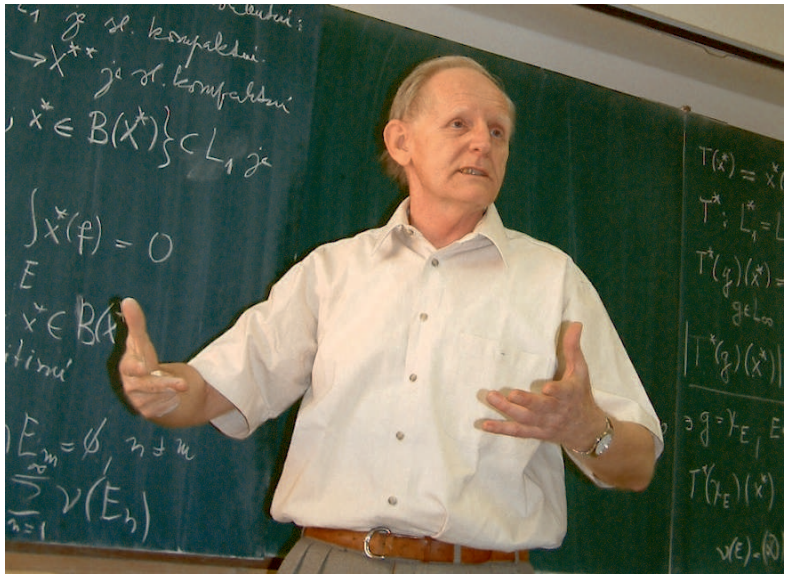
[82] The roads and aims of mathematics (with P. Krbec, in Czech). Rozhledy matematicko-fyzikální *67* (1988/89), 45–47.
[83] Some remarks to the development of calculus in the 19th century (in Czech). In: Matematika v 19. století. Sborník přednášek z letních škol. Dějiny matematiky 3, eds. J. Bečvář and E. Fuchs, Prometheus, Praha, 1996, 7–37.
[84] Pexider's mathematical publications. Czech version in: Jan Vilém Pexider (1874–1914). Dějiny matematiky 5, ed. J. Bečvář, Prometheus, Praha, 1997, pp. 37–44. English version in: Jan Vilém Pexider (1874–1914). History of Mathematics 38, eds. J. Bečvář and A. Slavík, Matfyzpress, Praha, 2009, 37–44.
[85] The second crisis of mathematics or the difficulties of growing differential and integral calculus (in Czech). In: Matematika v proměnách věků I (Dějiny matematiky 11), eds. J. Bečvář and E. Fuchs, Prometheus, Praha, 1998, 7–60.
[86] On the construction of the pendulum clock (in Czech). In: Matematika v 16. a 17. století (Dějiny matematiky 12), eds. J. Bečvář and E. Fuchs, Prometheus, Praha, 1999, 283–295.
[87] Bernard Bolzano and the calculus (in Czech). In: IX. seminář o filozofických otázkách matematiky a fyziky. Prometheus, Praha, 2000, 63–85.
[88] Czech contribution to modern integration theories (in Czech). In: 5. konference o matematice a fyzice na vysokých školách technických s mezinárodní účastí, Sborník příspěvků, část 1, Univerzita obrany, Brno, 2007, 11–20.

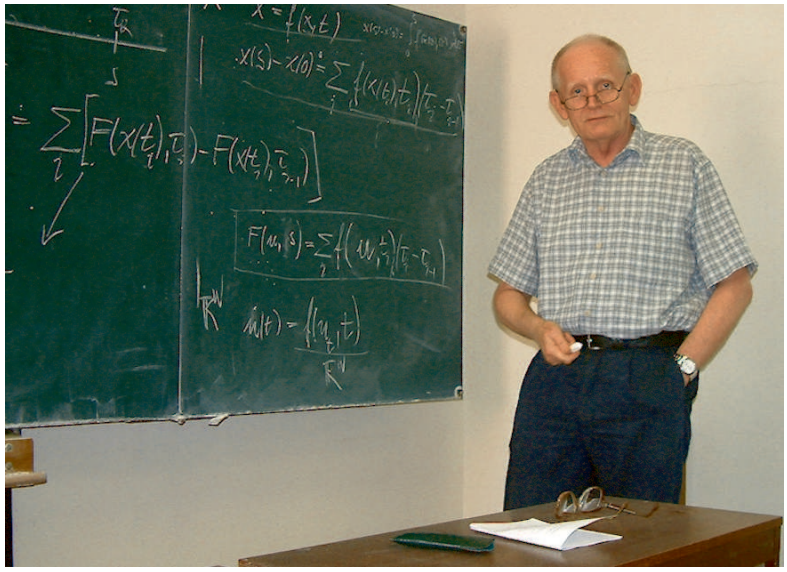
Other publications

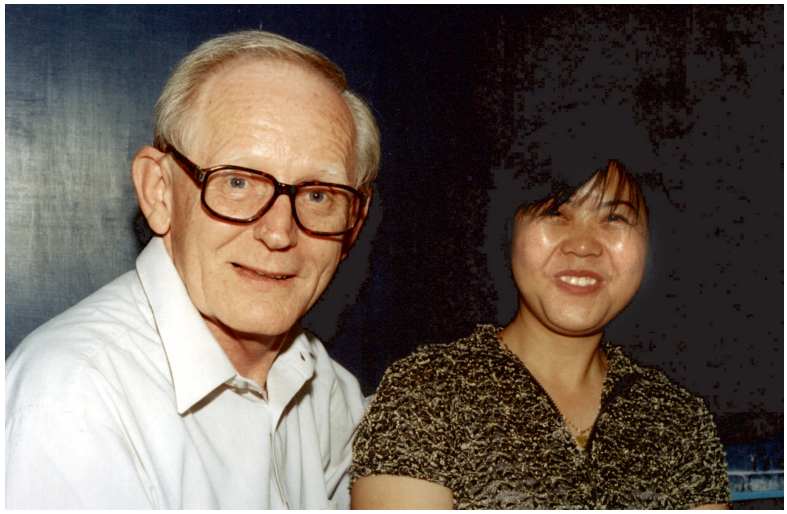
[89] Matematické základy optimální regulace (Mathematical foundations of optimal control). ČSVTS, Praha, 1974, pp. 42.
[90] Některé analytické metody popisu a studia chování diferenciálních dynamických systémů (Some analytic methods of describing and investigating the behaviour of differential dynamic systems) (with J. Nagy and E. Nováková). In: Metodika a aplikace simulace dynamických systémů. ÚVT ČVUT, Praha, 1974, 25–72.
[91] Babylónská věž (The Babylonian tower). Rozhledy matematicko-fyzikální *60* (1981/82), 371–372.
[92] Jaroslav Kurzweil šedesátníkem (with J. Jarník, M. Tvrdý and I. Vrkoč). Časopis pro pěstování matematiky *111* (1986), 91–111.
[93] Sixty years of Jaroslav Kurzweil (with J. Jarník, M. Tvrdý and I. Vrkoč). Czechoslovak Mathematical Journal *36* (1986), 147–166.
[94] Ivo Vrkoč šedesátiletý (with J. Fischer, P. Kolář, B. Maslowski and J. Seidler). Mathematica Bohemica *116* (1991), 412–424.
[95] Ivo Vrkoč sexagenarian (with J. Fischer, P. Kolář, B. Maslowski and J. Seidler). Czechoslovak Mathematical Journal *41* (1991), 737–750.
[96] Za profesorem Janem Mařík (with J. Král, J. Kurzweil and I. Netuka). Mathematica Bohemica *119* (1994), 213–215.
[97] In memoriam professor Jan Mařík (1920–1994) (with J. Král, J. Kurzweil and I. Netuka). Czechoslovak Mathematical Journal *44* (1994), 190–192.
[98] Jaroslav Kurzweil septuagenarian (with J. Jarník). Czechoslovak Mathematical Journal *46* (1996), 375–382.
[99] Jaroslav Kurzweil septuagenarian (with J. Jarník). Mathematica Bohemica *121* (1996), 215–222.
[100] Matematika a Mathematica (Mathematics and Mathematica). Vesmír *80* (2001), 587–588.
[101] Eighty years of Jaroslav Kurzweil (with J. Jarník, M. Tvrdý and I. Vrkoč). Mathematica Bohemica *131* (2006), 113–143.











With some of his many best friends