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Jiří Bouška

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On the Diameter-Depth Relationship of Lunar Craters

JIŘÍ BOUŠKA
Astronomical Institute, Faculty of Mathematics and Physics,
Charles University, Prague

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MacDonald's, Baldwin's, Markov's and present writer's results of the investigations of the diameter-depth relationship of lunar craters were analyzed. The relationship shows that all small craters are probably of impact origin. For many small lunar craters Ebert's rule is not valid.

I. Introduction

At the end of the last century Ebert (1890)* found that the ratios of the depths δ of lunar craters to their diameters Δ decreased with increasing diameter. This relationship between diameter and depth of craters was later known as Ebert's rule. At Ebert's time only a few depths of lunar craters were known with sufficient accuracy. Later many lunar photographs of high quality were obtained, particularly with the 40-inch refractor of Yerkes Observatory. Many of these photographs were also used for the determination of the diameters and depths of some lunar craters.

Until 1964 excellent photographs of the moon were also obtained at other observatories using big telescopes, e.g. McDonald, Mt Wilson, Lick, Pic-du-Midi etc. The smallest lunar craters visible in these photographs were up to 0.5 km in diameter. A great number of values of crater depths is contained in the well-known "Lunar Charts" (*LAC*) published for the United States Air Force and National Aeronautics and Space Administration by the Aeronautical Chart and Information Center, United States Air Force. These Charts represent a rich source of values of depth which are given for some tens of craters on each Chart. The diameters of these craters may be easily measured on the Charts and using this material the diameter-depth relationship may be studied for hundreds of craters, the diameters of which are larger than some kilometers.

The photographs obtained in 1964—1965 from the Ranger VII, VIII and IX missiles permitted the diameters and depths to be measured up to craters of some meters in diameter. The results of the Ranger photographs have been published in the Ranger Lunar Charts (Ranger VII: *RLC* 1–5, Ranger VIII: *RLC* 6–12, Ranger IX: *RLC* 13–17). The *RLC* were edited by the same Institution as the *LAC*. Similarly as *LAC*,

*) Ebert's paper (1889) which is usually cited, for instance by Fielder (1961), is not about Ebert's rule but about the experiments with artificial pits.

also *RLC* contain the values of depths of a large number of craters the diameters of which may be measured on the Charts. The *RLC* also represent a rich source of values for the study of the diameter-depth relationship, especially for craters of small diameters.

2. MacDonald's Investigations

T. L. MacDonald (1931a, 1931b) studied the diameter-depth relationship for many craters, the diameters and depths of which were known at that time. He found that the diameter-depth relationship may be expressed by the equation

$$\delta = a\,\Delta^{1/2} + b \tag{1}$$

where Δ is the diameter of crater, δ its depth (both in kilometers) and a and b are constants.

For craters with central mountains MacDonald found the following values of the coefficients

$$a = 0.378 \qquad b = 0.00$$

for craters without central mountains

$$a = 0.234 \qquad b = 0.00$$

and for continental craters around Tycho

$$a = 0.378 \qquad b = 0.95$$

For flooded craters it was found that $b < 0$.

MacDonald's empirical relationship holds for a large number of craters of large diameters. The mentioned relationship (shown also in Fig. 1) has evidently no physical meaning.

3. Baldwin's Investigations

The diameter-depth relationship was later investigated by R. B. Baldwin (1949) who found the following relation for explosive craters

$$D = Ad^2 + Bd + C \tag{2}$$

where D and d are the logarithmic diameters and depths ($D = \log \Delta$, $d = \log \delta$) of craters, both in feet, and

$$A = 0.1083 \qquad B = 0.6917 \qquad C = 0.75$$

Baldwin showed that the diameter-depth relationship given by equation (2) is valid not only for lunar craters but also for the earth's meteoritic craters and artificial explosion pits. For him, this fact was an evidence of the correctness of the meteoritic origin of lunar craters.

Later Baldwin (1963) studied the relationship between different crater parameters (radius, depth and rim of craters). For explosive craters, under the assumption of different values of scaled depth ($H/W^{1/3}$) of burst, the following coefficients of equation (2) were

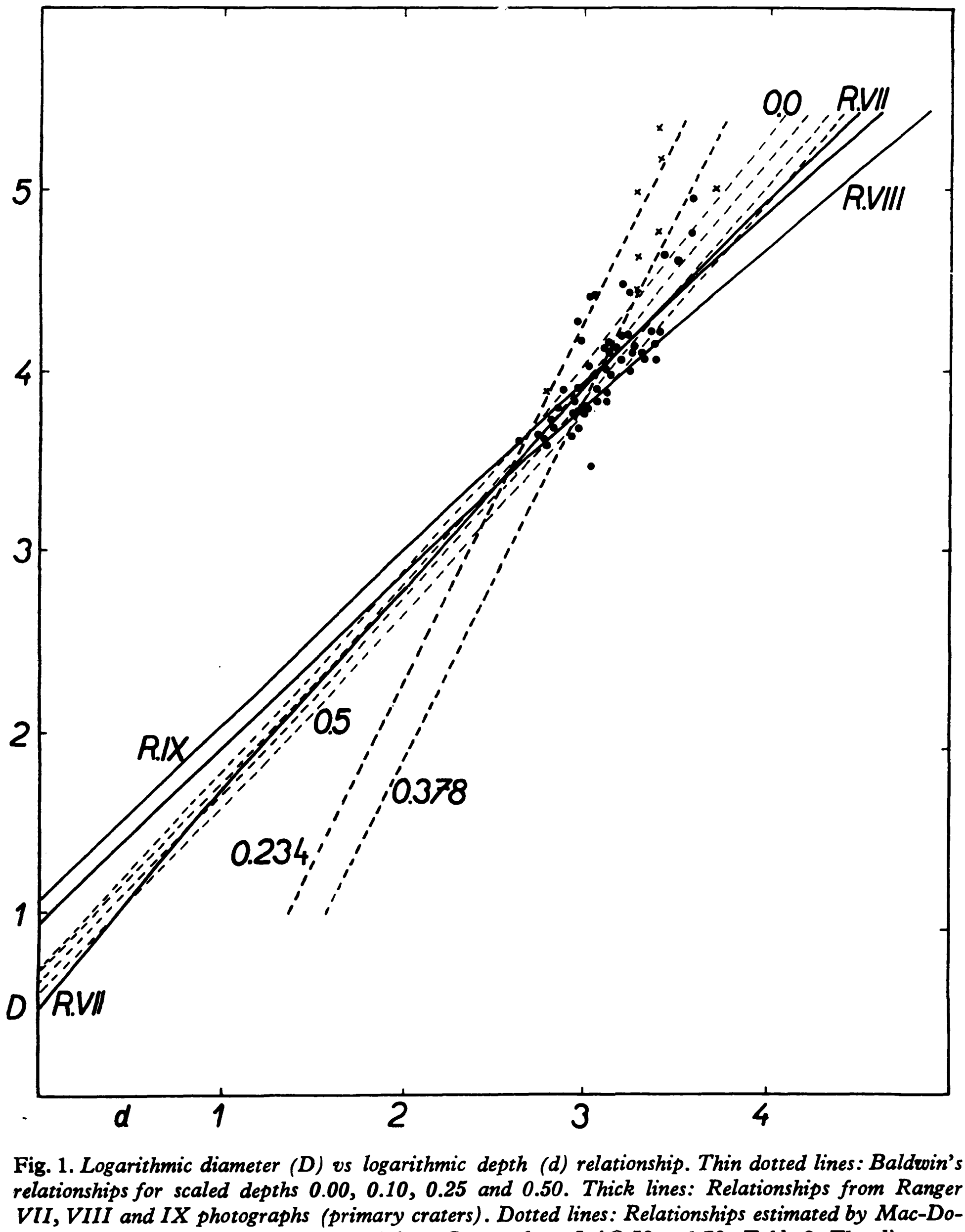

Fig. 1. *Logarithmic diameter (D) vs logarithmic depth (d) relationship. Thin dotted lines: Baldwin's relationships for scaled depths 0.00, 0.10, 0.25 and 0.50. Thick lines: Relationships from Ranger VII, VIII and IX photographs (primary craters). Dotted lines: Relationships estimated by Mac-Donald (for a = 0.234 and a = 0.378). Points: Craters from LAC 58 and 76 (Table 2. The diameters were estimated by the writer on the LAC, the given depths were used.) Crosses: Some other lunar craters (Table 2).*

found

$H/W^{1/3}$	A	B	C
0.00	0.0315	1.0363	0.6480
0.10	0.0256	1.0264	0.6539
0.25	0.0234	1.0211	0.5913
0.50	0.0225	1.0139	0.5370

if the logarithmic diameters and depths are expressed in meters. H is the depth of burst (in feet) below ground level and W the pounds equivalent in *TNT* of the energy release. The diameter-depth relationship for different values of $H/W^{1/3}$ is shown in Fig. 1.

Equations (2), with coefficients for different values of scaled depth, were compared by Baldwin with the diameter-depth relationships of artificial pits and meteoritic craters on the earth. It was found that equation (2), with the coefficients for $H/W^{1/3} = 0.10$, holds best for the earth's meteoritic craters and lunar craters of diameters between 1.6 and 32 kms.

In 1963 the original coefficients were revised by Baldwin (1963). For $H/W^{1/3} = 0.10$ the following values were obtained

$$A = 0.0256 \qquad B = 1.0000 \qquad C = 0.6300$$

if D and d are expressed in meters. Baldwin found that equation (2) with these new coefficients holds very well for all types of craters from the smallest explosion pit tested, about 6.5 cm in diameter, through the terrestrial meteoric crater range up to lunar craters at least 32 km in diameter. Baldwin notes that still larger lunar craters gradually depart from this relation in the sense that they become relatively shallower, but even here there is a close similarity and a continuity so that no large change in mode of origin is indicated (Baldwin 1965). From this fact Baldwin concluded that lunar craters were produced by impacts of meteors, not by volcanic actions.

In 1965 Baldwin studied the diameter-depth relationship for 130 craters of diameters between 8 m and 42 km from the Ranger VII photographs, using the data published in *RLC* 1–5. The relationship D vs d of these craters was compared with equation (2) using the revised coefficients (i.e. $A = 0.0256$, $B = 1.000$, $C = 0.6300$) and Baldwin found that this equation represents the Ranger data with great accuracy.

4. Some Other Investigations

Later, A. V. Markov (1966) studied the diameter-depth relationship using practically the same material as Baldwin (i.e. *RLC* 1–5). He found that the logarithmic relationship D vs d is nearly linear for craters of diameters between 8.5 m and 40 km. The diameter-depth relationship may be represented by the equation

$$D = Bd + C \qquad (3)$$

where $D = \log \Delta$, $d = \log \delta$ (both in meters) and

$$B = 1.0662 \qquad C = 0.6200$$

Recently, the present writer (Bouška 1967) measured the cross-sections of some primary craters from the Ranger VII, VIII and IX photographs. The diameters and

Table 1

d	*D*	*P/S*	*Crater*	*d*	*D*	*P/S*	*Crater*	*d*	*D*	*P/S*	*Crater*
RLC 7				2.95	3.78	P	Hypatia E	2.04	2.81	P	
				2.38	3.08	P		2.11	2.90	P	
2.38	3.18	P		2.54	3.43	S		2.11	3.00	P	Sabine CB
2.69	3.59	P	Sabine A	2.32	3.11	P		1.95	2.78	P	
2.52	3.40	P	Sabine AC	2.54	3.40	S		2.00	2.81	P	
2.61	3.46	P	Sabine AG	2.74	3.53	P	Hypatia CA	2.20	3.04	P	Sabine DG
2.54	3.57	S	Sabine AF	2.58	3.49	S	Hypatia CD	1.95	2.78	P	
2.08	2.90	P		2.40	3.34	P		2.00	2.88	P	
2.48	3.30	P		2.20	3.11	P		2.04	2.93	P	
2.45	3.30	P		2.23	2.95	P		2.20	2.98	P	
2.78	3.58	P	Sabine AD	2.53	3.28	P	Hypatia CB	2.15	2.95	P	Sabine DH
2.54	3.45	S		2.60	3.48	S	Moltke AD	2.15	2.95	S	
2.15	3.00	S						2.11	2.98	S	
2.18	2.90	P		*RLC 8*				2.20	3.10	S	Sabine DP
2.15	3.08	P						2.62	3.48	P	Sabine D
2.11	2.90	P		1.95	2.81	P		2.20	2.95	P	
2.15	2.95	P		2.00	3.02	P		2.30	3.28	S	Sabine DQ
2.18	3.04	P		2.11	3.08	P	Sabibe DE	2.11	3.00	P	Sabine DS
2.46	3.26	P	Sabine BA	1.78	2.74	P					
2.52	3.26	P		1.95	2.85	P		*RLC 9*			
2.48	3.23	S		2.34	3.12	P	Sabine DC				
2.38	3.15	P		2.11	3.10	S	Sabine DD	1.84	2.72	P	
2.45	3.28	P		1.84	2.70	P		1.78	2.74	S	
2.23	3.08	P		2.11	2.95	P	Sabine DF	1.84	2.74	S	
2.25	3.11	P		1.70	2.48	P		1.84	2.68	P	
2.23	3.11	P		1.84	2.73	P		1.72	2.67	P	
2.25	3.15	S		1.70	2.48	P		1.70	2.59	P	
2.23	2.95	P		1.60	2.48	P		1.95	2.81	P	Sabine DZ
2.41	3.21	S		2.11	3.15	P	Sabine DA	1.70	2.51	P	
2.54	3.32	S	Arago CB	2.00	2.93	P		1.72	2.62	P	
2.17	3.00	S		1.90	2.78	P		1.95	2.86	S	Sabine DU
2.04	2.90	P		1.84	2.72	P		1.60	2.52	P	
2.38	3.18	P	Sabine BB	1.84	2.70	P		1.90	2.80	S	
2.08	2.95	S		2.18	2.95	P	Sabine DL	1.65	2.60	P	
2.00	2.85	P		2.00	2.80	S		1.70	2.52	P	
2.70	3.50	S	Sabine B	1.90	2.73	P		1.60	2.51	P	
2.40	3.11	P		1.84	2.81	P		1.70	2.57	P	
2.20	3.08	P		1.78	2.67	P		1.70	2.63	P	
2.08	2.96	P		1.84	2.70	P		1.95	2.88	S	
2.56	3.48	S	Sabine C	1.95	2.83	S		1.90	2.76	P	
2.54	3.40	S	Sabine CA	2.00	2.95	P		1.60	2.51	P	
2.56	3.36	S		1.90	2.78	P		1.58	2.36	P	
2.64	3.60	P	Delambre FA	2.08	2.93	P		1.70	2.62	P	
2.60	3.40	S		2.00	2.84	S		1.70	2.86	P	Sabine DX
2.56	3.46	P	Sabine AB	1.84	2.78	P		2.15	3.05	P	Sabine DO
2.58	3.48	S		2.08	2.87	P	Sabine CD	1.60	2.70	P	

Table 1 (cont.)

d	*D*	*P/S*	*Crater*	*d*	*D*	*P/S*	*Crater*	*d*	*D*	*P/S*	*Crater*
1.60	2.65	P		1.78	2.70	P		0.90	1.84	P	
1.53	2.30	P		1.84	2.78	P	Sabine DY	0.78	1.65	P	
1.90	3.08	S	Sabine DM	1.84	2.84	P		1.57	2.42	P	
1.60	2.40	S		1.84	2.76	P		1.25	2.15	S	
2.00	2.88	P	Sabine DV	1.60	2.48	P		1.30	2.32	P	
1.30	2.15	P		1.90	2.83	P	Sabine DI	1.15	2.13	P	
1.28	2.15	P		1.78	2.62	P		0.95	1.90	P	
1.48	2.43	P		1.84	3.01	P	Sabine DN	1.15	1.95	S	
1.60	2.52	P		1.90	2.86	P		1.11	2.06	P	
1.60	2.38	P		1.78	2.73	P		1.00	1.90	P	
1.48	2.26	P		1.70	2.66	P		1.30	2.18	P	
1.54	2.36	P		1.65	2.53	P		1.04	1.90	P	
1.45	2.32	P		1.60	2.57	P		1.25	2.15	P	
1.60	2.51	P		1.70	2.53	P		0.60	1.48	P	
1.48	2.36	P		1.78	2.76	P		1.08	2.10	P	
1.48	2.30	P		1.70	2.58	S		1.15	2.15	P	
1.40	2.18	P		1.60	2.48	P		1.00	1.90	P	
1.32	2.15	P		1.78	2.64	P		0.90	1.90	P	
1.38	2.11	P		1.78	2.65	P		0.60	1.74	P	
1.60	2.60	S		1.74	2.63	P		1.20	2.16	P	
1.78	2.76	S		1.78	2.70	S		0.90	1.78	S	
1.54	2.43	P		1.48	2.45	P		1.53	2.53	S	Sabine EBA
1.60	2.45	P		1.70	2.60	P		1.51	2.51	S	Sabine EBB
1.23	2.23	S		2.00	3.05	P	Sabine EM	1.20	2.02	P	
1.70	2.60	P		1.60	2.53	P		1.15	2.08	P	
1.48	2.32	P		1.84	2.78	P	Sabine EK	1.45	2.28	P	
1.70	2.67	P		1.84	2.83	P	Sabine EI	1.25	2.18	P	
1.38	2.18	P						1.08	1.90	P	
1.54	2.49	P		*RLC 10*				1.38	2.25	P	
1.45	2.34	P						1.42	2.30	P	
1.52	2.34	P		1.20	2.11	P		1.08	1.90	P	
1.30	2.23	P		1.15	1.95	P		1.25	2.20	P	
1.48	2.34	P		1.00	2.00	P		1.52	2.49	P	Sabine EGA
1.42	2.30	P		1.04	1.84	P		1.40	2.40	S	
1.40	2.26	P		1.08	1.90	S		0.84	1.78	S	
1.60	2.53	P		2.00	2.94	P	Sabine EF	1.20	2.13	P	
1.48	2.42	P		0.78	1.95	P		0.90	1.93	P	
1.60	2.64	P		1.18	2.06	S		0.90	2.00	P	
1.90	2.84	P	Sabine EJ	1.23	2.15	S		0.90	2.00	S	
1.28	2.12	P		1.38	2.26	S		0.78	1.78	P	
1.48	2.50	P		1.28	2.10	S		0.90	1.81	P	
1.78	3.04	S	Sabine EE	1.75	2.49	P	Sabine EFC	1.11	2.11	P	
2.00	3.10	P	Sabine ED	0.78	1.70	P		0.78	1.70	P	
1.48	2.38	P		1.78	2.73	S	Sabine EFA	1.08	2.15	P	Sabine ELE
1.78	2.72	P		1.15	2.06	S					
1.75	2.68	P		1.18	2.18	P					

Table 1 (cont.)

d	*D*	*P/S*	*Crater*	*d*	*D*	*P/S*	*Crater*	*d*	*D*	*P/S*	*Crater*
RLC 11				0.78	1.68	P		0.00	1.08	P	
				0.90	1.76	P		0.48	1.34	P	
1.04	1.90	S		1.42	2.43	S	Sabine EBC	0.00	0.90	P	
1.30	2.22	P	Sabine EBE	0.60	1.58	P		0.00	0.90	P	
0.60	1.38	P		1.11	2.11	S		0.00	0.84	P	
0.48	1.40	P		0.78	1.67	S		0.00	0.90	P	
1.04	2.02	S		0.30	1.23	P		0.00	0.84	P	
0.30	1.26	P		0.48	1.42	S		0.30	1.08	P	
1.36	2.30	S	Sabine EBD	0.48	1.50	P		0.30	1.15	P	
0.48	1.48	P		0.90	1.95	S	Sabine EBG	0.30	1.28	P	
0.30	1.36	P		0.60	1.54	S		0.00	0.95	P	
0.30	1.23	P		0.70	1.49	S		0.00	0.90	P	
0.48	1.36	P		0.78	1.75	P		0.30	1.15	P	
0.48	1.15	P		0.48	1.32	S		0.30	1.15	P	
0.70	1.64	P		0.00	1.23	P		0.30	1.15	P	
0.48	1.45	P		0.48	1.28	P		0.30	1.11	P	
0.48	1.36	P		0.30	1.34	P		0.30	1.18	P	
0.30	1.26	P		0.00	1.23	P		0.30	1.15	P	
0.30	1.15	P		0.30	1.20	P		0.00	1.08	P	
0.48	1.43	P		0.60	1.53	S		0.30	1.23	S	
0.00	1.08	P		0.60	1.53	S		0.30	1.11	P	
0.30	1.15	P		1.08	2.15	S		1.23	2.16	S	Sabine EBF
0.30	1.15	P		0.48	1.38	P		0.48	1.34	P	
0.48	1.45	P						0.84	1.78	S	
0.30	1.23	P		*RLC 12*				0.30	1.08	P	
0.30	1.36	P						0.30	1.15	P	
0.70	1.62	P		0.30	1.15	P		0.30	1.15	P	
0.30	1.20	P		0.00	0.95	P		0.30	1.18	S	
0.30	1.23	P		0.00	0.95	S		0.30	1.08	P	
0.48	1.32	S		0.00	1.11	S		0.00	0.95	P	
0.30	1.23	P		0.00	1.11	S		0.00	1.00	P	
0.60	1.59	S		0.00	0.90	P		0.30	1.30	P	
0.30	1.26	P		0.00	0.90	P		0.30	1.26	S	
1.11	2.15	S		0.60	1.59	S		0.30	1.20	P	
0.30	1.26	P		0.30	1.25	S		0.30	1.08	P	
0.30	1.21	P		0.00	1.04	P		0.30	1.23	P	
0.30	1.28	S		0.30	1.28	P		0.30	1.12	P	
0.00	1.11	P		0.30	1.18	P		0.30	1.15	P	
0.30	1.34	P		0.30	1.20	S		0.30	1.11	S	
0.00	1.04	S		0.00	0.85	S		0.00	0.90	P	
0.30	1.26	P		0.48	1.51	P		0.30	1.26	S	
0.48	1.42	S		0.30	1.11	P					
0.48	1.43	P		0.00	0.78	P		*RLC 14*			
0.70	1.59	P		0.00	0.90	P					
0.78	1.73	P		0.00	0.85	P		2.41	3.36	P	
0.48	1.50	P		0.30	1.23	S		2.28	3.28	P	

Table 1 (cont.)

d	*D*	*P/S*	*Crater*	*d*	*D*	*P/S*	*Crater*	*d*	*D*	*P/S*	*Crater*
2.32	3.41	P		2.30	3.30	P	Alphonsus JB	1.38	2.43	P	
2.41	3.36	P	Alphonsus	2.63	3.62	P	Alphonsus A	1.68	2.75	S	
			CA	2.92	3.90	S	Alphonsus J	1.52	2.61	P	
2.63	3.63	P	Alphonsus L	2.36	3.38	P	Alphon. RA	1.48	2.57	P	
2.53	3.46	P		2.15	3.20	P		1.56	2.65	P	
2.18	3.15	P		2.11	3.15	P		1.73	2.72	P	
2.28	3.28	P		2.25	3.28	P		2.34	3.30	S	
2.18	3.32	P		2.46	3.45	P	Alphonsus Y	1.52	2.53	P	
2.36	3.40	P		2.45	3.52	P		1.67	2.62	P	
2.64	3.56	P	Alphonsus G	2.20	3.32	P		1.73	2.68	P	
1.95	3.04	P		2.57	3.50	P	Alphonsus BA	1.58	2.58	P	
2.11	3.08	P						1.58	2.62	P	
2.25	3.18	P		*RLC 15*				1.63	2.68	P	
2.00	3.08	P						1.49	2.51	P	
2.28	3.28	P		1.84	2.92	P		1.56	2.60	P	
3.04	3.92	S	Alphonsus α	1.82	2.95	S		1.57	2.62	P	
2.08	3.11	P		2.43	3.37	S	Alphonsus	1.71	2.74	P	
1.85	2.90	P					GA	2.16	3.15	P	Alphon. GK
2.15	3.32	S		2.20	3.17	S	Alphon. GG	1.63	2.68	P	
2.43	3.42	P		1.85	2.83	P		1.46	2.51	P	
2.18	3.15	P		1.75	2.93	S		1.90	3.04	P	
2.34	3.32	P	Alphonsus	2.21	3.18	S	Alphonsus	1.80	2.89	S	
			KC				GF	1.90	2.89	P	
2.20	3.24	P	Alphonsus	1.72	2.78	S		1.91	2.94	P	
			MA	1.53	2.58	P		2.40	3.36	P	Alphon. GB
2.28	3.20	P	Alphon. MB	1.51	2.56	P		1.62	2.81	P	
2.20	3.18	S		1.66	2.73	S		2.01	3.06	P	Alphon. GN
2.32	3.28	P		2.28	3.27	P	Alphon. GH	1.53	2.60	S	
2.60	3.62	S	Alphonsus	1.71	2.70	P		1.73	2.72	S	
			KA	1.99	2.97	S	Alphonsus GJ	1.54	2.78	S	
2.49	3.48	P	Alphon. KB	1.92	2.94	S	Alphonsus	1.83	2.92	P	
2.48	3.38	P					GL	1.48	2.58	P	
2.25	3.48	P		1.77	2.83	P		2.08	3.00	P	Alphon. GO
2.23	3.28	P		1.73	2.73	P		1.90	2.97	P	
2.20	3.20	P		1.76	2.72	P		1.57	2.64	S	
2.52	3.61	P	Alphonsus C	1.48	2.48	P		1.60	2.84	S	
2.46	3.40	S		1.89	2.94	P		2.02	3.11	S	
2.38	3.40	P		1.90	2.90	S		2.28	3.31	P	Alphon. GC
2.25	3.25	P		1.85	2.90	S		1.69	2.72	P	
2.15	3.20	P		1.58	2.58	P		1.53	2.58	P	
2.20	3.20	P		1.67	2.68	P		1.94	2.98	S	
2.08	3.15	P		1.79	2.83	S		1.68	2.64	P	
2.28	3.28	P		2.10	3.10	P	Alphon. ME	1.80	2.80	P	
2.23	3.32	P		1.64	2.78	P		1.73	2.76	P	
2.43	3.36	P	Alphon. JA	1.56	2.55	P		2.22	3.25	P	Alphon. GD
2.49	3.43	S		1.81	2.89	S		1.85	2.83	P	

Table 1 (cont.)

d	*D*	*P/S*	*Crater*	*d*	*D*	*P/S*	*Crater*	*d*	*D*	*P/S*	*Crater*
1.73	2.77	P		0.60	1.60	P		0.49	1.49	P	
1.76	2.74	P		0.70	1.60	S		0.23	1.23	P	
1.83	2.84	P		0.48	1.65	S		0.00	1.20	P	
1.73	2.78	P		0.90	1.95	P		0.00	1.11	S	
1.80	2.73	P		1.15	2.18	P		0.00	1.18	P	
1.56	2.65	P		0.78	1.84	P		0.00	1.30	S	
1.40	2.49	P		0.85	1.93	P		0.30	1.40	P	
2.02	3.08	P	Alphonsus	0.60	1.54	P		0.48	1.52	S	
			GE	0.85	1.85	P		0.48	1.52	S	
RLC 16				1.38	2.51	P	Alphon. GLA	0.30	1.26	S	
				0.95	2.04	P		0.30	1.36	P	
0.84	2.05	S		0.70	1.87	S		0.30	1.32	S	
1.11	2.28	S		0.60	1.74	S		0.00	1.26	S	
0.60	1.62	P		0.84	1.95	S		0.00	1.08	P	
0.84	1.93	P		0.70	1.78	S		0.48	1.43	S	
0.78	1.90	S		0.95	2.04	P		0.30	1.26	S	
1.25	2.34	S		1.18	2.21	S		0.78	1.75	S	Alphonsus
0.95	2.06	S		0.38	1.54	P					GLH
0.70	1.93	P		1.00	2.11	P		0.30	1.28	S	
0.90	1.93	P		0.60	1.62	P		0.30	1.41	S	
0.70	1.74	P		0.95	2.00	S		0.00	1.23	S	
0.78	1.85	P		1.12	2.28	S		0.48	1.42	S	
0.85	1.90	S		0.30	1.40	P		0.30	1.30	S	
0.60	1.70	P		1.67	2.73	S	Alphonsus	0.30	1.51	P	
1.11	2.25	S					GP	0.30	1.25	P	
0.60	1.70	P		1.69	2.78	S		0.00	1.18	P	
0.60	1.70	S		0.30	1.40	P		0.30	1.28	P	
0.30	1.40	P		0.48	1.70	S		0.60	1.58	S	
0.60	1.78	P		1.00	2.08	S		0.70	1.70	S	
0.95	2.11	S		0.60	1.70	S		0.30	1.46	S	
0.60	1.65	P		0.60	1.74	P		0.60	1.63	P	
0.70	1.95	S		1.00	2.02	S		0.70	1.68	P	
0.70	1.65	P		0.90	1.90	P		0.48	1.36	P	
0.70	1.74	P		1.12	2.32	S		0.30	1.38	S	
0.90	2.04	P		0.60	1.93	S					
0.85	1.98	P		0.70	1.81	S		*RLC 17b*			
0.70	1.70	P		1.18	2.30	S					
1.36	2.43	S						0.26	1.24	P	
1.78	2.79	S	Alphonsus	*RLC 17a*				0.08	1.09	S	
			GPD					0.48	1.40	P	
1.56	2.58	S	Alphonsus	0.43	1.43	P		0.11	1.04	P	
			GLG	0.30	1.32	S		0.04	1.02	S	
1.67	2.78	S	Alphon. GLC	0.23	1.23	P		0.15	1.10	S	
1.21	2.38	S		0.28	1.28	P		0.08	1.07	S	
1.08	2.23	S		0.70	1.76	S					
0.85	2.02	P		0.63	1.63	P					

depths of these craters were determined with great accuracy. From diameters and depths of 72 craters it has been found that the logarithmic diameter-depth relationship is linear for the craters used, the diameters of which were between 28 m and 17 km. The coefficients in equation (3) were found from this material as follows

$$B = 0.96 \qquad C = 0.98$$

if D and d are expressed in meters.

No substantial difference was found between the diameter-depth relationship for craters photographed by the three Ranger missiles in different lunar regions, but craters of small diameters ($D < 2.5$) only from Ranger IX photographs (formations inside the crater Alphonsus) could be used.

5. The Present Writer's Investigations

The differences between the present writer's results and the results obtained by Baldwin and Markov showed the possibility that the diameter-depth relationships may be somewhat different for craters in different lunar regions. For this reason the writer studied this relationship using the values published in *RLC* 7–12 (Ranger VIII) and *RLC* 14–17 (Ranger IX). Altogether 664 craters of diameters between 6 m and 8.3 km were used. These craters are shown in Table 1; P and S indicate primary (or regular) and secondary (or irregular) craters.

Assuming the relationship D vs d to be quadratic, the least squares method yielded the following coefficients of equation (2) together with their mean errors, if D and d are expressed in meters:

	N	A	B	C
Ranger VII –				
primary craters	299	—0.0201 ± 0.0092	1.0145 ± 0.0234	0.9256 ± 0.0126
secondary craters	92	—0.0191 ± 0.0156	1.0195 ± 0.0441	0.9446 ± 0.0259
Ranger IX –				
primary craters	164	—0.0179 ± 0.0106	1.0218 ± 0.0295	1.0520 ± 0.0174
secondary craters	89	—0.0475 ± 0.0156	1.0899 ± 0.0423	1.0575 ± 0.0238

(N denotes the number of craters used.)

Under the assumption of linear relationship D vs d the following coefficients of equation (3) were obtained using the least squares solution:

		B	C
Ranger VII –	*primary craters*	0.9657 ± 0.0065	0.9432 ± 0.0097
	secondary craters	0.9673 ± 0.0112	0.9677 ± 0.0179
Ranger IX –	*primary craters*	0.9736 ± 0.0068	1.0741 ± 0.0114
	secondary craters	0.9672 ± 0.0128	1.1084 ± 0.0177

The mean errors of the coefficients of the linear solution were smaller than those of the quadratic one and thus it may be supposed that the logarithmic diameter-depth relationship is linear for smaller lunar craters.

The differences between the coefficients A, B, C obtained by the present writer and by Baldwin (and also by Markov) from the Ranger photographs show that these coefficients must be somewhat dependent on the properties of the lunar surface material in which the craters have been formed. The negative sign of the coefficient A in all four cases is remarkable.

Figure 1 of Baldwin's paper (1965) shows that Baldwin's original equation relating D and d does not represent all the plotted points with sufficient accuracy. A new computation of the values of Baldwin's Table 1, using the least squares solution, yields the following values for the coefficients of equation (2)

$$A = -0.0135 \pm 0.0111 \quad B = 1.1644 \pm 0.0548 \quad C = 0.4976 \pm 0.0659$$

These coefficients differ substantially from the revised coefficients of Baldwin for the scaled depth $H/W^{1/3} = 0.10$. In this case the sign of the coefficient A is negative, too. The diameter-depth relationship with these coefficients is shown in Fig. 1.

Supposing that the logarithmic diameter-depth relationship is linear, we get, by means of the least squares method from the same Baldwin's values, the following coefficients of equation (3)

$$B = 1.0988 \pm 0.0299 \qquad C = 0.5710 \pm 0.0103$$

Also in this case the mean errors of the coefficients are smaller for the linear solution than for the quadratic one. The numerical values of the last mentioned coefficients do not differ much from those found by Markov.

6. Concluding Remarks

Baldwin (1965) asserts that equation (2), with the coefficients revised by him (1963), represents the diameter-depth relationship deduced with great accuracy from the Ranger VII photographs for craters the sizes of which are from about 8 m to 42 km in diameter. For him this fact is very strong evidence of the correctness of the meteoritic-impact theory of the origin of primary lunar craters. The coefficients A, B, C theoretically determined by Baldwin for different values of $H/W^{1/3}$ differ somewhat from those estimated from measured diameters and depths. Thus it may be supposed that the process of the formation of lunar impact craters must be more complicated than Baldwin thinks. But is evident that the differences between the diameter-depth relationship estimated theoretically and those determined from the measured values are not very large. Therefore, it may be supposed that the lunar craters photographed from the Ranger VII, VIII and IX missiles are impact craters. No differences between primary (or regular) and secondary (or irregular) craters have been found.

It is evident that Ebert's rule is valid if the coefficients $B > 1$ in equation (3); if $B \leq 1$ Ebert's rule is not valid. In all the cases considered, i.e. in all the relationships D vs d of craters published in *RLC*, B differs only very little from 1, but the differences $B \pm 1$ are evidently real, in view of the mean errors of the coefficients B. The craters in the region around Guericke and in Mare Cognitum (Ranger VII photographs) show that their ratio δ/Δ decreased with increasing diameter. On the other hand the ratio δ/Δ

increased with increasing diameter of craters situated near Sabine on the edge of Mare Tranquillitatis (Ranger VIII photographs) and inside the crater Alphonsus (Ranger IX photographs).

From Fig. 1 it is evident that two types of lunar craters must exist. For craters of type I the relationship D vs d may be expressed by the approximative equation

$$D = d + const.$$

For craters of type II this relation is not valid. All lunar primary and secondary craters

Table 2

Crater	*D*	*d*	*Crater*	*D*	*d*
LAC 58			*LAC 76*		
Copernicus	4.97	3.59	Lansberg**	4.62	3.52
Eratosthenes	4.78	3.58	Lansberg C	4.28	2.95
Reinhold	4.66	3.44	Darney	4.20	3.91
Tobias Mayer	4.50	3.20	Guericke B	4.18	2.98
GayLussac*	4.45	3.24	Parry A	4.16	3.13
Gambart	4.42	3.04	Darney C	4.15	3.16
Reinhold B	4.42	3.03	Euclides C	4.12	3.12
Tobias Mayer A	4.23	3.40	Euclides	4.11	3.12
Gay-Lussac A*	4.23	3.36	Turner	4.11	3.26
Tobias Mayer C	4.22	3.22	Guericke C	4.08	3.20
Hortensius	4.17	3.38	Lansberg B	4.04	3.01
Fauth*	4.15	3.27	Fra Mauro A	4.03	3.10
Gambart C	4.10	3.32	Bonpland E	3.92	2.95
Gambart B	4.08	3.29	Guericke D	3.91	3.07
Gambart A	4.08	3.33	Fra Mauro B	3.90	2.89
Tobias Mayer E	4.01	3.24	Turner F	3.85	3.08
Fauth A*	4.00	3.14	Darney J	3.85	2.93
Tobias Mayer D	3.90	3.12	Euclides D	3.81	2.98
Hortensius C	3.85	3.11	Bonpland D	3.79	2.95
Gambart BA	3.83	3.01	Opelt K	3.65	2.74
Stadius B*	3.83	3.00	Darney E	3.62	2.62
Gambart D	3.81	2.84	—		
Gambart G	3.79	3.00			
Stadius T*	3.78	2.93	Clavius**	5.36	3.41
Stadius M	3.74	2.82	Ptolemaeus**	5.20	3.43
Schröter M*	3.72	2.95	Theophilus**	5.02	3.70
Copernicus H	3.70	2.82	Gassendi**	5.00	3.28
Stadius R*	3.66	2.92	Bullialdus**	4.79	3.39
Gambart L	3.65	2.76	Plinius **	4.64	3.28
Gambart K*	3.60	2.77	Mädler**	4.45	3.27
Copernicus CA	3.50	3.03	Carlini	3.90	2.79

* Irregular craters.

** Crater with central mountain.

of small diameter ($\Delta < 10$ km) and most craters of larger diameter probably belong to type I. It may be supposed that these craters are of impact origin. Larger craters ($\Delta > 100$ km) and also some flooded craters, which are relatively shallower, probably belong to type II. From Table 2 it is evident that the formations Clavius, Ptolemaeus, Gassendi etc., which are probably not of impact origin, belong primarily to type II. Remarkable is the crater Copernicus *CA* (Table 2), the diameter of which is very small and the ratio δ/Δ very large (1 : 3); it belongs neither to type I nor to type II.

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